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WAVES REFLECTED AT THE "SURFACE" OF THE EARTH: P′P′P′P′

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ABSTRACT

Phases of the 600 km. deep earthquake of April 16, 1957, which have not been reflected at the earth's surface show very sharp beginnings arriving within the limits of error of a few seconds at the calculated times. However, phases which have been reflected at the surface, for example PP, pP′, pPP, have more or less emergent beginnings which arrive between a few seconds and as much as 30 seconds too early. These emergent beginnings are followed by an impulse which arrives at nearly the calculated time. In agreement with earlier findings, it is concluded that the emergent beginnings of such waves are caused by waves which have been reflected at the Mohorovičić discontinuity or other discontinuities below the surface. Observations of such early emergent arrivals also include waves of the types P′P′, PKSP′, P′P′P′ which have passed twice or more through the mantle and the core.

P′P′P′P′ has been observed for the first time. The travel times of P′P′P′P′ and the coefficient of absorption calculated from P′P′P′P′ and P′ agree within the limits of error with the expected values; however, these limits of error are so large that the results cannot be used to improve earlier findings. The periods of P′P′P′P′ which has traveled about 50,000 km. through mantle and core do not show a marked change from those observed in P waves which have traveled only a few thousand km. through the mantle.

Materials and Methods

It has been realized relatively early that in seismograms waves which have been reflected at the earth's surface may be preceded by waves reflected from discontinuities in and below the earth's crust. Gutenberg (1914, p. 183; p. 18 of reprint) found that PP frequently arrives about 10 seconds earlier than it was calculated. He pointed out that such waves with too early beginnings may be reflected at a discontinuity below the surface, and he calculated about 40 km. as its depth. Jeffreys (1926, pp. 344, 348) concluded that for reflection in continents "the wave reflected at the base of the granitic layer is probably stronger than that reflected at the outer surface when the observing station is less than 35° from the epicenter, and weaker, if the distance is greater." Such calculations assume constant velocity in each of two layers, extending to infinity from the boundary. From more recent calculations (Gutenberg, 1944, fig. 1*b*, p. 95) it appears that under conditions discussed in the present paper the amplitudes of longitudinal waves reflected at the Mohorovičić or similar discontinuities cannot be expected to be much greater than about 10 per cent of the amplitudes of the incident waves if the waves are short relative to the thickness of the crust. For longer waves there is no useful theory.

Travel times of waves reflected at the Mohorovičić and other discontinuities should be shorter than the times of those reflected at the surface by amounts depending on the usually varying depth of these discontinuities and the angle of incidence. Dipping or irregular form of these discontinuities must result in further complications for the travel times and amplitudes of such "early" reflected waves. Moreover, Gutenberg (1914, p. 183) had already pointed out that the latitude of the point of reflection of PP may produce differences amounting to as much as 6

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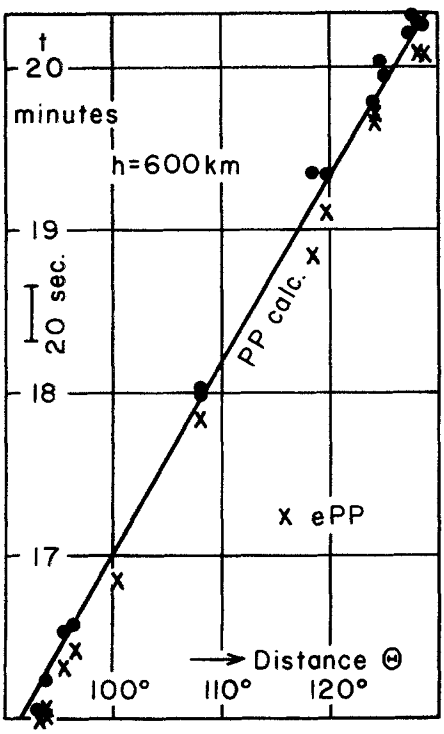
seconds in the observed travel times. We can therefore expect rather wide scattering in the travel times of the early arrival of such waves reflected at the earth's "surface," e.g., for PP, pP, pP′, P′P′, SS, etc. These differences are even greater if there are multiple reflections, e.g., for pPP, sPP, P′P′P′, SPS, etc.

