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# SOME PROBLEMS IN THE APPLICATION OF SPECTRUM TECHNIQUES TO STRONG-MOTION EARTHQUAKE ANALYSIS

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## ABSTRACT

A comparison of analog and digital computation of strong-motion earthquake response spectra is made, and it is shown that the preference as to method will depend mainly on availability of computing equipment. The accuracy of analog response spectrum computations is shown to be compatible with the limitations of the original ground acceleration data. The accuracy of the customary approximate relationship between the displacement response spectrum and the velocity response spectrum is investigated, and the validity of the simplifications are shown for typical strong-motion earthquake applications. The relationship between the response spectrum and the Fourier spectrum is developed, and a comparison as to suitability for earthquake engineering problems is given.


## Introduction

Since the introduction of the concept of the response spectrum into earthquake engineering investigations by Benioff (1934) and Biot (1941) in the 1930's, this technique has become a standard tool for the analysis of strong-motion acceleration records (Housner *et al.*, 1953; Hudson, 1956). The continued development of these methods has raised new questions as to applications and accuracy, and it is the purpose of the present paper to discuss three of these questions: (a) In the past, most response spectrum calculations have been carried out on specially developed electric analog computers. At present, many modern high-speed digital computers are available, and it is of interest to compare the two computer types for this particular application; (b) Certain approximations are usually made as to the relationship between displacement spectra and velocity spectra. The errors involved in these assumptions are examined, and data are given to show the accuracies that may be expected in typical situations; (c) Modern digital computers make it feasible to calculate the Fourier spectrum for complicated functions, and techniques for the application of spectrum techniques to data-processing problems have recently been highly developed (Blackman and Tukey, 1959). It is of interest to compare the calculation and application of Fourier Spectra and Response Spectra to earthquake engineering studies of strong-motion ground acceleration data.

## Analog and Digital Calculation of Response Spectra

The response spectrum is defined as the maximum response of a single degree of freedom linear system to a prescribed exciting ground acceleration, plotted versus the natural frequency or period for various fractions of critical damping. For example, the relative displacement response spectrum, $S_d$, for a ground acceleration $\ddot{y}\,(t)$ would be

$$S_d = \left[\frac{1}{\omega\sqrt{1-\zeta^2}} \int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \sin \omega\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau\right]_{\max} \qquad (1)$$

where $\omega$ is the natural frequency of the system and $\zeta$ is the fraction of critical damping.

The numerical evaluation of eq. (1) is rendered troublesome by the fact that $\ddot{y}(t)$ is a complicated function requiring some 500–1000 points to define it, and by the requirement that a maximum value at some unknown time must be determined by scanning the total response time for each point. To describe the complete set of response spectrum curves for the desired ranges of frequency and damping usually requires some 500 calculated points.

Analog computation techniques are particularly well suited to such response calculations, and the first complete earthquake spectrum calculations were made using a mechanical analog in the form of a torsion pendulum (Biot, 1941). A logical next step was to replace the mechanical torsion pendulum by the more convenient electrical analog, which greatly speeded up the process and increased the accuracy (Housner and McCann, 1949). These developments culminated in the development of the Electric Analog Response Spectrum Analyzer, now being regularly used at the California Institute of Technology and the U. S. Coast and Geodetic Survey for earthquake accelerogram analysis (Caughey *et al.*, 1960).

The recent increased availability of large, high-speed digital computers has raised the question as to how these digital computation methods would compare with the analog computer for earthquake response spectrum determinations. For certain problems the potential for increased accuracy of the digital machines might be an advantageous factor. More important, however, is the fact that almost any research group now has access to some kind of digital computer, whereas suitable analog devices are less generally available.

## Data Preparation

The most difficult part of the response spectrum determination for either digital or analog computer is the preparation of the input data in a proper form for the computer. The ground acceleration-time curve as recorded by the standard U. S. Coast and Geodetic Survey strong-motion accelerograph is in the form of a trace on a photographic paper, which must be converted in some way to a suitable electric signal.

