APPARENT ORIGIN TIME OF $\bar{S}$ *

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ABSTRACT

REPRESENTATIVE travel times for 50 shocks in southern California indicate that transverse waves appear to leave the focus about half a second earlier, on the average, than longitudinal waves. If fault rupture is propagated with a speed greater than that of transverse waves, the originating of such waves along a moderately extended fault segment will completely explain the observations. An exact theory is given on the simplest assumptions.

THE OBSERVED PHENOMENON

JEFFREYS (1927) found that observations of two British earthquakes led to linear travel-time equations for $\bar{P}$ and $\bar{S}$ in which the constant terms differed, as if $\bar{S}$ had originated two or three seconds earlier than $\bar{P}$. Similar results have been reached by other investigators whenever the observations have been good enough. Jeffreys (1928, 1929, 1937) has discussed the effect further and added other instances. For the Tauern earthquake the time difference amounts to only 0.4 sec., but for three others it ranges from 1.4 to 2.5 sec., with uncertainties from $\frac{1}{2}$ sec. to 1 sec.

Jeffreys' preferred explanation is that only transverse waves originate directly at the focus, while longitudinal waves arise from these by refraction at the surface. Even if the originating strain is supposed to be nearly pure shear, its limited extent makes it mechanically difficult to see how longitudinal waves can fail to originate at the source. It is possible that part of the observations may be accounted for by the arrival of a surface wave (Q) before the true $\bar{S}$; but this cannot apply generally. Jeffreys (1937) has suggested as an alternative that the transverse and longitudinal waves actually originate from the fracture, but at different times. This approaches the hypothesis which will now be considered, namely, that the early arrival of $\bar{S}$ is a consequence of the actual progress of faulting.

Reid (1918) pointed out that if the velocity of faulting exceeds that of the transverse elastic waves, the first $\bar{S}$ wave recorded at a given station may not be that directly from the focus, but instead may have originated at a nearer point on the fault line. This was applied to the present problem by Benioff (1938).

In a synthetic study of 50 recent earthquakes in southern California, now in progress, the senior author finds that average travel times for the region give an apparent origin time for $\bar{S}$ which is about 0.5 sec. earlier than that for $\bar{P}$. In working with shocks individually, as Jeffreys has done, the chief difficulty is that of eliminating systematic error such as may be occasioned by incorrectly assumed focal depth, inhomogeneity of the regional structure, and

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idiosyncrasies of the stations due to substructure or to instrumental causes. In the synthetic study these systematic causes of error are largely eliminated; but the method involves averaging the data of stations in all azimuths, which should result in reducing the apparent effect if the hypothesis that it is due to the progress of faulting is valid. The investigation of the theoretically expected mean value of the discrepancy in apparent origin time of $\bar{S}$ and $\bar{P}$ forms the principal subject of the present paper.

THEORY

In figure 1,* A represents the point of initial rupture on the fault, which will correspond to the instrumental epicenter as determined from the times of

Fig. 1

arrival of longitudinal waves. The effect of focal depth will be neglected. Faulting is taken to progress from A indefinitely along the positive Y axis, with constant speed w . The point C with coördinates (x, y) represents the recording station; AC is its epicentral distance, denoted as usual by Δ .

The velocity of transverse waves is taken to be v , which is supposed less than w . If the origin of time is taken at the beginning of faulting, the direct transverse wave from A reaches C at time Δ/v ; but the transverse wave originating at B when faulting begins at that point reaches C at the earlier time $c/w + d/v$, where c is the distance AB. The earliest transverse wave to reach C will be that for which the last expression has its smallest value. Let c' and d' be the values of c and d corresponding to this smallest time.

