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LONG-PERIOD SEISMOGRAPHS

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ABSTRACT

Descriptions and theories of a number of different seismographs developed particularly for recording of very long-period seismic waves are presented. These include (1) electromagnetic strain seismograph with galvanometer of 8 minutes period and photographic recording; (2) displacement transducer strain seismometer with resistance-capacitance network and short-period galvanometer photographic recorder or with ink-writing recorder; (3) electromagnetic pendulum seismometer with RC network having transfer characteristic of a long-period galvanometer recorder or a heated stylus visible writer; (4) electromagnetic pendulum with period increased tenfold or more using shunt capacitance; and (5) electromagnetic pendulum with condenser-lengthened period and triple RC integrating network recording with either heated stylus visible writer, ink writer, or short-period galvanometer photographic recorder.

STRAIN SEISMOGRAPH WITH GALVANOMETER RECORDING

WHEN operated with a velocity transducer and galvanometer recorder the strain seismograph has a frequency-response characteristic identical with that of a displacement transducer pendulum seismograph having a period and a damping constant equal to the period and damping constant of the galvanometer (Benioff, 1935). The construction of a strain seismograph with the response characteristic of a long-period pendulum thus becomes the relatively simple problem of construction of a long-period galvanometer. The response of such a combination is shown in figure 1 for two values of H , the galvanometer damping constant. $H = 1$ is critical damping, and $H = \frac{1}{2} \sqrt{2}$ is the value for optimum flatness of the response characteristic.

The first instrument of this kind was set up at Pasadena using a steel tube standard of length and a galvanometer of 40 seconds period. Later the galvanometer period was increased to 90 seconds. An additional recorder was installed on the N-S Pasadena strain with a galvanometer of 180 seconds period. In the Kamchatka earthquake of November 4, 1952, this combination recorded G waves G_1, G_2, G_3, G_4 , and Rayleigh waves with periods up to 8 minutes in addition to movements with periods of 52 minutes and longer (Benioff, 1955). In order to reduce disturbances caused by temperature variations of the standard a fused quartz extensometer was installed at Isabella, California, with a displacement transducer (Benioff, 1959). The output from this transducer was fed through a capacitance resistance differen-

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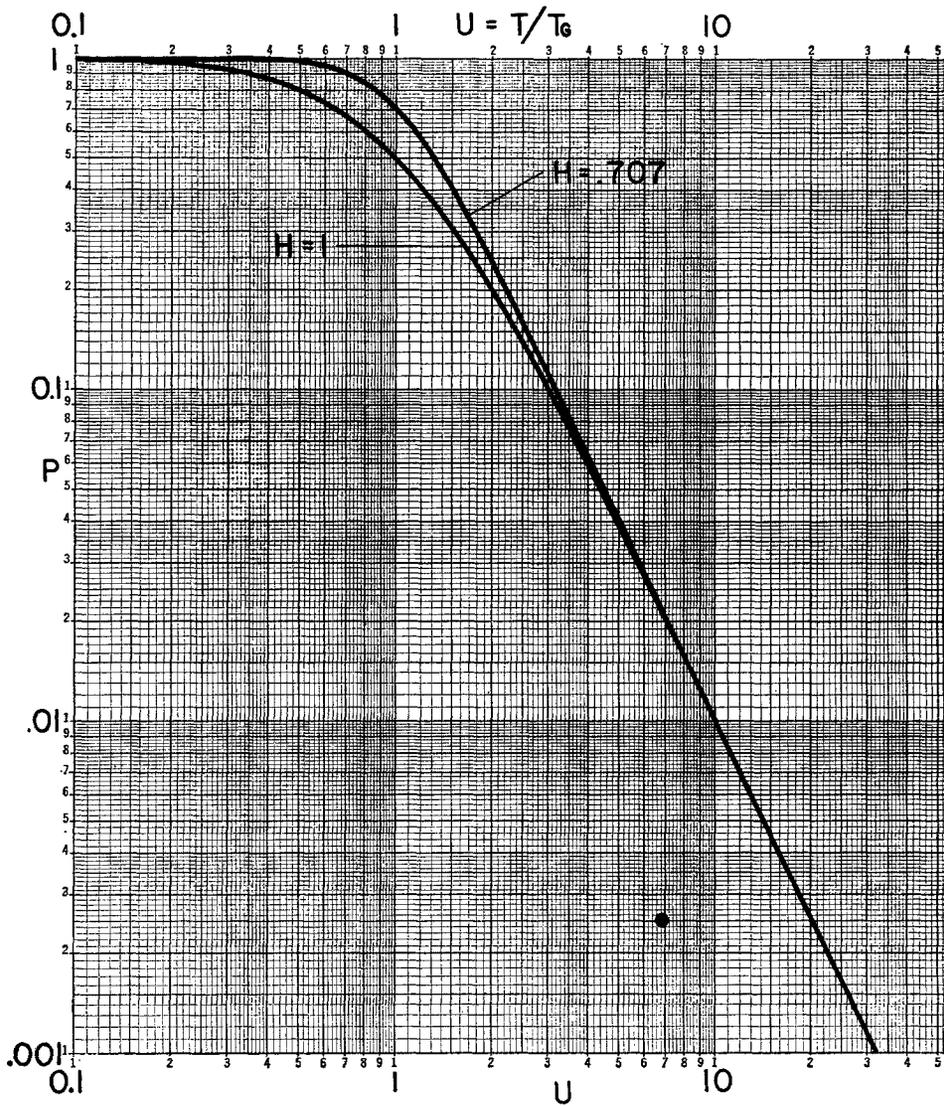


