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A LINEAR STRAIN SEISMOGRAPH*

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THE RESPONSE of hitherto known practical forms of seismographs depends upon the relative motion of a pendulum and the moving ground to which its supporting structure is fastened. The various kinds of seismographs differ in the type of pendulum used, such as gravity, spring, torsion, and/or they differ in the type of magnifying and recording elements which they employ. They all measure or indicate the vibratory motion of the ground at a given point. In

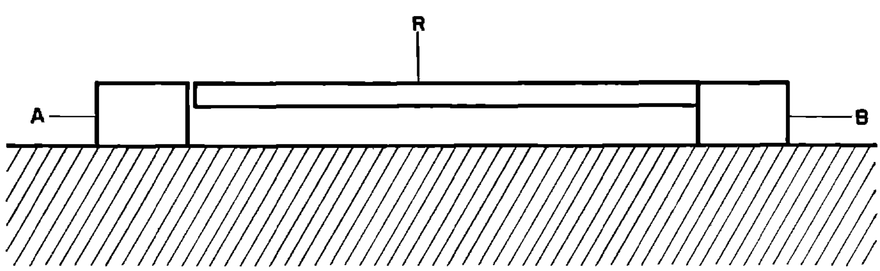

Fig. 1. Schematic representation of horizontal linear strain seismometer.

contrast to these earlier forms, the strain seismograph is a nonpendular instrument. It does not respond directly to vibratory movements. Its operation depends upon variations in the distance between two points of the ground. Such variations or linear strains are set up by seismic waves. A schematic drawing of the seismometer is shown in figure 1. A and B are two piers separated by a distance of 20 meters. One end of the rod R is rigidly fastened to the pier B. The other end extends to within a short distance of pier A. Earth strains re-

* The name first given to this instrument was "wave seismograph" because of its analogy to the wave antenna in radio. However, the analogy is not close and consequently it seemed best to adopt the more accurate name given above. The instrument is mentioned and described in the Carnegie Institution of Washington *Year Book* No. 29, 1929-30, and subsequent numbers. It was also described in a paper read before the meeting of the Seismological Society of America held in Pasadena in June, 1931.

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sulting from a seismic wave train produce variations in the separation of the two piers, and these variations are observable as changes in the distance between the free end of the rod and pier A. These small movements of the end of the rod relative to the adjacent pier actuate an electromechanical transducer which generates an e.m.f. proportional to the rate of change of the relative displacement. Recording is accomplished by means of galvanometers. A photograph of the seismometer is reproduced in figure 2.

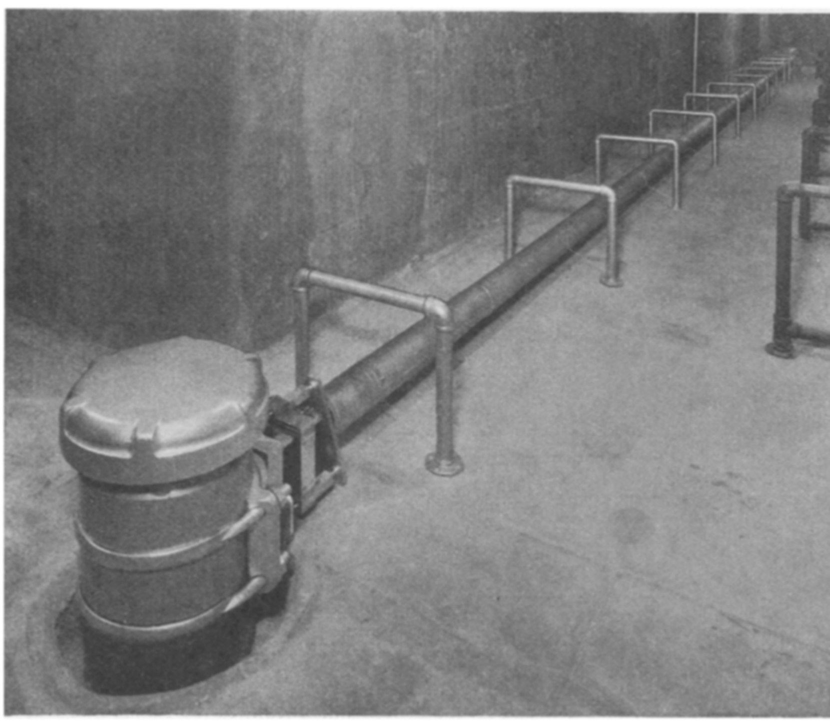


Fig. 2. Linear strain seismometer.