For analog computing, this is done by a special plotting device, which produces a variable width film trace on the periphery of a thin transparent circular film disk (Caughey *et al.*, 1960). This film disk is then rotated between a light source and a photocell to produce the analog signal. The whole process of modifying the original accelerogram and preparing the film disk takes approximately four hours for an average earthquake record.

For the digital computer, the accelerogram is approximated by a series of straight line segments, and the acceleration-time coordinates of the endpoints of each segment are punched on the input tape or on cards. For earthquake accelerograms it is usually sufficient to connect only the peaks and obvious points of slope discontinuity, although sometimes additional points must be picked to correctly indicate trace curvature. To read and manually punch the 400–500 points required for the average earthquake accelerogram takes about eight hours. If an automatic plotter which requires only a cross-hair setting on each point is available, this time can be reduced to about two hours.

An ideal solution to the above data processing difficulties would be to record the original data in digital form. Considering that the frequency range of the accelerometer must extend to at least 20 cycles per second, this would present some difficulties, of an economic rather than a technical nature. If a digital recording could be provided from the beginning, the superiority of digital computing techniques for response spectrum calculations would be unquestioned. If digital recording is not practicable, a considerable improvement over present practice could be attained by some recording method, such as magnetic tape, which would directly produce an analog electric signal. This electric signal could then be easily operated upon to produce input tapes having any desired characteristics.

The alteration of existing strong-motion accelerographs for digital or magnetic tape recording does not seem to be economically feasible at the present time. The strong-motion accelerograph networks throughout the world will need to be greatly expanded in the future, however, and there should be major design efforts to produce more suitable instrumentation embodying all modern advances in measurement and data processing techniques.

## Digital Response Spectrum Determinations

The digital computations to be reported here were made on a standard Burroughs 220 Digital Computer at the California Institute of Technology. This machine has a 5000 word memory and a 185 $\mu$s clear and add time.

The input accelerograms were represented by a series of straight lines as mentioned above. An exact expression can be written for the response of a single degree of freedom, damped, linear system for each straight-line segment of the applied excitation. By matching the initial conditions for solutions corresponding to each segment, an analytical expression for the total response can be obtained. These analytical solutions were solved by the digital computer in a purely arithmetical way; no numerical approximations were introduced other than those inherent in round off and in the evaluation of trigonometric functions.

In fig. 1 is shown the result of two different digital computations of a relative velocity response spectrum. In both cases the input data were that prepared by Professor Glen V. Berg of the Department of Civil Engineering, University of Michigan, from the original accelerogram. The solid line spectrum is that calculated at the California Institute of Technology using the Burroughs 220 computer and the above "exact" method. The dashed line is the spectrum calculated at the University of Michigan on an IBM 704 computer using a Runge-Kutta computation scheme. The time interval used in the Runge-Kutta program was $\frac{1}{24}$ of the response period, or the distance to the next accelerogram point, whichever was smaller. It was found that by reducing this time interval to one-half of the above, the spectrum curve coincided with the "exact" curve, at the expense of an increase of computation time of some 50–100%.

In order to investigate the errors that might be introduced in the process of reducing the original accelerogram to digital form, a completely independent reduction of the original accelerogram was carried through. In fig. 2 the solid line is the velocity spectrum as calculated on the Burroughs 220 computer with the "exact" method, using digital data as reduced from the original accelerogram at the California Institute of Technology. The dashed line shows the results of using exactly the same machine and method on the digital data produced at the University of Michigan. The deviations indicated in fig. 2 should thus be an indication of the accuracy inherent in the original accelerograph record, as far as response calculations are concerned. It would seem that deviations of the order shown in fig. 2 should therefore not be considered as significant in response spectrum work.