Fig. 1. Period magnification characteristic for electromagnetic strain seismograph with galvanometer recorder.

tiating circuit and then applied to a galvanometer of approximately 8 minutes period, designed by Francis Lehner. The damping of this galvanometer was substantially greater than critical owing to the effect of air viscosity. Nevertheless, this combination has provided some remarkable long-wave recordings, one of which is reproduced in figure 14.

STRAIN SEISMOGRAPH WITH RESISTANCE-CAPACITANCE NETWORK

However, such long-period galvanometers are subject to rather large drifts, and in addition they cannot be adjusted for critical damping except by operation in a rather high vacuum in which the molecular mean free path is of the order of the dimensions of the suspended system. Accordingly a resistance-capacitance network was designed which in operation with a displacement transducer extensometer provides a response very nearly identical with that of a velocity transducer extensometer and galvanometer. The network is shown diagrammatically in figure 2. E is the input emf from the extensometer displacement transducer, and I is the output current. The capacitors manufactured by the Aerovox Company come in units of

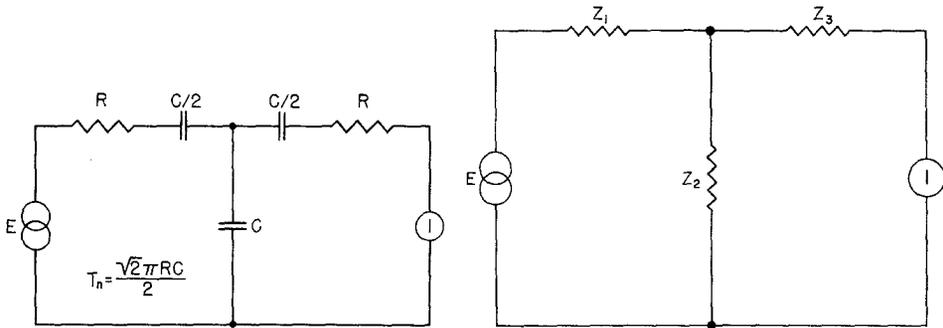


Fig. 2 (left). Resistance-capacitance network which with displacement transducer strain seismometer simulates pendulum type period response characteristic.

Fig. 3 (right). General 4-terminal network with 3 impedances.

50 mfd capacitance and are constructed with Mylar dielectric. They have time constants extending to weeks or months owing to their exceptionally low dielectric leakage. The network of figure 2 is a particular embodiment of the more general network shown in figure 3. For this network we may write

$$I/E = \frac{Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \tag{1}$$

When, as in figure 2, $Z_1 = Z_3$, equation (1) becomes

$$I/E = \frac{Z_2}{Z_1^2 + 2Z_1Z_2} \tag{2}$$

Furthermore,

$$Z_1 = R + 2/j\omega c \quad \text{and} \quad Z_2 = 1/j\omega c$$

Substituting these values in equation (2), we have

$$I/E = \frac{1}{6R + j(R^2\omega c - 8/\omega c)} \tag{3}$$

Solving for the real part of equation (3),

$$I/E = \frac{\sqrt{2} T_n}{4RT(T_n^4/T^4 + 2.5T_n^2/T^2 + 1)^{\frac{1}{2}}} \quad (4)$$

where $T_n = \pi RC \sqrt{2}/2$ and $T = 2\pi/\omega =$ the wave period. The emf across R , figure 2, is thus

$$e = RI = \frac{\sqrt{2} T_n E}{4T(T_n^4/T^4 + 2.5T_n^2/T^2 + 1)^{\frac{1}{2}}} \quad (5)$$