Earlier Forms

Subsequent to the development and construction of the instrument herein described, it was brought to the writer's attention that the basic principle had been used previously by John Milne and later by E. Oddone. It was Milne's opinion that relative movements of the ground upon which buildings rested were the causative factors of some of the damage caused by earthquakes. In order to exhibit these relative movements he set up a device[1] consisting of two piers separated by an interval of 3 feet. To one pier he fastened a rod which

[1] John Milne, "The Relative Motion of Neighboring Points of Ground," *Trans. Seis. Soc. Japan*, 12:63 (1888).

extended horizontally to within a small distance of the other pier. A lever system having a magnification of six recorded the relative motion of the free end of the rod with respect to the adjacent pier. With this apparatus he obtained traces of a few millimeters maximum amplitude in some thirteen large local earthquakes. It is apparent that his instrument was very insensitive; in fact, for earth periods of 1 second the ratio of recorded amplitude to actual earth displacement was 1:30. Oddone's instrument[2] was similar to Milne's. He increased the pier separation to 3 meters and employed a hydraulic device for indication. This hydraulic indicator was, in effect, an iron box of 18-liters capacity filled with water. A hole in one side of the box contained a piston flexibly fastened to the box by means of a diaphragm and at the same time rigidly bolted to the free end of the rod. Movements of the rod relative to the adjacent pier thus changed the level of the liquid in a glass tube of small bore which communicated with the water in the box. The area of the piston was approximately 3600 times that of the section of the glass tube, so that the magnification was 3600. The instrument was sensitive enough to show movements of the meniscus resulting from the passage of trains in the vicinity. The magnification for earth displacements having periods of 1 second was approximately 13. Oddone did not develop a satisfactory recording mechanism and as a consequence he did not observe the behavior of his instrument with respect to earthquakes. G. Agamenonne[3] adversely criticized the instrument in regard to questions which, in the light of present knowledge, are no longer of consequence.

It is clear that neither of these early instruments was satisfactory for routine operation—the one because of its extremely low sensitivity, and the other because of low sensitivity and lack of a suitable recording mechanism. A further difficulty lay in the fact that a theory for this type of instrument was not available.

The development of a highly sensitive electromagnetic transducer[4] for the writer's new vertical seismograph opened the way for the construction of a satisfactory strain seismograph. This transducer, in connection with suitable galvanometers, provides magnifications up to 1,000,000 with complete stability of operation. For example, with such a magnification, mechanical movements of 10^{-7} cm., or 1/2000 the wave length of sodium light, produce galvanometer deflections of 1 mm. Within its useful range of frequencies this device is considerably more sensitive to displacements than a Michelson interferometer. With the strain seismograph, magnifications of about 300,000 are sufficient to raise the sensitivity to the maximum value allowable by the microseismic activity of the ground in Pasadena.

[2] E. Oddone, "Ricerche Strumentali in Sismometria con Apparati non Pendulari," *Boll. Soc. Sismologica Italiana,* 11:168 (1900-01).

[3] G. Agamennone, "Sulla Pretesa Insufficienza degli Apparati Pendolari in Sismometria," *Boll. Soc. Sismologica Italiana,* 13:49 (1902-03).

[4] Hugo Benioff, "A New Vertical Seismograph," *Bull. Seism. Soc. Am.,* 22:155 (1932).