Figure 3 shows a comparison of response spectra calculated by ditigal and analog methods. The solid line was calculated on the Burroughs 220 computer using the

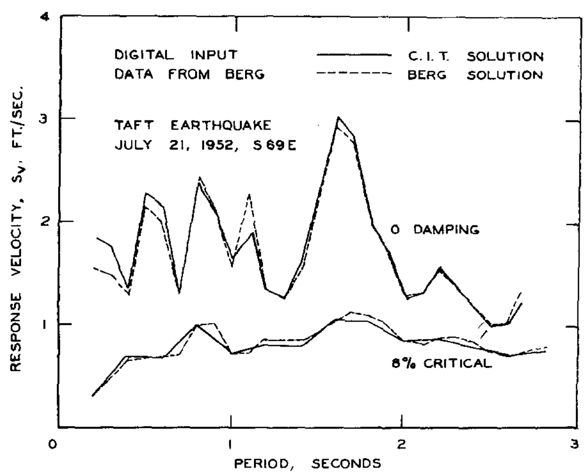


FIG. 1. Comparison of different digital response spectrum calculations from same input data.

"exact" method. The dashed line is a previously published response spectrum calculated on the original model of the electric analog response spectrum analyzer (Housner, 1953). Comparing fig. 3 with fig. 2, we see that the deviations between the digital and analog solutions are a little larger than the deviations between the two digital results. As would be expected, the analog process introduces a somewhat greater error, since not only the errors in the function generation enter, but also additional computational errors such as those involved in reading amplitudes on the screen of a cathode ray tube. The deviations indicated in fig. 3 are believed to be a realistic picture of the accuracies to be expected in the published response spectrum curves. Part of the difference in fig. 3 can be explained by the fact that different period coordinate points are used in the two methods, and in the vicinity of sharp peaks this might introduce a large local deviation which would not in fact be significant for the general shape of the spectrum curves.

Although the past analog calculations have of course introduced additional errors over those inherent in the basic data, figs. 2 and 3 indicate that these analog determinations have been adequate to define the significant features of the spectrum curves.

## Digital and Analog Computing Time

To compare the computation and plotting time by digital and analog methods, consider a typical set of earthquake response spectrum curves for five different damping values, requiring a total of about 400 points. On the electric analog re-

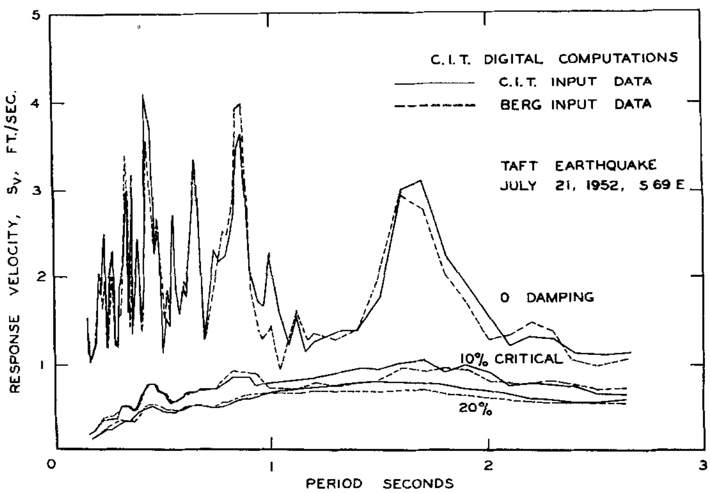


FIG. 2. Digital response spectrum calculations for independently reduced input data.

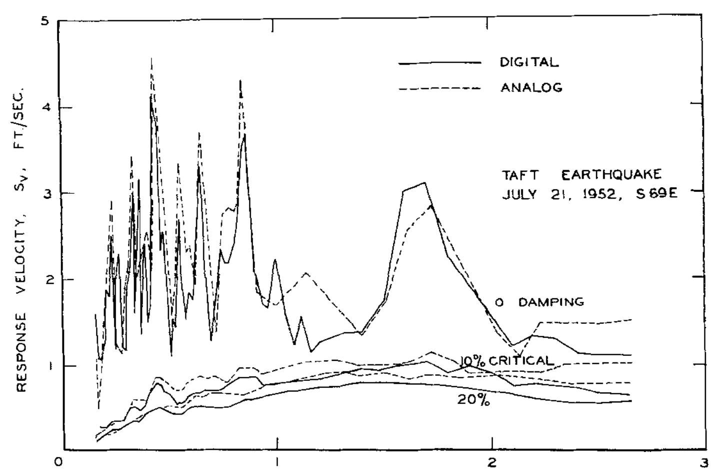


FIG. 3. Comparison of digital and analog response spectrum calculations.

