ON FOCAL POINTS OF SKS*

By B. Gutenberg

Although SKS is one of the most important phases in investigating the core of the earth, apparently it has not yet been noted that it may have one or more focal points. The full lines in figure 1 show the observed parts of the travel-time curves (epicentral distance θ, time t) of S, ScS, and SKS in the region between 73 and 88 degrees. The problem is to find the first part of the SKS curve at distances preceding the intersection of the travel-time curves of S and SKS.

As the velocity of the longitudinal waves in the outer parts of the core apparently shows no discontinuity, and as its value just below the surface of the core seems to be greater than the velocity of the transverse waves just above the core, the travel-time curve of SKS must begin at a point on the travel-time curve of ScS. At this point both curves must have a common tangent as in all such cases. This follows from the fact that

$$t' = \frac{\partial t}{\partial \theta} = \frac{\sin i}{V_0} \tag{1}$$

As at this critical point the velocity V_0 and the angle of incidence i are the same for both curves, it follows that there t' must be the same for both waves, too. The radius of curvature of a travel-time curve is given by

$$\rho = \frac{(1+t'^2)^{\frac{3}{2}}}{t''} \quad \text{or} \quad \rho = \frac{(1+t'^2)^{\frac{3}{2}} \tan i}{t'} \left| \frac{d\theta}{di} \right| \tag{2}$$

as[1]

$$t'' = \frac{t'}{\tan i} \frac{di}{d\theta}$$

As t' and $\tan i$ are the same for ScS and SKS at the point where the two waves separate, the difference in curvature there is determined by $d\theta/di$. To find this difference we consider that SKS consists of two segments in the mantle which together form the path of an ScS wave, and a segment K in the core. The total distance of SKS thus may be given by

$$\theta_{\mathrm{SKS}} = \theta_{\mathrm{ScS}} + \theta_{\mathrm{K}} \tag{3}$$

where the three quantities θ have to be chosen in such a way that the quantity $\frac{r \sin i}{V}$ is the same for all points forming a given ray of SKS and the corresponding segments of ScS and K. Thus the calculated angle of incidence i_0 of the

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[1] B. Gutenberg and C. F. Richter, "On Seismic Waves" (second paper), *Gerlands Beitr. z. Geophysik*, 45:300 (1935).

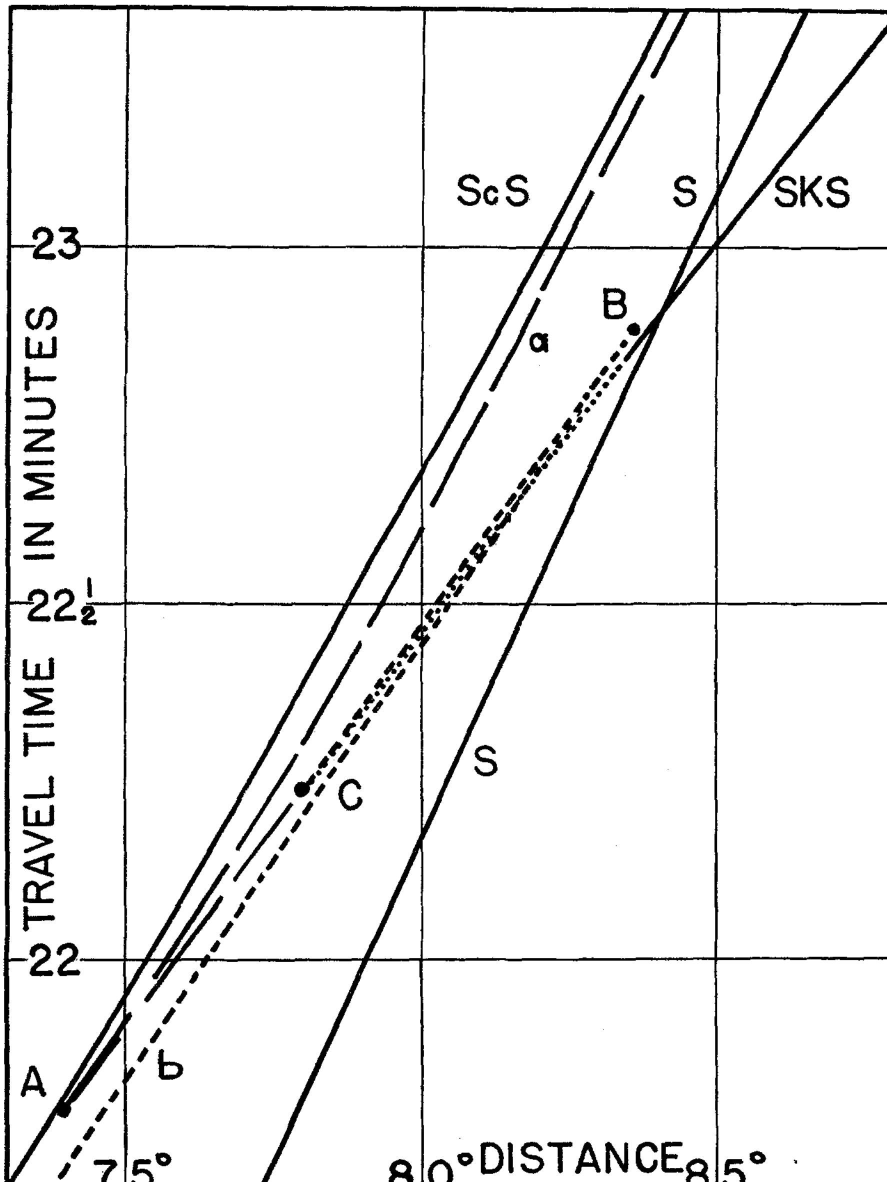

Fig. 1. Travel times for a focus in the surface.