Construction

The piers are made of sections of 12-in. standard iron pipe 2 m. in length. These are sunk approximately 1.5 m. into the weathered granite underlying the Laboratory, and are cemented in with concrete. The submerged sections of the pipes are filled with concrete to the level of the floor (see fig. 4). The distance between the piers is approximately 20 m. The rod is made of 2-in. inside diameter standard iron pipe and is rigidly attached to one of the piers. In order to reduce short-period temperature variations, the rod is surrounded by a layer of asbestos insulation 2 cm. thick. It is supported by 12 structures of the type shown in figures 3 and 4. These are distributed at equal distances along its length. H (fig. 3) is constructed of 1.25-in. iron pipe and is bolted securely to the concrete floor. R is a ring of steel with set screws S bearing on the seismometer rod P. The ring R is supported by three taut bicycle spokes B. These 12 structures effectively constrain the rod so that it can move only in the longitudinal direction.[5] Since the rod is clamped at one end, its longitudinal period is equal to the time required for a longitudinal wave to travel four times the length of the rod. Thus for this instrument the period is $80/5000 = 0.016$ second approximately.

Electromechanical Transducer

The transducer is a modified form of the one devised for the writer's electromagnetic pendulum seismographs. A detailed description with theory will appear in a future paper; consequently only a brief description is given here. As the schematic drawing of the transducer, figure 5, shows, M is a permanent magnet which supplies magnetic flux to the pole pieces B, B. From the pole pieces the flux crosses the air gaps, dividing equally between the two armatures A, A. The structure consisting of magnet and pole pieces is bolted to the free end of the seismometer rod, and the armature assembly is fastened rigidly to the adjacent pier. Movement of the rod relative to the pier thus varies the lengths of the air gaps, one pair increasing while the other is decreasing. The resulting change in flux through the armatures induces an e.m.f. in the coils C, C surrounding the armatures, which is proportional to the rate of change of the relative displacement. The coils are connected in series aiding. Since a flux

[5] When the rod is unclamped from the pier and allowed to oscillate longitudinally as a pendulum, its period is approximately 0.25 sec. The total mass of the rod is approximately 100 kg. Hence the restoring force resulting from the combined effects of the 12 supporting structures is $m\omega^2 = 6 \times 10^7$ dynes per cm. displacement (m = mass, $\omega = 2\pi/T$, and T = free period of pendulum). When the bar is clamped to the pier, the force required to elongate it 1 cm. is YA/L, where Y is Young's Modulus, A the effective section of the bar, here 6.6 cm., and L the length of the bar. Thus $YA/L = 5 \times 10^9$ dynes approximately. The effective restoring force of the 12 supports acting on the clamped rod is one-half the value for the unclamped rod. Hence, the ratio of the restoring force of the supports to the longitudinal stiffness of the rod is $3 \times 10^7 / 5 \times 10^9 = 6 \times 10^{-3}$ or approximately 0.5 per cent. Therefore the longitudinal restraining effects of the supports may be neglected.