Then, if the earliest $\bar{S}$ waves arriving at C are wrongly supposed to have originated at A, the origin time calculated for them will be earlier than the true origin time by an amount

$$\tau = \frac{\Delta}{v} - \frac{c'}{w} - \frac{d'}{v} \quad (1)$$

Here c' and d' , consequently τ , will be determined by the position of C. If faulting extends indefinitely from A, then the problem of finding τ reduces to

* The figures have been drawn by Mr. John M. Nordquist.

finding a true minimum of $c/w + d/v$. This is equivalent to the problem of refraction in geometrical optics. c' and d' are then determined by the condition that angle $YBC = i_2$, with the definition

$$\cos i_2 = \frac{v}{w} \quad (2)$$

so that $c' = y - x \cot i_2$, $d' = x \csc i_2$. Noting further that $x = \Delta \sin i$ and $y = \Delta \cos i$, and substituting all these values in (1), the expression reduces to

$$\tau = \frac{\Delta}{v} [1 - \cos (i_2 - i)] \quad (3)$$

If the fault fracture has a limited linear extent f , then (3) still gives the complete solution of the problem when $\Delta \leq f$. Usually we may expect to have $\Delta > f$, and further discussion is needed. The details are apparent from figure 2, where Δ is chosen larger than f . Faulting is assumed to extend to F, so that $AF = f$. The successive sketches are drawn for increasing values of the angle $i = \text{FAC}$.

Fig. 2

If $i = 0$ (fig. 2, a), τ has its maximum value

$$\tau_m = f \left(\frac{1}{v} - \frac{1}{w} \right) \quad (4)$$

For small i (fig. 2, *b*) the earliest transverse wave reaches C from the point F, so that $c' = f$. Since always

$$d^2 = c^2 + \Delta^2 - 2c\Delta \cos i \quad (5)$$

(1) then becomes

$$\tau = \frac{\Delta}{v} - \frac{f}{w} - \frac{\sqrt{f^2 + \Delta^2 - 2f\Delta \cos i}}{v} \quad (6)$$

This result applies from $i = 0$ to $i = i_1$ (fig. 2, *c*), where i_1 is so defined that $i = i_1$ when angle YFC = i_2 . If we write p for f/Δ , then

$$\sin(i_2 - i_1) = p \sin i_2 \quad (7)$$

which gives

$$\sin i_1 = \sin i_2 \sqrt{1 - p^2 \sin^2 i_2} - p \sin i_2 \cos i_2 \quad (8)$$

and

$$\cos i_1 = p \sin^2 i_2 + \cos i_2 \sqrt{1 - p^2 \sin^2 i_2} \quad (9)$$

If $i_1 \leq i \leq i_2$, then $0 \leq c' \leq f$, FBC = i_2 , and τ is given by (3) (fig. 2, *d*). For $i = i_2$ (fig. 2, *e*), $\tau = 0$. For larger values of i , τ continues to be zero; the effect we are discussing does not exist. All the preceding results are repeated symmetrically on the left side of the Y axis.

The quantity required for comparison with observation, which we will call a , is the mean value of τ at constant Δ . This is given by

$$a = \frac{1}{2\pi} \int_{-\pi}^{\pi} \tau di \quad (10)$$

which because of the last remarks can be replaced at once by

$$a = \frac{1}{\pi} \int_0^{i_2} \tau di \quad (11)$$

If $\Delta \leq f$, τ is given for the whole range by (3), whence

$$a = \frac{\Delta}{\pi v} (i_2 - \sin i_2) \quad (12)$$

If $\Delta > f$, (11) may be separated to read

$$a = \frac{1}{\pi} \int_0^{i_1} \tau di + \frac{1}{\pi} \int_{i_1}^{i_2} \tau di = \frac{1}{\pi} (I + J) \quad (13)$$

In the first integral of (13) τ is given by (6), in the second by (3). The latter is a simple integration, giving

$$J = \frac{\Delta}{v} [(i_2 - i_1) - \sin(i_2 - i_1)] \quad (14)$$

which includes (12) as the special case $i_1 = 0$.