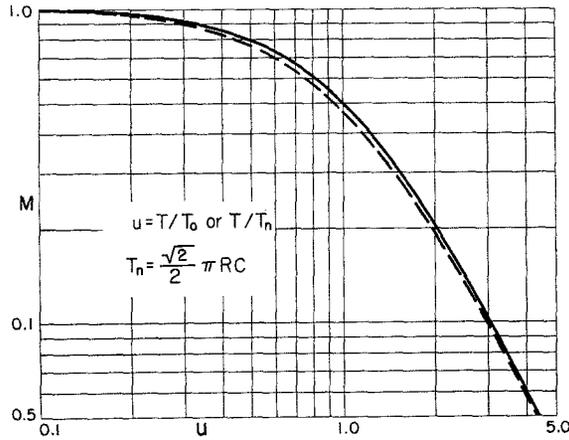


Fig. 4. Period magnification characteristic of displacement transducer strain seismometer with network of figure 2 (dashed line) and period magnification characteristic of critically damped pendulum seismometer.

If the network is connected to an extensometer with displacement transducer the input voltage is (Benioff, 1935)

$$E = ka\omega \cos \omega t$$

Where $a \sin \omega t$ is the wave displacement amplitude, $\omega = 2\pi/T$, and k is a constant. Hence, taking maximum values of the sinusoidal term,

$$e = \frac{\sqrt{2} ak\pi}{2T_n(T^4/T_n^4 + 2.5T^2/T_n^2 + 1)^{\frac{1}{2}}} \quad (6)$$

Let $T/T_n = u$ and $(\sqrt{2} ak\pi)/2T_n = M$, so that equation (6) may be written

$$e = \frac{M}{(u^4 + 2.5u^2 + 1)^{\frac{1}{2}}} \quad (7)$$

The period response characteristic of the strain seismometer with the network of figure 2 is thus

$$G = 1/(u^4 + 2.5u^2 + 1)^{\frac{1}{2}} \quad (8)$$

For a pendulum seismograph with critical damping $h = 1$, the period response characteristic is

$$G_p = 1/(u^4 + 2u^2 + 1)^{\frac{1}{2}} = 1/(u^2 + 1) \quad (9)$$

Comparing equation (8) with the general pendulum equation shows that the response of the network combination corresponds exactly to that of a pendulum with damping constant $h = (3\sqrt{2})/4 = 1.06$. Figure 4 is a plot of equations (8) and (9) showing the response of the network of figure 2 and that of a pendulum seismograph with critical damping.

Two types of recorders have been used with this network at Isabella. Figure 5 shows the circuit for use with a photographic galvanometer recorder. The instrument is a Cambridge Instrument Company short-period galvanometer ($T_g = 3.7$

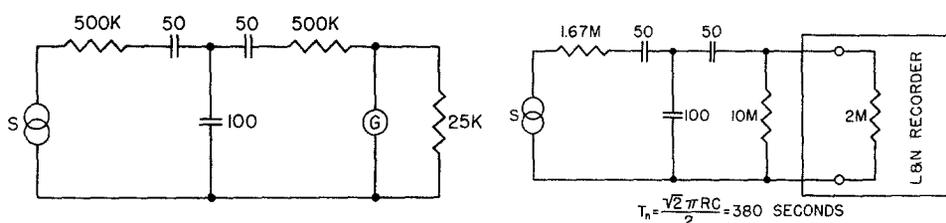


Fig. 5 (left). Long-period strain seismograph circuit using RC network with short-period photographic recorder.

Fig. 6 (right). Long-period strain seismograph circuit using RC network with Leeds and Northrup ink-writing recorder.

seconds, critical damping resistance = 25,000 ohms). The network constant is $T_n = 111$ seconds. This recorder operated at paper speed of 7.5 mm. per minute. The other recorder is a type G high impedance Leeds and Northrup ink-writing recorder. It operates at a paper speed of 6 inches per hour with the circuit given in figure 6, which also shows the response characteristic. The 2 megohm input resistance of the recorder and the 10 megohm shunt resistance form the output resistive arm of the network. The equivalent pendulum period T_n of the network is $T_n = (\sqrt{2}/2) \pi RC = 380$ seconds. It is hoped that this combination will be effective for recording the free vibrations of the earth when and if they occur with sufficient amplitude.

