Bulletin of the Seismological Society of America. Vol. 51, No. 1, pp. 29–34. January, 1961

AUTOMATIC COMPUTATION OF IMPULSE RESPONSE SEISMOGRAMS OF RAYLEIGH WAVES FOR MIXED PATHS

By Keiiti Aki and John M. Nordquist

ABSTRACT

A program has been devised to compute theoretical seismograms of Rayleigh waves for a given epicenter and a given station entirely automatically on an electronic computer.

The earth's surface is divided into three regions; continents, Pacific Ocean, and oceans other than the Pacific. Allowance can be made for differences in structure in these regions. This simple division seems satisfactory at present for Rayleigh waves of periods longer than 35 sec.

Introduction

This paper describes a method of obtaining an impulse response seismogram of Rayleigh waves for a mixed path. The method is entirely automatic using the Bendix G-15D electronic digital computer at the Seismological Laboratory, Pasadena.

The earth's surface is divided into three regions; Atlantic Ocean (includes Indian and other oceans, excludes Pacific), Pacific Ocean and Continent. The phase velocity data for the three regions are now available from theoretical and observational studies by various authors. This simple division of the earth's surface seems, at present, satisfactory for Rayleigh waves in the period range longer than 35 sec.

The impulse response is useful for the study of the earthquake mechanism (Aki, 1960). Cross correlation of the impulse response with the actual record removes the effect of phase delay due to propagation and instrument recording, yielding information about the nature of forces exerted at the earthquake source.

Fig. 1 shows the flow diagram of the computation. The boundaries between the three regions and the phase and group velocity curves for each region are stored in the memory of the computer. Input to the computer is the longitude and latitude of epicenter and station. The type of seismograph is specified before the computation.

The great circle path is first determined, and the position of each consecutive point on the path is put into the selection subroutine. The subroutine finds out whether the point lies in Atlantic, in Pacific, or in continents. Next, the phase and group delay times for the entire path are obtained as the sum of those for each segment of the path. Finally, the total delay times are obtained by the addition of those due to the instrument. The last portion of the program computes the impulse response for given phase and group delay times.

Great Circle Path

The great circle path between the epicenter and the station is divided into segments of equal length δ (5° in this case). The position of the center of the nth segment from the epicenter is given by the following formulas. The direction cosines

Manuscript received for publication April 26, 1960.

of the nth point are

$$\left.\begin{aligned} x_n &= x_2 \sin\{(n - \tfrac{1}{2})\delta\} + x_0 \cos\{(n - \tfrac{1}{2})\delta\}, \\ y_n &= y_2 \sin\{(n - \tfrac{1}{2})\delta\} + y_0 \cos\{(n - \tfrac{1}{2})\delta\}, \\ z_n &= z_2 \sin\{(n - \tfrac{1}{2})\delta\} + z_0 \cos\{(n - \tfrac{1}{2})\delta\}, \end{aligned}\right\} \tag{1}$$

where (x_0, y_0, z_0) are the direction cosines of the epicenter, and (x_2, y_2, z_2) are those of the point which is on the great circle from the epicenter to the station and

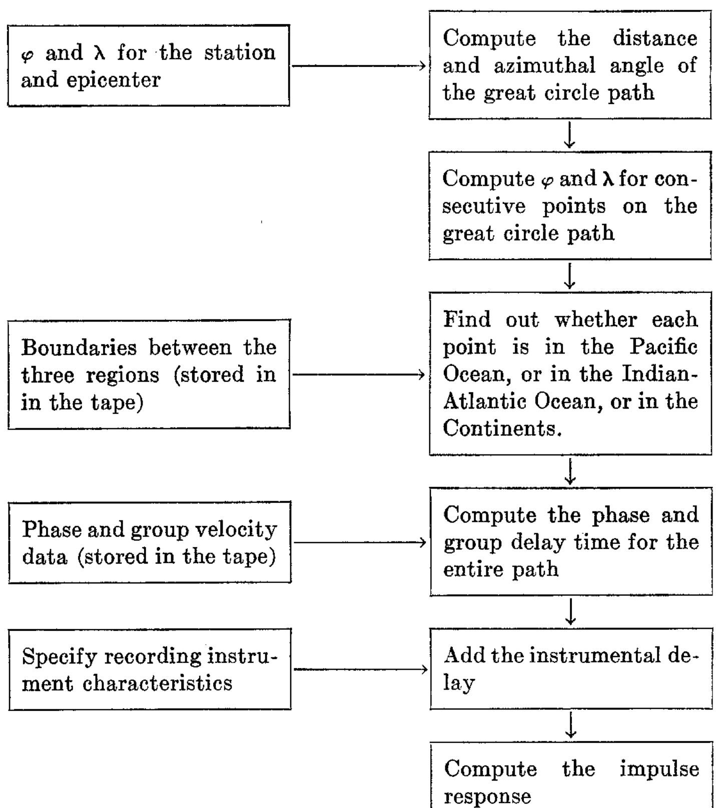

FIG. 1. Flow diagram of computation.

is distant from the epicenter by 90°. These are given by

$$\left.\begin{aligned} x_2 &= \frac{x_1 - x_0 \cos \Delta}{\sin \Delta}, \\ y_2 &= \frac{y_1 - y_0 \cos \Delta}{\sin \Delta}, \\ z_2 &= \frac{z_1 - z_0 \cos \Delta}{\sin \Delta}, \end{aligned}\right\} \tag{2}$$

where (x_1, y_1, z_1) are the direction cosines of the station, and Δ is the epicentral distance.