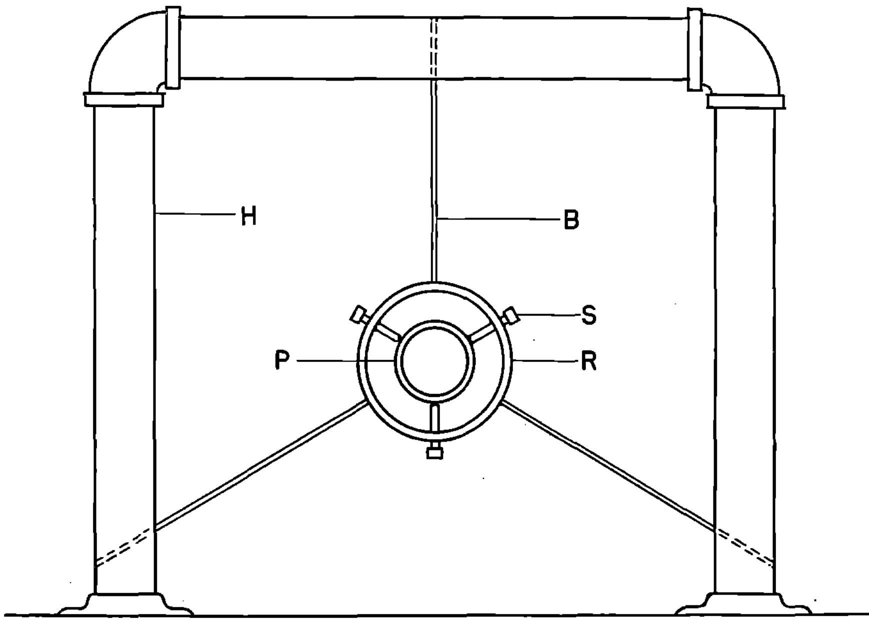


Fig. 3. Rod supporting structure.

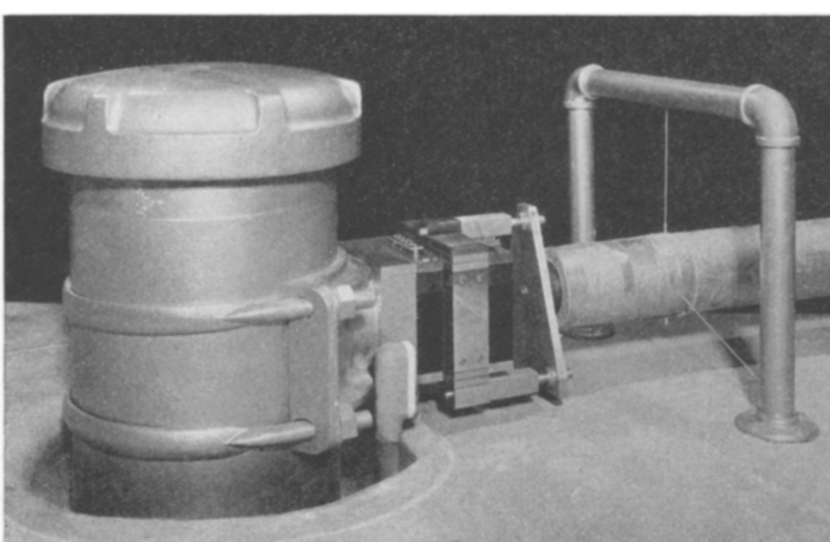


Fig. 4. Pier, transducer structure, and rod supporting structure of strain seismometer.

increase in one pair of gaps is offset by a corresponding decrease in the other pair, the total flux through the magnet remains constant. In this way, difficulties resulting from the high reluctance and hysteresis of the permanent magnet circuit are entirely avoided. By virtue of the push-pull structure of the

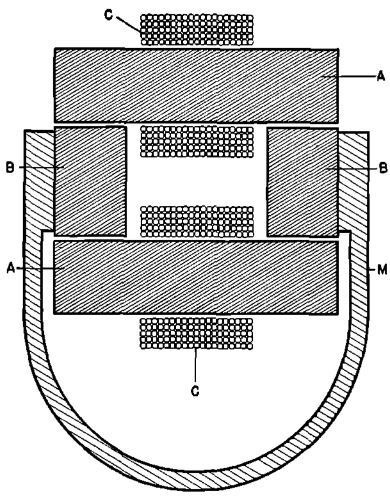


Fig. 5. Schematic diagram of electromechanical transducer.

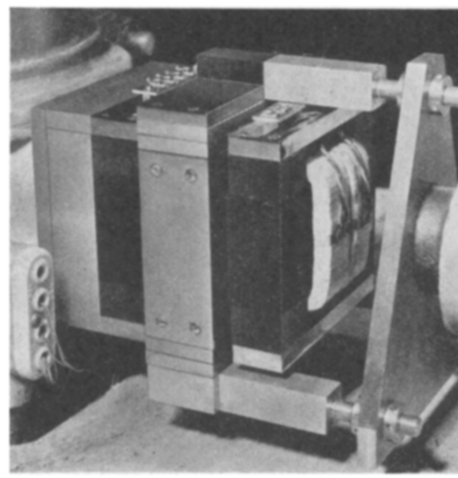


Fig. 6. Electromechanical transducer.

transducer, the output e.m.f. is linear up to terms of the third order of the displacement divided by the air-gap length. A photograph of the transducer is shown in figure 6.