To evaluate I , transform (6) by the identical substitution

$$\cos i = 2 \sin^2 \left(\frac{\pi}{2} - \frac{i}{2} \right) - 1 \quad (15)$$

Then

$$I = \int_0^{i_1} \left[\left(\frac{\Delta}{v} - \frac{f}{w} \right) - \frac{\Delta + f}{v} \sqrt{1 - \frac{4\Delta f}{(\Delta + f)^2} \sin^2 \left(\frac{\pi}{2} - \frac{i}{2} \right)} \right] di \quad (16)$$

Introducing

$$k = \sin \alpha = \frac{2\sqrt{\Delta f}}{\Delta + f} \quad \text{and} \quad \phi = \frac{1}{2}(\pi - i_1) \quad (17)$$

$$I = \left(\frac{\Delta}{v} - \frac{f}{w} \right) i_1 - \frac{2}{v} (\Delta + f) \left[E \left(k, \frac{\pi}{2} \right) - E(k, \phi) \right] \quad (18)$$

where $E(k, \phi)$ is a standard elliptic integral of the second kind. In calculating from (18) interpolation in the usual tables of elliptic integrals is laborious; it is often convenient to select α as an even number of degrees, and then determine f and Δ by

$$p = \frac{f}{\Delta} = \frac{1 - \cos \alpha}{1 + \cos \alpha} \quad (19)$$

From (2), (7), (13), (14), and (18), given $f < \Delta$,

$$a = \frac{\Delta}{\pi v} \left\{ i_2 - p i_1 \cos i_2 - 2(1 + p) \left[E \left(k, \frac{\pi}{2} \right) - E(k, \phi) \right] - p \sin i_2 \right\} \quad (20)$$

where i_2 is calculated from (2) and i_1 from (8) or (9).

If $\Delta \rightarrow \infty$ for finite f , then $i_1 \rightarrow i_2$, and in the limit

$$\tau_{\infty} = f \left(\frac{\cos i}{v} - \frac{1}{w} \right) \quad (21)$$

and

$$a_{\infty} = \frac{f}{\pi} \left(\frac{\sin i_2}{v} - \frac{i_2}{w} \right) \quad (22)$$

DISCUSSION

In applying the theory some assumptions must be made regarding the velocities. The velocity v of transverse elastic waves may be taken as 3.25 kilometers per second, which is characteristic of $\bar{S}$ for the southern California region. It has already been supposed that w , the velocity of faulting, exceeds this. It is difficult to see how it can exceed the velocity of longitudinal elastic waves. The velocity for $\bar{P}$ which corresponds to the assumed value 3.25 for $\bar{S}$ is near 5.58 km/sec. However, since most earthquakes in this region originate close to the base of the granitic layer, it is conceivable that the initial fracture might be propagated with a velocity approaching that of longitudinal elastic waves in the next lower layer, which is about 6.0 km/sec. The velocity w will presumably vary from instance to instance according to the initial conditions. Two values, $w = 4.5$ and $w = 5.5$, have been used as representative in the calculations.

Figure 3 shows the calculated time effect τ as a function of the angle i . The curves lettered a to d are drawn for extent of faulting $f = 50$ kilometers. Curve a , derived from (21), represents the case when Δ , the distance, is infinite. Curves b , c , and d are respectively for $\Delta = 500$, 50, and 40 km., using equations (3) and (6). The heavy dots on curve b mark the points of transition between the ranges of applicability of these two equations, where $i = \pm i_1$. For curves c and d equation (3) applies throughout; the dots on d refer to the partly coincident curve f . Curves c and d are applicable without change to larger values of f with the given Δ . Curves e , f , and g are drawn for $f = 20$ km.; for g , $w = 4.5$ km/sec., while for all other curves $w = 5.5$ km/sec. Hence all the curves except g end at the same points $i = \pm i_2$, where $\cos i_2 = 3.25/5.5$. All curves except a are tangent to the axis of abscissas at these points. Larger values of i_2 are seen to be physically unlikely, as they require large values of w .

The calculation shows that observed travel times may be expected to lead to values of τ up to a few seconds in a comparatively few instances. The mean value a , as rendered by a large group of observations, will be smaller. This is

Fig. 3

apparent from figure 4. The curves ABCD and EFGH represent the conditions when $\Delta \leq f$, as given by (12). The curves branching off to the right include $\Delta > f$, since they are drawn for $f = 5, 20$, and 50 km.; they are computed from (20).

Fig. 4

For local earthquakes the majority of observations are usually for Δ from 50 to 200 km. The figure indicates that for the smaller shocks, with presumably moderate extent of faulting, $\bar{S}$ should appear to arrive early by a few tenths of a second. In greater shocks, with larger f , the effect may exceed one second. If a considerable fraction of the stations are in azimuths close to the trend of the fault, the observed effect may be still larger, approaching the value of τ_m . If faulting proceeds in opposite directions from the initial rupture, the two values of a resulting must be added. This effect may partly account for improbably small values of focal depth occasionally derived from $S - P$ observed at short epicentral distances.