sponse spectrum analyzer this job can be done in about two hours. The Burroughs 220 digital computer using the "exact" method requires about seven hours. By using approximate numerical techniques, which, as shown above, have an accuracy consistent with the original data, the digital time can be reduced to about two hours. If a higher speed machine such as an IBM 709 is available, another factor of 1.5 to 2 improvement can be realized.

Considering the above computing times, and the data preparation time previously mentioned, the following conclusions can be reached. If a high speed computer equivalent to an IBM 709 is available along with an automatic data reduction and plotting system, the digital computation would have a clear advantage in speed and accuracy over the analog methods. In the absence of automatic data reduction and plotting devices, analog computing would be a somewhat quicker method, provided that suitable function generation equipment were available. The Mark II electric analog response spectrum analyzer is still the most feasible method of producing response spectrum curves on a routine basis. Research groups who are making such calculations on only an occasional basis would probably find it more practicable to use digital computing rather than to develop the equivalent of the electric analog response spectrum analyzer.

An advantage of the digital method that should be mentioned is the ease with which other calculations than response determinations can be carried out on the earthquake accelerograms. For example, the double integration of accelerograms to give displacements, which requires various corrections and adjustments of the data, can easily be handled on the digital machine (Berg and Housner, 1961).

## Displacement and Velocity Response Spectra

Depending upon the application, it may be convenient to use displacement, velocity, or acceleration response spectra. Since there are simple approximate relationships between these various spectra for earthquake-like excitations, it has not ordinarily mattered which was originally calculated (Hudson, 1956). The exact relationships are somewhat complicated, however, and questions sometimes arise as to the errors involved in using the simplified equations.

The exact expression for the relative displacement of a single degree of freedom, damped, linear system acted upon by a ground acceleration $\ddot{y}(t)$ is:

$$\text{relative displacement} = -\frac{1}{\omega\sqrt{1-\zeta^2}} \int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \sin \omega\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau \tag{2}$$

where, as before, $\omega$ is the undamped natural frequency, and $\zeta$ is the fraction of critical damping. The exact expression for the relative velocity may be obtained by differentiating eq. (2):

$$\begin{aligned}\text{relative velocity} = &-\int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \cos \omega\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau \\ &+ \frac{\zeta}{\sqrt{1-\zeta^2}} \int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \sin \omega\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau\end{aligned} \tag{3}$$

A further differentiation will give the *absolute* acceleration of the mass:

$$\begin{aligned}\text{absolute acceleration} = &\frac{\omega(1-2\zeta^2)}{\sqrt{1-\zeta^2}} \int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \sin \omega\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau \\ &+ 2\omega\zeta \int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \cos \omega\sqrt{1-\zeta^2}\,(1-\tau)\, d\tau\end{aligned} \tag{4}$$

Referring to the definition of eq. (1), and comparing eqs. (2) and (3), we see that

$$S_v \approx \omega S_d \tag{5}$$

where

$$S_v = \left[ \int_0^t \ddot{y}(\tau)\, e^{-\omega\zeta(t-\tau)} \sin \omega\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau \right]_{\max} \tag{6}$$

Equation (5) is obtained by considering first that the damping is small, so that $\sqrt{1-\zeta^2} \approx 1$ (in which case the $\sqrt{1-\zeta^2}$ term might just as well be dropped from eq. (1) and eq. (6) as well), next by dropping the second term of order $\zeta$ in eq. (3), and finally by replacing the cosine in eq. (3) by a sine. The advantage of this replacement of a cosine by a sine is that displacements, velocities, and accelerations can then all be expressed in terms of the same integral, of the form of that in eq. (6).