Theoretical Behavior of the Rod

In the early stages of the development of this instrument, the theoretical problem of the behavior of the rod was discussed with Dr. P. S. Epstein. As a result of an investigation which he will report in another paper, he showed that the movement of the free end of the rod relative to the bound end depends upon the damping of the rod. Having under consideration seismic waves with periods which are long in comparison with the natural period of the rod, he found that with small damping the free end of the rod moves essentially with the same phase and amplitude as the bound end. In other words, the undamped rod behaves as a rigid body in accordance with our everyday experience. But when the damping is not negligibly small, the motion of the free end differs from that of the bound end in both phase and amplitude. Thus a seismometer constructed with a damped rod would exhibit some very interesting properties, one of which would be an asymmetrical response to waves arriving from opposite directions. However, in order to be effective, the damping must be of a type which is independent of frequency, and this condition cannot be met except with the aid of a mechanical frame of reference which does not partake of the seismic wave motion; hence it has not been possible to build a damped instrument. A simple theory of an undamped rod is given in an appendix to this paper.

Theory of the Horizontal Linear Strain Seismometer

In the following discussion it is assumed that the rod behaves as a rigid body. The origin of coördinates is taken at the undisturbed position of the free pier. The line joining the piers defines the x-axis. Let ξ be the horizontal displacement of the ground at the point x. If β is the angle between ξ and the direction of x, the component of the displacement parallel to the rod is $\xi \cos \beta$. The linear strain at any point x is, therefore, $\cos \beta \frac{\delta \xi}{\delta x}$. The total strain or relative displacement of the piers parallel to the line joining them is

$$y = \int_0^L \cos \beta \frac{\delta \xi}{\delta x} dx \tag{1}$$

the e.m.f. induced in the transducer coils is

$$k \frac{\delta y}{\delta t} = k \frac{\delta}{\delta t} \int_0^L \cos \beta \frac{\delta \xi}{\delta x} dx \tag{2}$$

k is the e.m.f. induced in the coils for unit relative velocity of rod and pier. L is the distance between the piers. Equations (1) and (2) represent the mechanical and electrical response, respectively, of the undamped strain seismometer for all conditions in which the proper motions of the rod can be neglected. When the disturbance consists of plane waves which are long in comparison to L it is clear that β and $\frac{\delta \xi}{\delta x}$ are essentially constant over the interval L. Under these conditions equations (1) and (2) may be integrated immediately so that we may write

$$y = L \cos \beta \frac{\delta \xi}{\delta x} \tag{3}$$

and

$$k \frac{\delta y}{\delta t} = k\, L \cos \beta \frac{\delta^2 \xi}{\delta x \delta t} \tag{4}$$

In common with pendulum seismometers, the strain seismometer does not respond to the true earth waves, but rather to the apparent surface waves which appear on the ground as a result of the incidence of the true waves. True waves which propagate horizontally at the surface of the ground, such as Rayleigh and Love waves, are identical with their corresponding apparent waves.

Other waves which arrive at the surface with angles of incidence less than $\pi/2$ give rise to apparent waves which in general differ from the originals in type, amplitude, and velocity. For example, an SV wave, that is, a vertically polarized transverse wave, is accompanied by an apparent wave of the longitudinal type. The velocity of the apparent wave is greater than that of the true wave for all angles of incidence less than $\pi/2$. If the incidence is vertical, the apparent velocity is infinite. If C is the true wave velocity and c is the velocity of the apparent wave, we may write $c=C\ csc\ i$, where i is the angle of incidence. In general the apparent velocity is given by the slope of the travel-time curve.