PENDULUM SEISMOGRAPH WITH RC NETWORK

The strain seismograph with long-period real or equivalent galvanometer has a response that is very nearly flat for ground displacements for periods less than the period of the galvanometer. However, a response of this character is inadequate to record many of the long-period phases of seismograms, owing to masking by the larger amplitudes of short-period waves—especially in smaller earthquakes. Moreover, the strain seismograph is not suitable for vertical component recording. Accordingly, a new pendulum seismograph system has been developed which permits wide adjustment of the response characteristics to very long periods. The pickup is an electromagnetic pendulum of 30 seconds period of the Press-Ewing type. This is provided with a winding of 138,000 turns of fine wire with a resistance of approximately 450,000 ohms. The seismometer output is fed to a double integrat-

ing RC network as shown in figure 7. The transfer characteristic of this network approximates that of a galvanometer. The output emf from the network goes to a Hewlett-Packard model 425A DC microvolt ammeter used as a DC amplifier. The maximum peak output of the amplifier is 1 volt. This is sufficient to drive either a photographic recorder with a short period insensitive galvanometer or a visible writing recorder such as the Geotechnical Corporation Helicorder which writes with a heated stylus on heat-sensitive paper.

The network of figure 7 is a particular form of the general 5-element 4-terminal network shown in figure 8. In this network the current I_5 in Z_5 is given by

$$I_5/E = \frac{Z_2 Z_4}{Z_1(Z_2 + Z_3)(Z_4 + Z_5) + Z_4 Z_5(Z_1 + Z_2) + Z_2 Z_3(Z_4 + Z_5)} \quad (10)$$

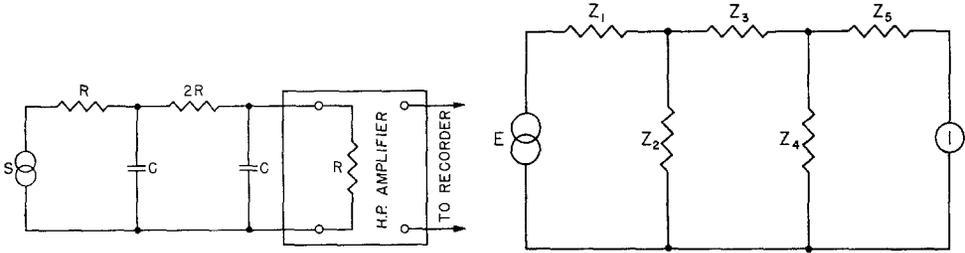


Fig. 7 (left). Circuit of dual integrating network with DC amplifier.

Fig. 8 (right). General 5-element 4-terminal network.

From figure 7 we see that $Z_1 = Z_5 = R$; $Z_2 = Z_4 = 1/j\omega C$; $Z_3 = 2R$. Substituting these values in equation (10), we obtain

$$I_5/E = \frac{1}{2R(R^2\omega^2C^2 - 3jR\omega C - 2)} \quad (11)$$

Setting $2\pi RC = T_n$ and $2\pi/\omega = T$ and solving equation (11) for the real part, the result is

$$I_5/E = - \frac{1}{4R(T_n^4/T^4 + 2.5T_n^2/T^2 + 1)^{\frac{1}{2}}} \quad (12)$$

If the output emf e is taken across Z_5 (fig. 8), equation (12) becomes

$$e/E = - \frac{1}{4(T_n^4/T^4 + 2.5T_n^2/T^2 + 1)^{\frac{1}{2}}} \quad (13)$$

The period transfer characteristic of this network is thus

$$G_n = \frac{1}{(T_n^4/T^4 + 2.5T_n^2/T^2 + 1)^{\frac{1}{2}}} \quad (14)$$

The period transfer characteristic of a galvanometer having a period T_g and a damping constant h in terms of the deflection for constant input voltage is

$$G_g = \frac{1}{(T_g^4/T^4 + T_g^2[4h^2 - 2]/T^2 + 1)^{\frac{1}{2}}} \quad (15)$$

Setting $4h^2 - 2 = 2.5$ and solving for h , we find that the transfer characteristic G_n of the network of figure 7 is exactly equivalent to that of a galvanometer having a period $T_g = T_n$ and a damping constant $h = 1.0606$, slightly greater than the critical value.