For earthquake-like excitations the above approximations can be made plausible, although no rigorous demonstration of the errors involved seems possible (Hudson, 1959).

The best way of evaluating the accuracy of eq. (5) is to calculate the exact relationships from eq. (2) and eq. (3) and to compare the result with the simplified expression. In fig. 4, the solid line is the digital calculation of the velocity response spectrum $S_v$ for the Taft earthquake of July 21, 1952, S 69°E component. The dashed line is the digital calculation of the quantity $\omega S_d$, so that a direct comparison of the two sets of curves will indicate the approximation involved in eq. (5). Referring to eqs. (2) and (3), it will be seen that for the zero damping curve the only approximation is that caused by the replacement of the cosine by the sine, and this evidently does not introduce a significant error except possibly at the relatively long periods. Below a period of 1 second, the curves cannot be distinguished.

As the amount of damping increases, the effect of the second term of eq. (3) is felt, so that at 20% damping, deviations of the order of 20% might be expected. It will be seen from fig. 4 that a progressive increase in the deviations does in fact occur as the damping is increased.

In order to check the extent to which the differences of fig. 4 are typical of earthquakes, similar comparisons were made for two other earthquakes, using calculations made on the electric analog response spectrum analyzer. The Golden Gate Park recording of the San Francisco earthquake of March 22, 1957, S 80°E component, of fig. 5 shows a slightly different pattern than fig. 4, although the overall level of the deviation magnitudes is about the same. This Golden Gate Park accelerogram was a very short time duration ground motion containing relatively short period peaks (Hudson and Housner, 1959). For such an excitation, more difficulties might be expected in replacing the cosine by the sine. The velocity response spectrum for Golden Gate Park has been recalculated for purposes of this comparison, and the curve varies slightly from that previously published because of normal errors in reading the cathode ray oscilloscope (Hudson and Housner, 1959).

In fig. 6 is shown a similar comparison for the El Centro earthquake of May 18, 1940, E-W component. This $S_v$ spectrum was calculated on the Mark II electric analog response spectrum analyzer, whereas the originally published El Centro spectra were calculated on the general purpose analog computer of the California Institute of Technology Analysis Laboratory (Housner *et al.*, 1953). The new com-

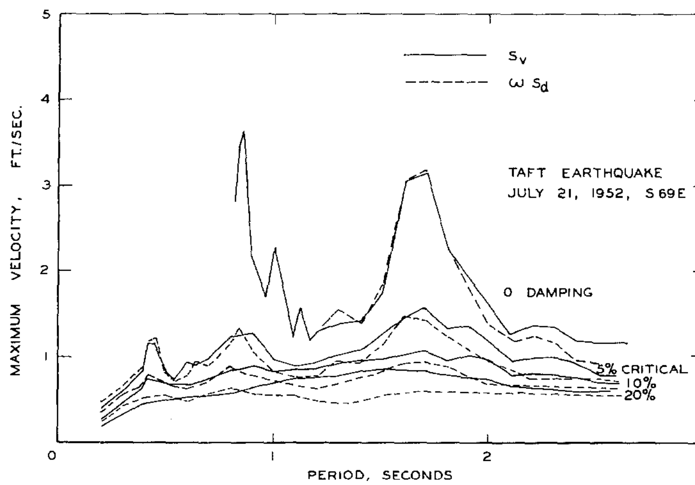


FIG. 4. Comparison of velocity spectrum and frequency times displacement spectrum for Taft earthquake.