For an investigation of waves reflected at the earth's surface, records of the earthquake of $4^h\,04^m\,04^s$, April 16, 1957, have been used. Its epicenter has been found to



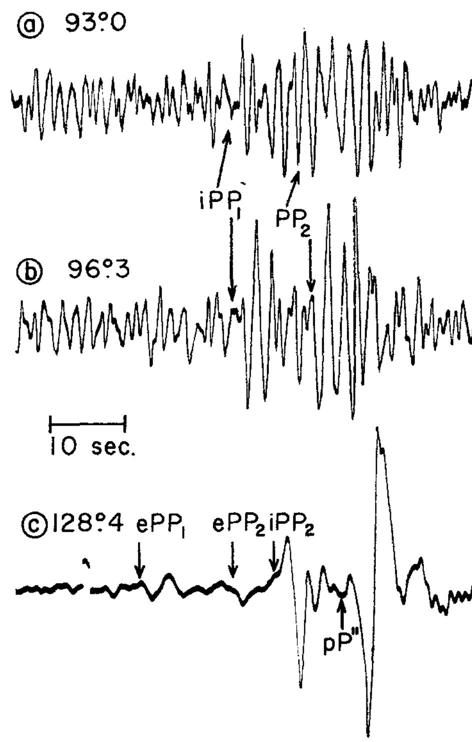


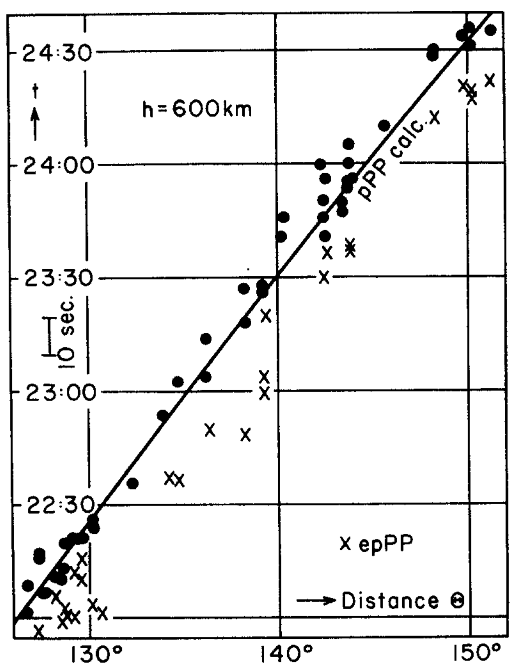
Fig. 1 (left). Travel times t of PP for epicentral distances θ in degrees, observed for earthquake of April 16, 1957 at 4:04:04, focal depth h = 600 km. and calculated travel-time curve for h = 600 km. (Figures are drafted and seismograms are arranged by Mr. L. Lenches.)

Fig. 2 (right). Portions of records of shock of April 16, 1957 4:04:04, h = 600 km., showing waves of the PP group. *a* and *b*, Grenet vertical; *c*, long-period Benioff vertical. *a*, Kiruna, distance 93°0; *b*, Skalstugan, 96°3; *c*, Pasadena, 128°4.

be very close to 4°5 S, 107°5 E, and its focal depth close to 600 km. Records of this shock from three stations at distances between 49° and 80° and from 65 stations at distances beyond 90° have been used previously (Gutenberg, 1959) for the investigation of waves reflected at the inside of the boundary of the core. The direct waves of this shock, e.g., P, PKP beyond the focal point (Gutenberg, 1958*a*, figs. 4E to 4I), start at the calculated time with a large, sharp impulse which even on records of highly magnifying instruments is not preceded by any visible motion produced by the shock.

In the present investigation, theoretical travel times have been calculated on the basis of the tables for surface foci of Jeffreys and Bullen (1940) for the portions of waves traveling through the mantle, while the portions through the core are based on travel times of Gutenberg (1958*b*) as in his publication (1959) mentioned above. The effect of the focal depth has been taken either from Jeffreys and Bullen (1940) or from Gutenberg and Richter (1936, pp. 342–345).

To avoid too long symbols, waves which have traveled through the outer but not through the inner core will be indicated by one prime, and those which, in addition, have entered the inner core, by two primes; thus, P′ indicates PKP, P″ = PKIKP, P′P′ = PKPPKP, etc.