If the period T_n is adjusted to 90 seconds and the network is interposed between a Press-Ewing seismometer and the Hewlett-Packard amplifier as mentioned earlier, the period response characteristic of the combination as written by the heated stylus recorder is very nearly identical with that of the Press-Ewing seismograph ($T_0 = 30$; $T_g = 90$). A combination of this kind was set up and operated and the resulting seismograms were indistinguishable from those of the standard Press-Ewing instrument. The use of this network in place of a galvanometer thus provides a highly stable seismograph with equivalent galvanometer periods extending to 10 minutes or more.

PENDULUM PERIOD INCREASE WITH SHUNT CAPACITANCE

Another approach to the long-period seismograph problem makes use of a condenser shunt for an electromagnetic pendulum (Bernard, 1955). Using the Press-Ewing pendulum with a period of 30 seconds and the high impedance winding of 138,000 turns, it was found that the period can be increased to 412 seconds with a shunt capacitance of 200 microfarads. The theory for the increase in period has been given by Coulomb (1952). The exact expression relating pendulum period with shunt capacitance is rather complicated, but an approximation sufficiently good for practical purposes has the form

$$T_c = T_0 (g^2C/k + 1)^{\frac{1}{2}} \quad (16)$$

where T_c and T_0 are the periods with and without the condenser C , respectively, and g and k are the electrodynamic constant and the moment of inertia of the pendulum. On the Press-Ewing pendulum, the coil diameter is small compared with the radius of gyration. The inertia reactor is also fairly small relative to the radius of gyration, hence equation (16) can be written

$$T_c = T_0(CL^2H^2/m + 1)^{\frac{1}{2}} \quad (17)$$

where L is length of wire of the winding in cm., H is the field strength in gauss, m is the mass of the inertia reactor in grams, and C is the capacity of the condenser in abfarads. Measurements of the period of a Press-Ewing pendulum with different values of shunt capacitance are shown in figure 9, in which T_c^2/T_0^2 is plotted versus the capacitance C . The straightness of the curve shows that the approximate relation given by equation (16) is adequate. The period T_0 of the pendulum was 29.6 seconds. The coil had 110,000 turns, with a mean diameter of 3.2 cm. and a re-

sistance of 365,000 ohms. The mass of the inertia reactor was 6.8 kg. The measured value of L^2H^2/m from the curve is 0.85 when C is given in microfarads, or 8.5×10^{14} when C is measured in abfarads. The length of the coil was 2.8 cm. and over this distance the magnetic field strength varied considerably so that an accurate value could not be obtained. Using an estimated value of 2,000 gauss, the calculated value of L^2H^2/m comes out 7.1×10^{14} , showing that the average field was more nearly 2,200 gauss. An easier way for calculating L^2H^2/m in equation (17) is to use the relation

$$\frac{L^2H^2}{m} = \frac{4\pi rh}{T_0} \quad (18)$$

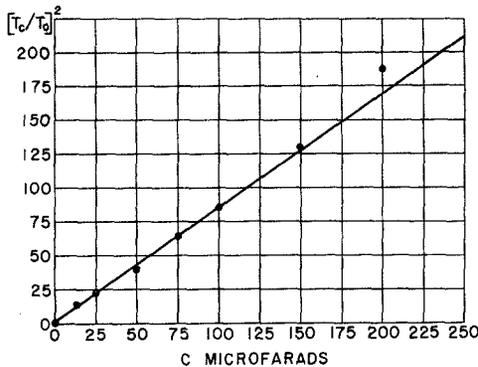


Fig. 9 (left). Measured values of the square of the ratio of period of condenser-shunted pendulum to free period (29.6 seconds).

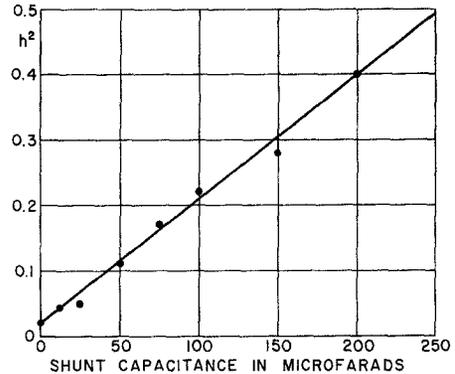


Fig. 10 (right). Measured values of the pendulum damping constant h squared versus shunt capacitance in microfarads.