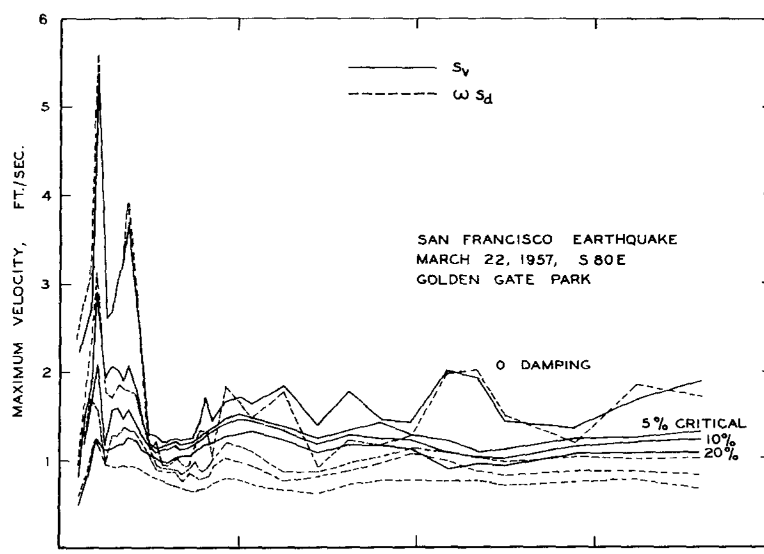


FIG. 5. Comparison of velocity spectrum and frequency times displacement spectrum for San Francisco earthquake.

putation will differ in some details because different period coordinates were used, and in regions of rapidly changing $S_v$ values, some of the peaks might thus be altered. The El Centro earthquake was a long duration earthquake of relatively large magnitude, and the maximum acceleration peaks occurred late in the record. This response spectrum is likely to be more typical of a major destructive earthquake than the other two spectra. It will be noted that the approximation of eq. (5) is better for this earthquake than for the others.

We conclude that for the types of earthquakes and the values of structural damping of most concern, the approximation of eq. (5) has an order of accuracy consistent with the original data and the calculation methods. Such an approximation, however, should only be used for earthquake-like excitations having the general character of a random distribution of peaks, and should not be used for other types of excitation without special study.

It has recently been proposed by Fung (1960) that the quantity ($\omega S_d$) should be called the "pseudo-velocity spectrum" to distinguish it from the true velocity

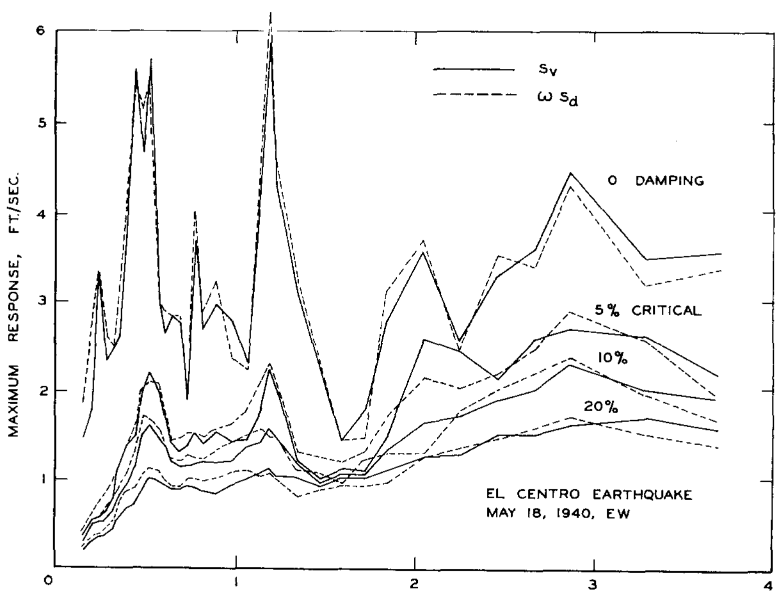


FIG. 6. Comparison of velocity spectrum and frequency times displacement spectrum for El Centro earthquake.

spectrum $S_v$. This nomenclature has been adopted by some investigators in the earthquake engineering field (Lycan, 1960; Wiggins, 1961).

## Fourier Spectra and Response Spectra

The Fourier spectrum of an input function will not only show directly the significant frequency characteristics of the function, but from it the system response can be calculated. The response spectrum has, however, been preferred for structural engineering studies of strong-motion earthquakes, because it, in a sense, combines both the representation of the exciting force and the response calculations. It thus brings together under one representation the major parameters of interest to the structural engineer.