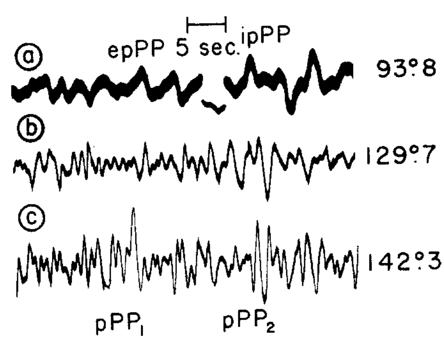


Fig. 3 (left). Travel times *t* of pPP. Details as in figure 1.

Fig. 4 (right). Portions of records of shock of April 16, 1957, h = 600 km., showing waves of the pPP group. *a*, long-period, *b* and *c*, short-period Benioff verticals. *a*, Uppsala, 93°8; *b*, Palomar, 129°7; *c*, Weston, 142°3.

PP, pPP AND pP′

Contrasting with the sharp impulse with which P begins (Jerusalem and Rome report i!!P, several other stations i!P, and on several records available here the light spot disappears at the first wave of P− or P′), PP frequently shows definite ePP phases followed by iPP. In all such instances the time of iPP fits the calculated travel-time curve, while ePP, which is sometimes relatively large, arrives earlier. Figure 1 shows a portion of the travel times, and figure 2 gives examples of records. The time difference iPP − ePP varies between 6 and 16 seconds. This agrees with the findings of Gutenberg (1914) and corresponds well to the range to be expected, if ePP results prevailingly from a reflection from the Mohorovičić discontinuity. It should be noted that the original measurements (before the present research had been intended) had been made without looking for any specific phenomenon and without use of a calculated travel-time curve.

The data are probably not accurate enough to determine the depth of the Mohorovičić discontinuity at the respective points of reflection. In the present research, the data for distances between 93° and 97° are from records at the Swedish stations, and the points of reflection are in Central Asia. Records corresponding to distances of 100° to 128° have been written in Alaska, British Columbia, and the western United States. In all these the reflections occur at points from the northern Philippines to Japan or very close to Japan. At distances between about 128° and 156° PP arrives too closely to other phases to be useful for the present research. There are possibly one or two early PP arrivals at stations in the West Indies at distances over 160° with points of reflection in the Arctic Ocean near the coast of Siberia.

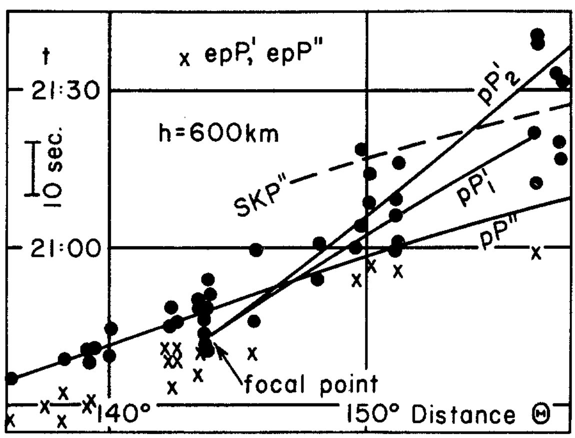


Fig. 5. Travel times t of pP′ group. Details as in figure 1.

A portion of the travel-time curve of pPP is shown in figure 3, and examples of records of pPP are reproduced in figure 4. The scattering of the early observations of epPP is noticeably greater than for PP. There are two reflections, the first near the epicenter in the Sunda Sea at or near Borneo, Sumatra, or Java, the second near the same points as for PP. It is not possible to disentangle the effects of the two reflections for calculation of the depth of the Mohorovičić discontinuity. Finally, figure 5 shows a portion of the travel-time curve of pP′. While P′ and P″ at distances over 140° show no indication of early phases, they are fairly frequently recorded in the corresponding pP′ and pP″ waves. All points of reflection are near the source of the shock.