It will be assumed that a seismic disturbance consists of apparent waves of the form

$$\xi=\phi\left(t-\frac{r}{c}\right) \tag{5}$$

where r is the coördinate in the line of propagation and c is the apparent wave velocity. In longitudinal waves, the displacement is parallel to the line of propagation and according to the usual convention the sign is positive when the displacement is in the direction of propagation. Thus, for longitudinal waves $r=x\cos\beta$ and hence the above equation becomes

$$\xi=\phi\left(t-\frac{x\cos\beta}{c}\right) \tag{6}$$

From equation (6) it follows that

$$\frac{\delta\xi}{\delta x}=-\frac{\cos\beta}{c}\,\frac{\delta\xi}{\delta t} \tag{7}$$

Substituting the value of $\frac{\delta\xi}{\delta x}$ from equation (7) into equation (3), we find that the response of the strain seismometer to longitudinal apparent waves is

$$y=-\frac{L}{c}\cos^2\beta\frac{\delta\xi}{\delta t} \tag{8}$$

In transverse waves the displacement is normal to the direction of propagation and therefore $r=x\cos\left(\beta-\frac{\pi}{2}\right)=x\sin\beta$. Thus, for transverse waves equation (5) is written

$$\xi=\phi\left(t-\frac{x\sin\beta}{c}\right) \tag{9}$$

Here

$$\frac{\delta\xi}{\delta x}=-\frac{\sin\beta}{c}\,\frac{\delta\xi}{\delta t} \tag{10}$$

If this value of $\frac{\delta\xi}{\delta x}$ is substituted into equation (3) we find that the response of the strain seismometer to transverse apparent waves is

$$y = -\frac{L}{c}\sin\beta\cos\beta\frac{\delta\xi}{\delta t} \tag{11}$$

In equations (8) and (11), β is the angle between the rod and the direction of the earth displacement. The equations are more useful if given in the form containing α, the angle in the plane of the ground surface between the rod and the direction of propagation. For longitudinal waves $\alpha = \beta$, and the response of the strain seismometer to longitudinal apparent waves is therefore

$$y = -\frac{L}{c}\cos^2\alpha\frac{\delta\xi}{\delta t} \tag{12}$$

With reference to apparent transverse waves, $\alpha = \beta - \pi/2$, so that the response of the strain seismometer to these waves is

$$y = \frac{L}{c}\sin\alpha\cos\alpha\frac{\delta\xi}{\delta t} \tag{13}$$

It is evident from inspection of equations (12) and (13) that the response of the strain seismometer differs in a number of ways from that of the pendulum seismometer. Attention will be given first to differences in the directional characteristics. Figure 7 shows a polar graph of the function $\cos^2\alpha$, the directional response characteristic of the strain seismometer to longitudinal apparent waves. The response of the pendulum seismometer varies as $\cos\alpha$ and is shown in dotted line for comparison. Similarly, in figure 8 are shown polar graphs of the functions $\sin\alpha\cos\alpha$ and $\sin\alpha$ which are the directional characteristics of the strain and pendulum seismometers to transverse apparent waves. It will be noticed that the strain seismometer exhibits four directions of zero response for transverse waves as compared with two for the pendulum instrument. The most striking difference in the directional characteristics of the two instruments is found in the fact that the pendulum response to a given seismic wave reverses when the direction of propagation of the wave is reversed, whereas the strain response remains the same. Thus, for example, if two exactly similar earthquakes originate at equal distances from the two seismometers but in opposite directions, 0° and 180° say, the responses of the strain instrument to the two shocks are the same, while those of the pendulum instrument are opposite in sign.[6] Hence, comparison of a strain-seismograph record

[6] An interesting verification of this behavior was observed on records of an Indian earthquake. Surface waves which traveled over the long arc were recorded, in addition to those which traveled over the short arc. Thus, in a single seismogram of a single earthquake, waves were recorded on the short-period galvanometer combination which arrived from opposite directions. Upon comparison with the N-S torsion seismogram it was found that the southern group of waves (long arc) were in phase on the two seismograms, whereas in the northern group (short arc) the waves were exactly opposite in phase on the two seismograms.