The latitude φ_n and longitude λ_n of the nth point on the great circle path are obtained from the direction cosines by

$$\begin{aligned} x_n &= \cos \varphi_n \cos\lambda_n , \\ y_n &= \cos \varphi_n \sin\lambda_n , \\ z_n &= \sin \varphi_n \end{aligned} \tag{3}$$

Selection Subroutine

The selection subroutine is designed to accept the latitude and longitude of a point, to determine whether that point is located in an Atlantic (includes Indian Ocean and other oceans than Pacific), Pacific, or a Continental region, and to add a constant to one of three sums, corresponding to the region thus determined. For this purpose, the earth is divided into 36 belts by circles of latitude spaced 5° apart. Each belt is divided into segments of Atlantic, Pacific, or Continental structure by meridians which, for convenience in computation and checking, were chosen at multiples of 5° longitude. As far as possible, the regional boundaries thus defined were made to approximate the location of the two-kilometer depth contour in the oceans. The lengths of the segments, in degrees of longitude, are stored in the computer memory in convenient locations.

The selection program transforms the longitude to a positive number between zero and 360°, subtracts a segment length from the result, and tests whether the result is less than zero. If so, a constant is added to the sum corresponding to the structure in the first segment of the belt and the machine returns to the main program to compute the coordinates of the next point on the great circle. If the remainder is positive, the length of the next segment of the belt is subtracted and another inquiry, "less than zero?" is made leading similarly to incrementing the sum for the corresponding structure or to subtraction of the length of the next belt segment. The process of subtraction and test for negative is repeated until the proper regional sum has been incremented and a return has been made to the main program.

In the Bendix G-15D, provision is made for temporary storage of four numbers at a time in a short line for rapid access. By putting four segment lengths into temporary storage, it is possible to test for inclusion in four consecutive belt segments in one drum cycle (about 0.03 sec). As long as the Atlantic, Pacific, and Continental segments appear in the same order on different belts, the same general test program can be used with different sets of segment lengths. These orders differ as one goes east from Greenwich in different belts, and extra boundaries appear where the belts cross the Indian Ocean, so that a single sequence of tests will not suffice for all belts. However, by choosing 20° W as the starting longitude, and by using two sequences of tests consecutively where the Indian Ocean is involved, it is possible to test along any belt by choosing from three different general test sequences.

Fig. 2 shows the boundaries of the belts and segments and the domains in which each test sequence applies. Note that Belt 0 (85°–90° N) is entirely Atlantic, and Belts 32–35 (70°–90° S) is here considered to be entirely continental. The location in the computer of the segment lengths, and the choice of the applicable test sequence, are determined from the latitude input.

Impulse Response

The phase delay due to propagation over each segment of the great circle path is equal to $\omega\delta/c(\omega)$, where ω is the angular frequency, δ is the length of the segment, and $c(\omega)$ is the phase velocity of Rayleigh waves for the segment. We get the total phase delay $\phi(\omega)$ by summing up $\omega\delta/c(\omega)$ for each segment and adding the instrumental delay. Then, the impulse response $g(t)$ is computed according to the formula

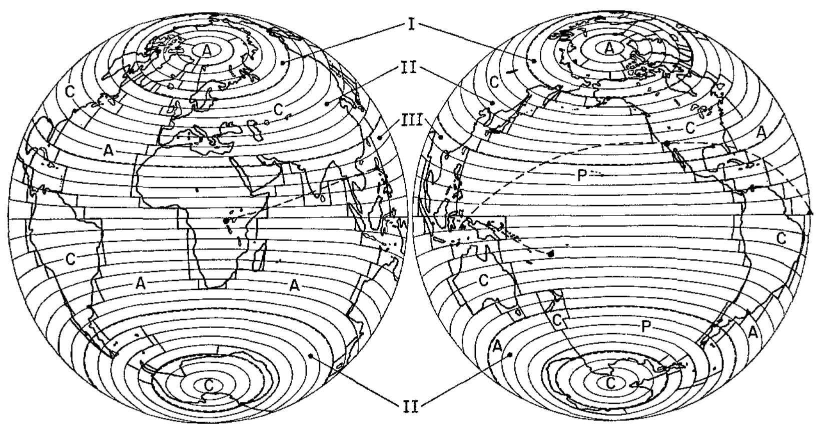

Fig. 2. Boundaries between the three regions; the Continents, Pacific Ocean and Oceans other than Pacific. Roman numbers I, II, III, indicate the domains in which the three selection routines are valid (see text).