where $r = r_c + r_1$ and h is the damping constant of the pendulum when only an external resistance r_1 is shunted across the coil winding with resistance r_c . The constants r , h , and T_0 can all be measured from the electrical terminals of the seismometer without opening the case. Along with the increase in period, the condenser-shunted seismometer also exhibits an increase in the damping constant h , which varies approximately as the square root of the shunt capacitance (or to T_c/T_0), as is shown by the measurements plotted in figure 10. A pendulum with a condenser-lengthened period responds to earth motion exactly the same as it would if its unshunted mechanical period were equal to the lengthened period except for an increase in the damping constant h and a decrease in magnification. Since the condenser-shunted seismometer is used with an amplifier, the decrease in magnification is not of serious consequence. This method thus permits the construction of pendulums with periods increased to 5 or 10 minutes having the mechanical stability of a 30-second pendulum.

LONG-PERIOD PENDULUM RC NETWORK SEISMOGRAPH

In the greatest earthquakes the amplitudes of mantle Rayleigh waves and G waves are large in relation to those of the shorter-period waves (<30 seconds) so as to be clearly distinguishable on a seismogram with flat magnification to 180 seconds or more (Benioff, 1958). In other, smaller, shocks the long-period waves are usually

masked by the shorter ones. A seismograph combination that has been found very satisfactory for the longer waves in all earthquakes is shown in schematic form in figure 11, along with its measured period response characteristic. The seismometer is a Press-Ewing 30-second horizontal pendulum with 450,000-ohm coil. (The measurements were made on an early model with a 365,000-ohm coil.) The pendulum is shunted with a 100-mfd condenser which increases its period to 275 seconds. Following the shunt condenser is an RC integrating network with a series arm of 0.5

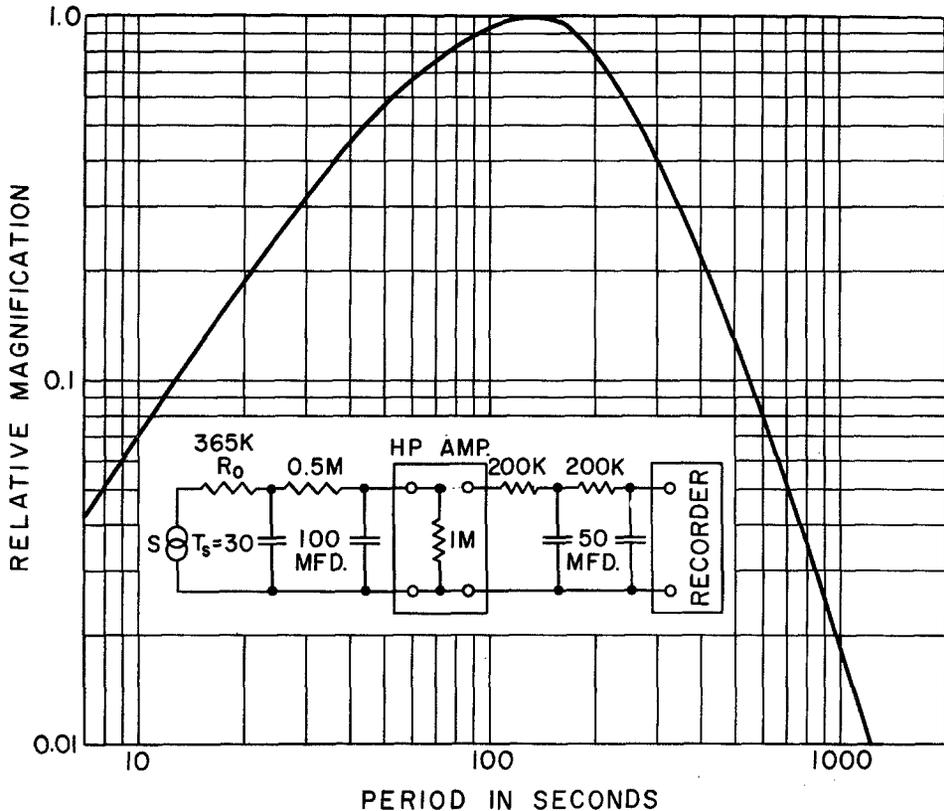


Fig. 11. Circuit and period response characteristic of long-period seismograph with condenser shunted pendulum, triple integrating RC network, and DC amplifier. R_0 = resistance of transducer coil.