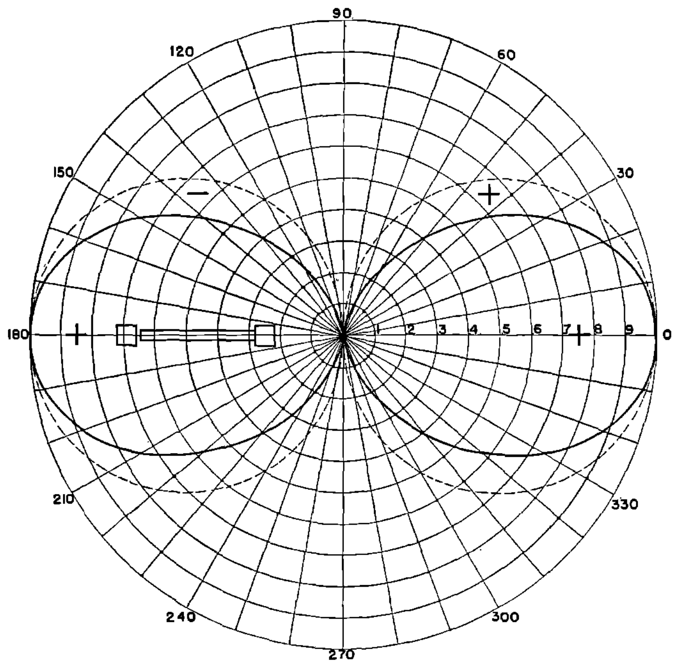


Fig. 7. Directional response characteristic of linear strain seismometer for longitudinal apparent waves. Pendulum directional response characteristic is shown in dotted lines.

with a similar component record from a pendulum seismograph results in the elimination of the 180° ambiguity in the determination of epicentral azimuth. The initial movement in the pendulum response to an earthquake indicates the direction of the initial earth movement, such as up or down, east or west, north or south, whereas the initial movement of a strain response indicates compression or rarefaction.

Another difference in the behavior of the two types of instrument is that the response of the strain seismometer is inversely proportional to the apparent wave velocity, and the pendulum response is independent of wave velocity. Accurate comparison of the records of the two instruments therefore provides the data for the determination of apparent wave velocities from observations at a single station.

If standing waves are set up in the ground upon which the instruments are placed, the strain response is maximum at a node and zero at an antinode, while the pendulum response is zero at a node and maximum at an antinode.

If the ground executes pendular vibrations such as the movements of a shaking table, there is zero response with a strain seismometer and full response with a pendulum seismometer.

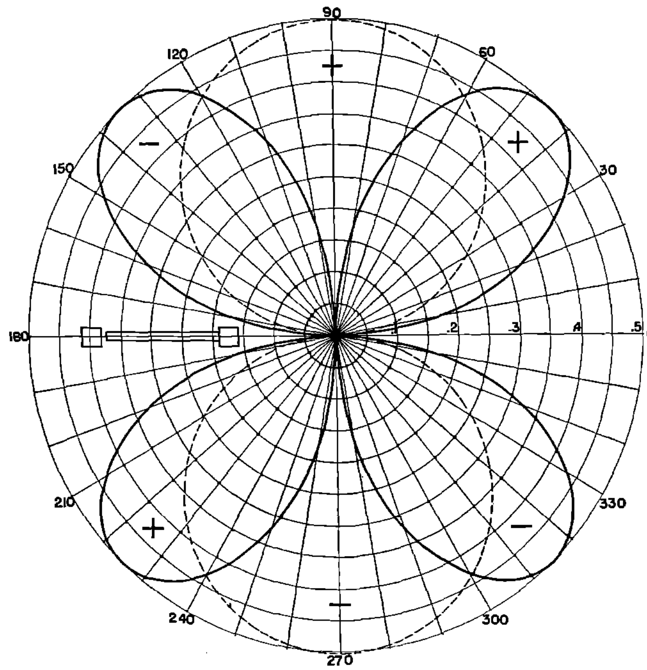


Fig. 8. Directional response characteristic of linear strain seismometer for transverse apparent waves. Pendulum directional response characteristic is shown in dotted lines.