P′P′, P′P′P′, SKPP′ and Related Phases

Waves which have traveled twice or more through the whole earth, either on the same type of path, like P′P′, or on different types, like SKPP′, can be expected to show early beginnings similar to those of PP and the other phases discussed above as a result of reflections from the Mohorovičić, the Conrad, and similar discontinuities. Calculated travel times of such phases, as well as some observed travel times, are shown in figure 6, while figure 7 gives a few examples of such records. No attempt has been made to collect seismograms of these waves; those used here have been

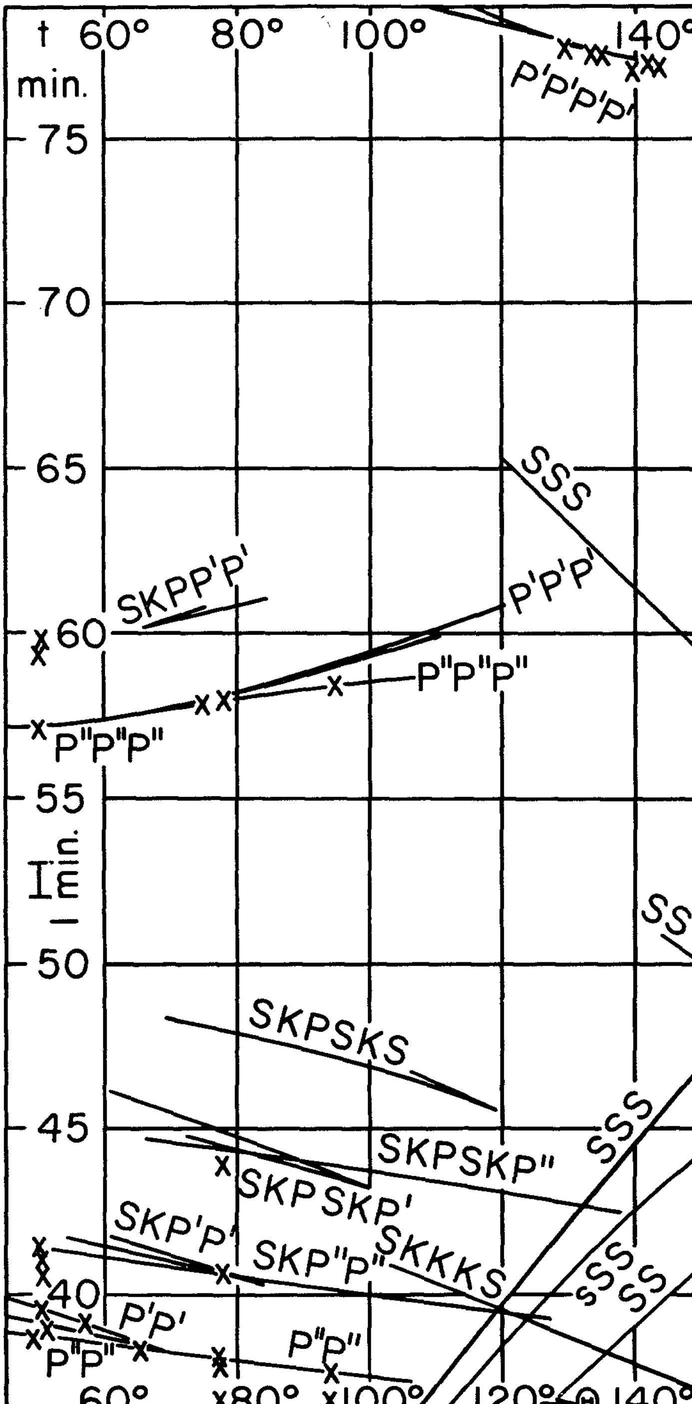


Fig. 6. Calculated travel-time curves for focal depth h of 600 km. of waves which have been reflected twice or more at the earth's surface, and observed travel times, earthquake of April 16, 1957, 4:04:04, h = 600 km.

accidentally among the records of the shock of April 16, 1957, which have been used for other purposes. In addition to the complications resulting from reflections at discontinuities in the crustal layers of the earth, there may be effects of the transition zone between outer and inner core, corresponding to the early arrival of short-period waves in P″ at distances smaller than about 140° (Gutenberg, 1958*b*). *Vice versa*, early arrivals of waves belonging to the P″P″, P″P″P″, SKPP″, etc. groups cannot be considered as additional evidence for effects in the boundary zone between inner and outer core. For example, at Jerusalem, distance 77°.4 (fig. 7, *b*), the observed beginning of P″P″ is even earlier than that calculated on the basis of the earliest short-period P″ waves, and the same is true for the earliest SKPP″ (fig. 7, *c* and *d*); the largest wave of this group corresponds within a second or two to the