megohms resistance and a shunt condenser of 100 mfd capacity. The output of the network goes to a Hewlett-Packard model 425A DC amplifier which has an input resistance of 1 megohm. The output of the amplifier is fed to the recorder through a two-stage RC integrating network having constants as shown. Essentially the combination represents a pendulum of 275 seconds period followed by an integrating network of 50 seconds time constant and two additional integrating networks of 10 seconds time constants. Although so far as the response characteristic of the combination is concerned all the integrating networks could be together in series, there are compelling reasons for separating them as shown. It is necessary to have at least one of the networks precede the amplifier to prevent overloading by the large-amplitude shorter-period waves and microseisms. On the other hand, if all

the networks precede the amplifier, the signal level falls too low in comparison with the amplifier internal noise level. The response characteristic discriminates strongly against periods shorter than about 20 seconds and also against periods longer than about 400 seconds. The attenuation of the longer periods is dictated by the increase in ground noise level at these periods, which appears to be caused by barometric pressure variations, temperature variations of the ground, vehicular traffic, and other agents (Benioff, 1959). At the Seismological Laboratory the recordings from this seismograph are useless during daylight working hours, owing to disturbance from automobiles and other vehicles. For this reason the instrument has been in-

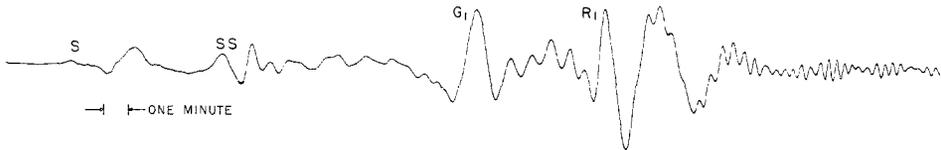


Fig. 12. Part of seismogram of Mongolian earthquake of December 4, 1957, recorded at Isabella by strain seismograph with displacement transducer, RC differentiating network and overdamped galvanometer of 8 minutes period. $\Delta = 94^\circ$; $M = 8.5$.

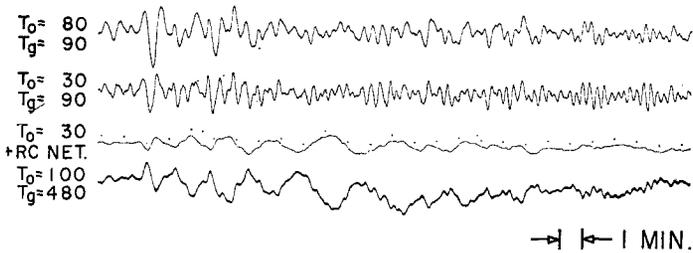


Fig. 13. Part of Rayleigh wave train of the New Guinea earthquake of March 1, 1959, recorded at Pasadena on four different E-W seismographs as indicated. $\Delta = 105^\circ$; $M = 7.0$; $h = 100$ km. The phase of the third trace is reversed relative to other ones.

stalled in the Dalton Tunnel (Benioff, 1959). Three types of recorders have been successfully operated with this combination: (1) A photographic recorder with a 1.2 second galvanometer connected through a 1 megohm resistance and a paper speed of 7.5 mm. per minute, (2) a Geotechnical Corporation heated stylus visible writing recorder with a paper speed of 7.5 mm per minute, and (3) a Leeds and Northrup high impedance type G ink-writing recorder with a paper speed of 6 inches per hour. The maximum usable magnification of this seismograph is determined by the level of ground noise. Thus at Dalton the peak of the magnification curve (fig. 11) is limited to about 750.

SEISMOGRAMS

In figure 12 a part of a seismogram of the Mongolian earthquake of December 4, 1957, is reproduced. This recording was made at Isabella with the strain seismograph operating an overdamped galvanometer of 8 minutes period through a differentiating network (Benioff, 1959). This earthquake was assigned a magnitude of 8.5, and the seismogram demonstrates that in the largest earthquakes a flat magnification characteristic is satisfactory for observing the mantle Rayleigh waves and G waves

as well as long-period body waves. The Rayleigh wave begins with a period of approximately 80 seconds, corresponding to the peak in the continental Rayleigh wave dispersion curve. Following the onset, the wave train exhibits components increasing in period with time superposed on others decreasing in period with time in accordance with the decrease in velocities on either side of the 80 second peak in the dispersion curve.