OBSERVATIONS AND CONCLUSION

Table 1 summarizes the observational results bearing on the phenomenon being discussed, as derived from the study of fifty southern California shocks already cited (Gutenberg, 1943). The first column gives the identification

TABLE 1

VALUES OF a (IN SECONDS) IN SOUTHERN CALIFORNIA SHOCKS, WITH STANDARD ERRORS

Identification numbers of shocks	Numbers of observed $\bar{S}-\bar{P}$	$-a$ from observed $\bar{S}-\bar{P}$	Numbers of observed $\bar{S}$	$-a$ from origin time and observed $\bar{S}$	Numbers of observed $\bar{P}$
1 to 7.....	5	-0.2 ± 0.5	14	-0.1 ± 0.5	53
8, 9, 13 to 17.....	20	-0.4 ± 0.1	24	-0.8 ± 0.3	46
10, 11, 12.....	4	-0.5 ± 0.8	5		14
18, 19, 28 to 31.....	19	-0.5 ± 0.1	22	-0.6 ± 0.2	39
20 to 27.....	21	-0.2 ± 0.2	22	$+0.2 \pm 0.3$	46
32 to 35.....	10	-0.6 ± 0.2	12	-1.1 ± 0.3	22
36 to 45.....	22	-0.9 ± 0.2	25	-0.6 ± 0.6	50
46 to 50.....	7	-0.9 ± 0.3	9	-1.0 ± 0.6	31
1 to 50.....	108	-0.51 ± 0.08	133	-0.64 ± 0.06^a	301

^a For this average it has been assumed that the reciprocal of the velocity of S is 0.3068 sec/km.

number of each shock in the eight geographically selected groups; the second and fourth columns show the number of observations in each group; and the third and fifth columns show the computed value of a , together with its mean or standard error (the usual probable error may be obtained from these by multiplying by 0.6745). In the third column, a is derived from the interval $\bar{S} - \bar{P}$, on the assumption that the difference of the reciprocals of the two velocities is numerically 0.1275, which the complete study gives as best for the entire region. The second column shows the number of such intervals measured in each group. The fifth column shows a as derived from a least-square reduction for each group in which both the velocity of $\bar{S}$ and the value of a were

retained as unknown quantities, using the origin time known from study of the longitudinal waves. The number of observations of $\bar{S}$ alone are entered in the fourth column. The last line shows general solutions over the entire body of fifty shocks, giving the same weight to each observed time, taking only the single quantity a as unknown.

A further solution was made for the best representation of the 108 observed values of $\bar{S} - \bar{P}$ in the form $\bar{S} - \bar{P} = -a + bD$, taking $D^2 = \Delta^2 + 18^2$. The result is

$$\bar{S} - \bar{P} = -0.51 \pm 0.13 + (0.1275 \pm 0.0010)D$$

The indicated value $b = 0.1275$ coincides exactly with that calculated from the entire set of observations on these shocks as best representing the velocities of $\bar{P}$ and $\bar{S}$ for the region, which had already been assumed in calculating the third column of table 1.

The indicated value of half a second for a falls so well within the reasonably expected range as indicated by figure 4 that we may conclude that the early arrival of $\bar{S}$ is fully explained by the hypothesis of the present paper.

Upon a suggestion of Professor P. Byerly, the correlation between the value of a from $\bar{S} - O$ and the magnitude M of the shocks used in table 1 was investigated, with the following results.

M	Number of observations	a (with standard error)
3.4 to 3.8	9	-0.3 ± 0.2
4.0 to 4.4	31	-0.6 ± 0.1
4.5 to 4.9	51	-0.7 ± 0.1
5.0 to 5.4	35	-0.7 ± 0.1
5.5 to 6.3	9	-0.9 ± 0.1

There is strong evidence that a increases with magnitude. Since the extent of faulting may be expected to increase with magnitude, this is a further support of the hypothesis.

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