transverse waves at the surface of the earth as well as i_K at the surface of the core is equal for the three distances θ. From equation (3) we find

$$\frac{d\theta_{SKS}}{di_0} = \frac{d\theta_{ScS}}{di_0} + \frac{d\theta_K}{di_0} \tag{4}$$

$\frac{d\theta_{ScS}}{di_0}$ is positive, $\frac{d\theta_K}{di_0}$ is negative, for the distances involved, and therefore

$$\left|\frac{d\theta_{SKS}}{di_0}\right| = \left|\frac{d\theta_{ScS}}{di_0}\right| - \left|\frac{d\theta_K}{di_0}\right| \tag{5}$$

If $\left|\frac{d\theta_K}{di_0}\right| > \left|\frac{d\theta_{ScS}}{di_0}\right|$, θ_{SKS} increases with decreasing i_0. If this happens at the beginning of the travel-time curve of SKS, t''_{SKS} is negative there; the travel-time curve of SKS must turn away from ScS with a segment concave towards the θ-axis, while ScS is convex towards it.

If $\left|\frac{d\theta_{ScS}}{di_0}\right| > \left|\frac{d\theta_K}{di_0}\right|$ at the point where SKS begins, the first segment of SKS must be convex towards the θ-axis. From equation (5) it follows that at the beginning of SKS in the case just mentioned $\left|\frac{d\theta_{SKS}}{di_0}\right| < \left|\frac{d\theta_{ScS}}{di_0}\right|$ and from equation (2) that SKS at its point of beginning has a larger radius of curvature than ScS. Thus it follows that the first segment of SKS is always below the ScS curve.

If SKS initially is convex towards the θ-axis, it follows from equation (4) that θ_{SKS} begins with a decrease in distance, as i_0 decreases. Since for large distances $d\theta_{SKS}/di$ is negative, and as it cannot be infinite,[2] it must pass through zero. This happens where $\left|\frac{d\theta_K}{di}\right| = \left|\frac{d\theta_{ScS}}{di}\right|$. At this point, t'' reverses its sign,[3] and the amplitudes of SKS are infinite.[4] SKS has a cusp with a focal point (A, curve a in fig. 1).

If SKS begins with a segment concave towards the θ-axis, it may have two focal points (B and C, curve b, in fig. 1).

Examples of possible types of the beginning of the travel-time curves of SKS are indicated by the curves marked a and b in figure 1. (The dotted section from C to the beginning of the observed part of SKS is approximately the same for a and b). The figure shows quite clearly that the travel-time curve of SKS is very sensitive to the assumptions regarding the velocity in the outer part of the core, since a has been calculated under the assumption that the velocity in the uppermost layer of the core is about 7.4 km/sec., while for curve b a velocity of about 7.9 km/sec. just below the boundary of the core has

[2] *Ibid.*, p. 301.

[3] This follows from equations (3) and (6), *ibid.*, p. 300.

[4] *Ibid.*, p. 301 and equations (19), p. 300.

been assumed. The following distances, D, of the beginning of SKS are found assuming the same ScS curve but various velocities, V_C, below the boundary of the core:

V_C	7.4	7.5	7.6	7.7	7.8	7.9	8.0	km/sec.
D	87°	81°	77°	73°	70°	67°	65°	

Other types of SKS curve than a and b in figure 1 are possible; for example, SKS may begin at the ScS curve with a focal point. This happens if, at the beginning of SKS,

$$\left|\frac{d\theta_{ScS}}{di}\right| = \left|\frac{d\theta_K}{di}\right|.$$

Focal points similar to those of SKS are to be expected for SKKS. Under the assumption corresponding to curve a in figure 1, this phase should have two focal points, the first at an epicentral distance of about 90° with a travel time of about 24 minutes, the second at a slightly larger distance.

The observations indicate strong waves at the epicentral distances and with travel times corresponding to the focal regions from the calculations. It is not unusual that a strong phase is reported as "iS" at a distance between 70° and 80° with a travel time which is one-half minute to one minute too late for S, but which may correspond to the focal region of SKS.

A more detailed study of this problem is being undertaken by the author jointly with Dr. C. F. Richter.

Summary

The travel-time curve of the first section of SKS depends much on the velocity in the outer part of the core. It begins on the travel-time curve of ScS at an epicentral distance somewhere between 65° and 90°, depending on the velocity of longitudinal waves just below the surface of the core. The extreme values correspond to velocities there of approximately 8.0 and 7.4 km/sec., respectively. If the distance where SKS begins is relatively large, its first section extends to decreasing distances and is convex towards the axis of distance, and SKS must have an odd number of cusps with focal points (at least one) where it reverses in direction and changes from convex to concave or vice versa. If SKS begins at a relatively short distance, its first segment extends to increasing distances and is concave towards the axis of distance; in this case the number of cusps is even (possibly zero). In an intermediate case, SKS begins with a focal point. In any case, the first segment of the travel-time curve of SKS is below the travel-time curve of ScS. Similar conclusions are correct for SKKS. A preliminary study of the observations seems to indicate a focal point of SKS at a distance between 70° and 80°. More detailed investigations which are under way may be used to draw conclusions respecting the velocity of longitudinal waves in the outer part of the core.

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