Figure 13 is a reproduction of a part of the Rayleigh wave train of the New Guinea earthquake of March 1, 1959, as written on four different E-W seismographs. The upper trace is from a pendulum of 80 seconds period with a galvanometer of 90 seconds period. The next trace was written with a Press-Ewing seismograph $T_0 = 30$, $T_g = 90$. The fourth trace came from the Gilman seismograph

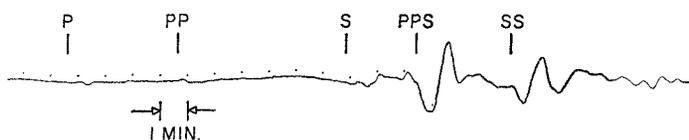


Fig. 14. Initial part of New Britain earthquake seismogram of May 16, 1959, written at Pasadena with the E-W pendulum RC network seismograph of figure 11. $\Delta = 92^\circ$; $M = 6.7$; $h = 60$ km.

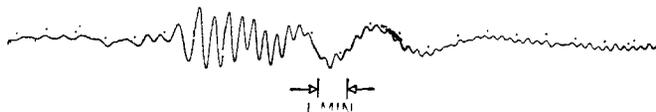


Fig. 15. Later part of seismogram of figure 14, showing Rayleigh wave train.

(Benioff and Press, 1958) with $T_0 = 100$, $T_g = 480$. The third trace, written with the pendulum RC integrating network seismograph of figure 11, demonstrates the advantage in long-wave recording of the steep short-wave part of the response characteristic provided by this combination.

A part of the seismogram of the New Britain earthquake of May 16, 1959, written with the E-W pendulum RC network seismograph of figure 11 at Pasadena is reproduced in figure 14. The prominence of the phases PPS and SS with periods of about 95 seconds is noteworthy. Figure 15 shows the Rayleigh wave group from the same seismogram. The mantle waves with periods increasing with time occur together with the crustal waves that decrease in period with time. Figure 16 shows parts of seismograms of the Mexican earthquake of May 24, 1959. The upper one was written at Pasadena with the E-W pendulum RC network, and the lower one was recorded with the strain RC network seismograph of figure 5 at Isabella. The equivalent pendulum period of the strain combination was 111 seconds and, consequently, the magnification was nearly constant to 111 seconds, where it was down one-half. In the pendulum RC network seismogram the P wave group appears with a dominant period of 51 seconds. On the lower seismogram this period is just discernible.

For ground motion periods longer than about one hour the instruments described in the preceding paragraphs are rather low in sensitivity. To record these, the fused quartz strain seismograph (Benioff, 1959) with displacement transducer is superior.

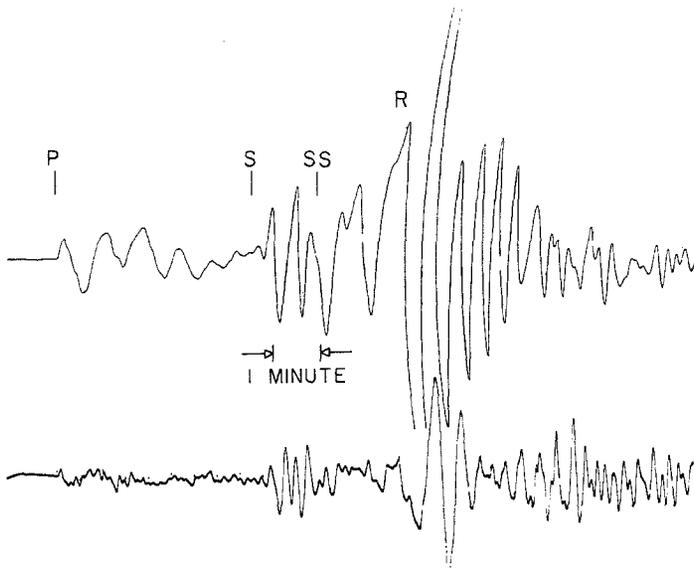


Fig. 16. Initial parts of seismograms of Mexican earthquake of May 24, 1959, written at Pasadena (upper) with E-W pendulum RC network seismograph of figure 11 and at Isabella (lower) with strain RC network seismograph of figure 5, $T_n = 111$ seconds. $\Delta = 25^\circ$; $M = 6.8$; $h = 100$ km.

Figure 17 is a reproduction of a recording made at Isabella with a quartz instrument and an Esterline-Angus ink-writing recorder operating at a paper speed of 1.5 inches per hour. The trace amplitude is proportional to ground strain with the indicated scale. The earthquake occurred in Mongolia on December 4, 1957. The seismic wave strains are superposed on the earth tidal strain of approximately 12 hours period.

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