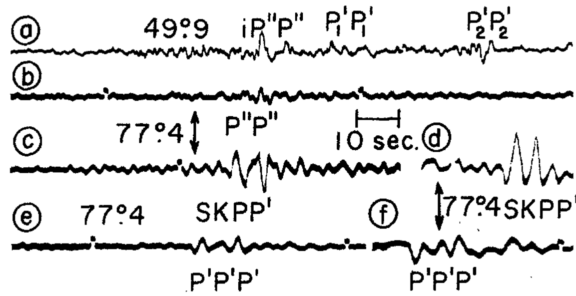


Fig. 7. Portions of records of shock of April 16, 1957, h = 600 km., showing waves through the core, reflected twice or three times at the earth's surface; *d* and *f*, long-period, the others, short-period verticals; *a*, Matsushiro, distance 49°.9, all others Jerusalem, 77°.4, originals on film.

calculated time for SKPP′. At Matsushiro (distance 49°.9, fig. 7, *a*) the observed times of iP″P″ are 5 seconds later, those of $P_1'P_1'$ about 4 seconds earlier, and those of $P_2'P_2'$ about 9 seconds earlier than the calculated times. However, these differences may well be considered to be within the limits of error on account of the accumulation of errors in the calculated times as well as the uncertainty in the time of the recorded wave. All these phases are preceded by small waves, some of which emerge by as much as 30 seconds (fig. 7, *a*) before the main phase. These complications are probably produced by various processes, including the reflection at discontinuities in the crust as well as diffraction in the transition zone from the outer to the inner core.

Well-recorded P′P′P′ waves at Jerusalem (fig. 7, *e* and *f*) and eP′P′P′ at Uppsala arrive within a few seconds of the calculated times. They are possibly preceded by small waves for about 12 seconds, but the visibly recorded motion in P′P′P′ is much less complicated than that in P′P′.

P′P′P′P′

Gutenberg and Richter (1939, p. 133) had calculated that the focus of P′P′P′P′ should be near a distance of 150° with a travel time of 78 minutes for surface foci. The new calculations give the focal point near 140° (580°) in shallow shocks, and about 142° with a travel time of about 77 min. 35 sec. for a 600 km. deep shock. Apparently, heretofore P′P′P′P′ has not been observed. In the shock of April 16, 1957, there are at least six stations which show more or less definite waves about the time at which P′P′P′P′ is expected. The travel times of these phases are plotted in figure 6, and in figure 8 for each of these stations one record is reproduced. The records are enlarged by different factors to show the same minute length. The times of the beginnings of P′P′P′P′ are somewhat uncertain. The relatively long waves recorded at several stations do not belong to P′P′P′P′, but to trains of other late phases which are recorded for about two hours at some stations. At Ottawa, the beginning measured with some doubt as that of P′P′P′P′ may belong to some other phase. Otherwise, the measured times of the apparent beginnings of these waves are

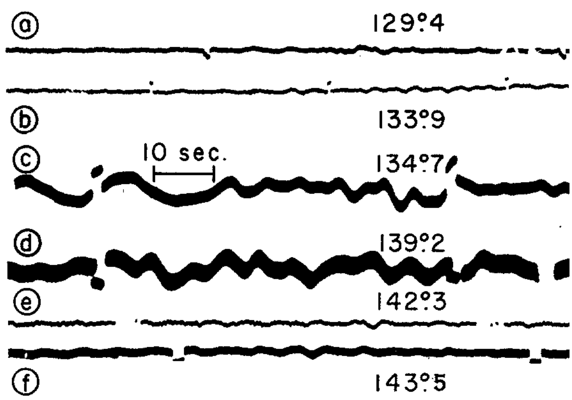


Fig. 8. Portions of records of shock of April 16, 1957, h = 600 km., showing vertical-component records of P′P′P′P′. *a*, Salt Lake City, 129°4, Wilson-Lamison medium-period; *b*, Boulder, Colorado, 133°9, short-period Benioff on film; *c*, Tucson, 134°7, long-period Benioff; *d*, Ottawa, 139°2, long-period Benioff; *e*, Cleveland, 142°3, medium-period Sprengnether; *f*, Fayetteville, 143°5, medium-period Benioff.