As would be expected, there is a close relationship between the Fourier spectrum and the undamped response spectrum (Kawasumi, 1956; Rubin, 1961).

The exact relative velocity response of an undamped system may be found from eq. (3) to be:

$$\text{relative velocity} = \int_0^t \ddot{y}(\tau) \cos \omega(t - \tau)\, d\tau$$

which can be written as

$$\cos \omega t \int_0^t \ddot{y}(\tau) \cos \omega\tau\, d\tau + \sin \omega t \int_0^t \ddot{y}(\tau) \sin \omega\tau\, d\tau$$

The maximum value of this expression is by definition the undamped response spectrum $(S_v)_0$:

$$(S_v)_0 = \sqrt{\left[\int_0^t \ddot{y}(\tau) \cos \omega\tau\, d\tau\right]^2 + \left[\int_0^t \ddot{y}(\tau) \sin \omega\tau\, d\tau\right]^2} \tag{7}$$

If it is now supposed that the duration of the earthquake excitation is $T$, and that the maximum response occurs at the end of the earthquake at $t = T$, eq. (7) becomes:

$$(S_v)_0 = \sqrt{\left[\int_0^T \ddot{y}(\tau) \cos \omega\tau\, d\tau\right]^2 + \left[\int_0^T \ddot{y}(\tau) \sin \omega\tau\, d\tau\right]^2} \tag{8}$$

This is, of course, a very special case, since the maximum response might often occur before the end of the earthquake.

We next note that the Fourier spectrum for a truncated function differing from zero for $0 < t < T$ would be defined as:

$$F(\omega) = \int_0^T \ddot{y}(\tau)\, e^{-i\omega\tau}\, d\tau \tag{9}$$

Writing this in terms of sines and cosines:

$$F(\omega) = \int_0^T \ddot{y}(\tau) \cos \omega\tau\, d\tau - i \int_0^T \ddot{y}(\tau) \sin \omega\tau\, d\tau$$

The Fourier amplitude spectrum would then be given by the square root of the sum of the squares of the real and imaginary parts of $F(\omega)$;

$$|F(\omega)| = \sqrt{\left[\int_0^T \ddot{y}(\tau) \cos \omega\tau\, \omega\tau\right]^2 + \left[\ddot{y}(\tau) \sin \omega\tau\, d\tau\right]^2} \tag{10}$$

Equation (10) is identical to eq. (8), so that for the special case in which the maximum system response occurs at the end of the earthquake, the undamped velocity response spectrum is identical to the Fourier amplitude spectrum of the ground acceleration.

The above shows how the electric analog response spectrum analyzer could be used to determine the Fourier amplitude spectrum. By reading the undamped response at the end of the earthquake for each frequency setting instead of picking the maximum response in each case, the Fourier amplitude spectrum could be directly determined.

## Fourier Spectra of Strong-Motion Earthquakes

The Fourier spectra of the ground accelerations of strong-motion earthquakes can be computed either digitally, using the same straight-line approximation for the accelerogram as discussed above for the digital response spectrum calculations, or by analog techniques, as pointed out above. These calculations will be simpler to make than the response spectrum calculations because there is no need to search for maximum values, and because only one curve corresponding to the undamped response spectrum is involved.

There are certain difficulties in the computation of Fourier spectra for functions that do not have a clearly distinguishable beginning and end, or which may be derived from measurements in which signal strength is not much greater than background noise. These difficulties are not important for strong-motion accelerogram analysis, since the beginning of the record is clearly indicated, the amplitude levels are large compared to any extraneous signals, and an appropriate end of record can usually be assigned.

In fig. 7 is shown a comparison of the zero-damped response spectrum and the Fourier spectrum for the Taft earthquake of July 21, 1952, S 69°E component. The response spectrum curve of fig. 7 is the same as that shown in figures 2 and 3. The Fourier spectrum was computed on the Burroughs 220 digital computer from the same input data used for the Response Spectrum, using a similar "exact" solution of eq. (10).