$$g(t) = \int_{\omega_1}^{\omega_2} \cos(\omega t - \phi(\omega))\, d\omega \tag{4}$$

This is rewritten as

$$g(t) = \sum_i \int_{\omega_i - \Delta\pi_i/2}^{\omega_i + \Delta\omega_i/2} \cos(\omega t - \phi(\omega))\, d\omega$$

and is approximated by

$$\sum_i \Delta\omega_i \frac{\sin\left\{\dfrac{\Delta\omega_i}{2}(t - t_i)\right\}}{\dfrac{\Delta\omega_i}{2}(t - t_i)} \cos(\omega_i t - \omega_i \tau_i) \tag{5}$$

where

$$\tau_i = \phi(\omega_i)/\omega_i \qquad \text{(phase delay time)}$$

$$t_i = \frac{d\phi}{d\omega i} \qquad \text{(group delay time)}$$

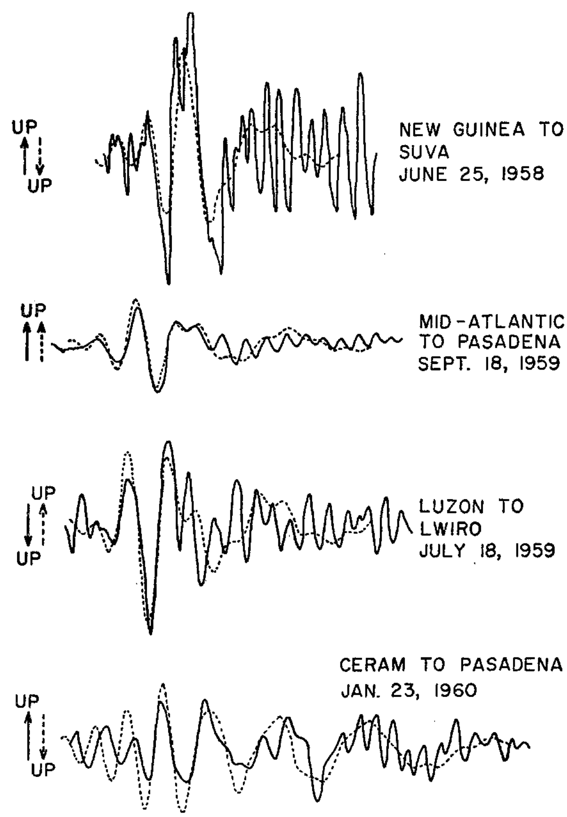

FIG. 3. Impulse response seismograms (broken line) and corresponding actual seismograms (solid line). The periods of seismograph and galvanometer are 80 sec and 90 sec respectively at Pasadena, and 15 sec and 80 sec at Lwiro and Suva. Lwiro and Suva are IGY stations where the Columbia type seismographs are operated. Copies of the records were sent from the Lamont Geological Observatory.

In our case, the cutoff frequencies $\omega_1/2\pi$ and $\omega_2/2\pi$ are taken as 0.0045 cps and 0.03 cps respectively, and the interval between them is divided into 11 portions in the computation by use of equation (5). The validity of this approximation was confirmed by the application to model seismograms (Aki, 1960a).

Three different phase velocity curves are used for the three regions. For the Pacific Ocean, we use the curve for the model 8099 obtained by Dorman et al (1960); for the Continent, we use the one which is based on Lehman's shear velocity data and computed by Dorman et al (1960) for the long period branch, combined with Press' curve (1960) for the short period branch; for the Oceans other than Pacific, we use the curve corresponding to a slightly modified model 8099.

Example

Some examples of impulse response are given in fig. 3 with the corresponding actual seismograms. The epicenters and the great circle paths are shown in fig. 2. The agreement between the actual record and the theoretical is good, except that the higher frequency waves are cut off in the theoretical impulse response.

It takes about 5 minutes on the Bendix G-15D computer to determine the great circle path and to sum up the phase and group delay times for an epicenter near the antipodes of a station. After this is done, the computation of $g(t)$ by the equation (5) takes about 30 sec per time point.

Acknowledgments

The authors are grateful to Dr. Frank Press for his advice and encouragement in carrying out the present work. This research was supported by grants from the IGY interdisciplinary Program and the American Petroleum Institute.

References

Aki, K.

1960. "The Study of Earthquake Mechanism by a Method of Equalization Applied to Rayleigh and Love Waves," *Jour. Geophys. Res.*, 65, 729–740.

1960a. "Interpretation of Source Functions of Circum-Pacific Earthquakes Obtained from Long-Period Rayleigh Waves," *Jour. Geophys. Res.*, 65, 2405-2417.

Dorman, J., M. Ewing, and J. Oliver

1960. "Study of Shear Velocity Distribution in the Upper Mantle by Mantle Rayleigh Waves," *Bull. Seism. Soc. Am.*, 50, 87–115.

Press, F.

1960. "Crustal Structure in California-Nevada Region," *Jour. Geophys. Res.*, 65, 1039–1051.

Seismological Laboratory,
California Institute of Technology,
Pasadena, California.
(Division of the Geological Sciences, contribution no. 977).