within less than 10 seconds of the calculated times for P″P″P″P″ at distances less than 135° and about 15 to 20 seconds earlier than the time calculated for P′P′P′P′. At the two stations at distances between 142° and 144°, beyond the calculated focus at 142°, where the extrapolated travel time of P′P′P′P′ is close to that calculated for P″P″P″P″, the observed times of arrival are respectively 11 and 17 seconds earlier than calculated. There can be little doubt that actually waves of the P′P′P′P′ group have been observed. The periods of the waves are between about 3 and 6 seconds; in addition, waves with periods of about 1 sec. recorded at Salt Lake City may belong to P′P′P′P′.

The amplitudes a and periods T of P′P′P′P′ and those of P′ may be used to calculate the "average" coefficient of absorption k for amplitudes of waves traveling through the mantle and core of the earth. If H is the theoretical ratio of a/T for P′P′P′P′ to a/T for P′, neglecting the effect of absorption, and R the actually observed ratio, k can be calculated (Gutenberg, 1945, p. 58, omitting a factor 2, since the earlier coefficient had been calculated for the energy) from

$$k = 2.3(\log H - \log R)/(D_2 - D_1) \tag{1}$$

where D_1 is the length of the path of P′, D_2 that of P′P′P′P′. If we calculate k per 1,000 km., $D_2 - D_1$ is about 37 (thousand km.). If we use P′P′P′P′ and P′, the theoretical value of H is given by

$$H = 0.5F^3 \tag{2}$$

according to Gutenberg (1945, p. 57), where F is the factor of reduction of the amplitudes of P′ at the Mohorovičić discontinuity and the core boundary and that for P′P′ at the earth's surface. The factor 0.5 (square root of 0.25) results from the fact that $di/d\Delta$ is four times greater for P′ than for P′P′P′P′. All other quantities in the original equation for the ratio a/T may be assumed to be the same for P′ and for P′P′P′P′.

The factor giving the decrease of the amplitudes for the reflection at the earth's surface and the refraction across the Mohorovičić discontinuity is close to 1 (Gutenberg, 1944, fig. 1, p. 95) in both directions. Unfortunately, the reduction of the amplitudes at the core boundary depends much on the ratio of the densities there (Dana, 1944). Gutenberg (1951, pp. 387–388) has found that the ratio assumed by Dana leads to results which do not agree with the observations, and that the density ratio is probably somewhat greater than he has assumed. On Dana's assumptions, F would be about 0.7, but a value of between 0.8 and 0.9 seems more probable (Gutenberg, 1951, p. 388). Since F^3 enters into the calculation, even so small a change affects noticeably the resulting value of k. If we assume that $F^3 = 0.6$, we find $H = 0.3$.

Another source of error results from the fact that the magnification of the instruments which have recorded P′P′P′P′ is not accurately known for the time of the shock of April 16, 1957 (fig. 8). However, no great error can be introduced if it is assumed that $a = 0.8$ microns, $T = 4$ sec., $a/T = 0.2$ microns/sec. On the other hand, the average value of a/T for P′ for a shallow shock of magnitude 7.2 follows from Gutenberg (1951, fig. 10, p. 383):

$$\log (a/T) = 7.2 - 6.5 = 0.7\,, \quad \text{which gives } R = 0.2/5 = 0.04$$

This figure is based on several assumptions, for example, that the energy flux which leaves the source is equal in all directions. Thus, we find that within noticeable limits of error

$$k = 2.3\,(-0.5 + 1.4)/37 = 0.06 \text{ per } 1{,}000 \text{ km.}$$

This agrees well with previous calculations (Gutenberg, 1958*c*, p. 279), but it confirms only the order of magnitude of k.

It is of interest to note that P′P′P′P′ after traveling about 50,000 km. through the mantle and core of the earth still shows periods in the same range as P. Possibly, periods of less than 2 seconds are less abundant in P′P′P′P′ than in P, but there are too few data to establish this as a fact.

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