It will be seen that the two spectra of fig. 7 are very similar. As shown above, the curves should be identical for any periods for which the maximum response occurs at the end of the earthquake, which evidently happens at many points. The Fourier spectrum values can of course never be greater than the response spectrum values. Periods for which the Fourier spectrum is markedly different from the response spectrum would represent situations in which the maximum response during the earthquake was different from the response at the end of the earthquake, which is of course easily possible.

There may sometimes be a question as when to terminate the strong-motion portion of the accelerogram. To investigate the effects of record length on the Fourier spectrum, two different earthquake time durations were computed. In fig. 8 this comparison can be seen, and it will be noted that while there are differences, there is perhaps no significant difference in the general picture of the major frequency distribution of the function. It will be noted that in fig. 8 not as many points were calculated in the low period region as for fig. 7, and this lack of definition in the curves may account for some of the differences at the low periods.

It is clear that if the only object of the spectrum calculations is to visualize the predominant frequencies in the record, the Fourier spectrum will serve this purpose as well as the response spectrum. The digital computation of the Fourier spectrum takes only $\frac{1}{3}$ to $\frac{1}{2}$ the time that the response spectrum calculation would take, although it must be kept in mind that it is the data preparation which is troublesome in either case. The damped response spectrum curves contain of course more infor-

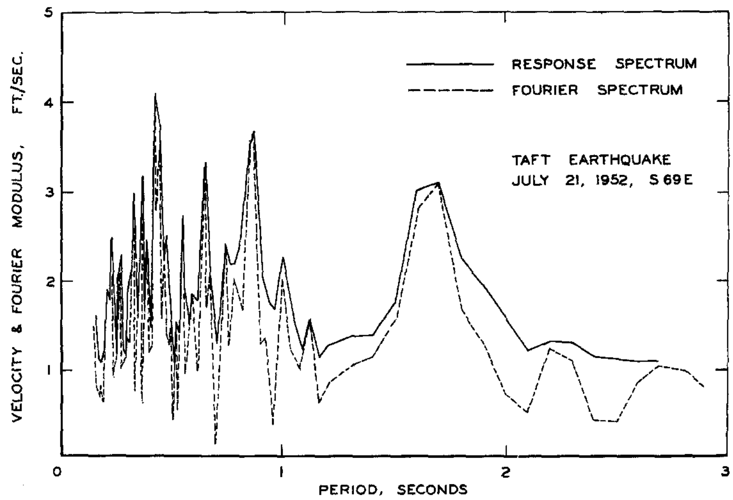


FIG. 7. Comparison of Fourier spectrum and response spectrum.

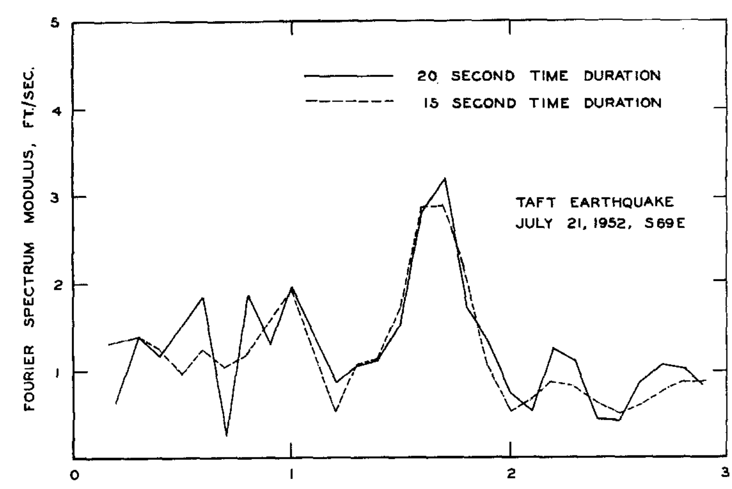


FIG. 8. Comparison of Fourier spectra for two earthquake time durations.

mation than the Fourier spectrum curves, and hence for most earthquake engineering purposes would be well worth the small additional calculation time.

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