Another striking difference is the greater sensitivity of the strain seismometer for the situation in which the instrument period is small compared to the wave period. The approximate ratio of sensitivities is readily calculated. If $a=0$, the response of the strain seismometer to longitudinal waves propagating horizontally is

$$y=-\frac{L}{c_p}\frac{\delta\xi}{\delta t} \tag{14}$$

The natural period of the rod is $T_o=\frac{4L}{c_r}$ where c_r is the velocity of longitudinal waves in the rod.To a first approximation $c_p=c_r=5\times10^5$ cm/sec., where c_p is the velocity of longitudinal waves in the ground. If we substitute these values for $\frac{L}{c_p}$ in equation (14), the response of the strain seismometer in terms of the free period of the rod is $y=-\frac{T_o}{4}\frac{\delta\xi}{\delta t}$ approximately. If $\xi=a\sin\frac{2\pi}{T}t$, in which

T is the period of the wave, the maximum instantaneous value of the strain response is

$$y_m = \frac{a\pi T_o}{2T} \tag{15}$$

The maximum instantaneous response of a pendulum seismometer having a period T_o, short in comparison with the seismic wave period, is

$$Y_m = \frac{aT_o^2}{T^2} \tag{16}$$

If the two instruments have the same period, T_o, the ratio of the strain response to the pendulum response is

$$\frac{y_m}{Y_m} = \frac{\pi T}{2T_o} \text{ approximately} \tag{17}$$

As a concrete example, it will be assumed that $T_o = 1.5 \times 10^{-2}$ sec., which is approximately the value for the present strain instrument. With this value for T_o, the ratio of the strain response to that of a pendulum having the same period is for earth periods of 1 sec., $\frac{\pi}{2 \times 1.5 \times 10^{-2}} = 100$, approximately. With longer earth periods the ratio is still greater. Another way of expressing this difference between the two instruments is to note that the response of a strain seismometer with a period of 1.5×10^{-2} sec. to earth waves of 1-sec. period is equivalent to that of a pendulum having a period of 1.5×10^{-1} sec. approximately.

Theory of the Horizontal Electromagnetic Linear Strain Seismograph

Neglecting the directional factor, we find that the response of the horizontal linear strain seismometer to longitudinal apparent surface waves is, from equation (12),

$$y = -\frac{L}{c}\,\frac{\delta\xi}{\delta t}$$

The e.m.f. induced in the coils of the transducer is

$$E = k\frac{\delta y}{\delta t} = -\frac{Lk}{c}\,\frac{\delta^2\xi}{\delta t^2} \tag{18}$$

Neglecting the back m.m.f. of the transducer and also the reactances of the transducer and galvanometer coils, we may assume that the current is in phase with and proportional to the e.m.f. Hence the differential equation of a galvanometer connected to the transducer coils is

$$\frac{d^2\theta}{dt^2}+2\epsilon\frac{d\theta}{dt}+\omega_g{}^2\,\theta=\frac{gE}{mr} \tag{19}$$

where

θ = angular deflection of galvanometer in radians

ϵ = damping constant

$\omega_g = \frac{2\pi}{T_g}$, T_g = free period of galvanometer

g = electrodynamic constant of galvanometer—the product of the area of the coil, the number of turns, and the field strength

m = moment of inertia of galvanometer

r = the sum of the galvanometer and transducer resistances

The mechanical damping of the galvanometer is neglected in comparison with the electromagnetic damping. Setting

$$b=\frac{kgL}{mrc}$$

and introducing the value of E from equation (18) into (19), we find that the differential equation of the electromagnetic strain seismograph is

$$\frac{d^2\theta}{dt^2}+2\epsilon\frac{d\theta}{dt}+\omega_g{}^2\,\theta=-b\frac{\delta^2\xi}{\delta t^2} \tag{20}$$

Equation (20) discloses a most remarkable property of the electromagnetic strain seismograph, for it is evident by inspection that this equation is identical in form with the differential equation of the simple pendulum seismograph.[7] The significance of this identity is that the frequency response characteristic of an electromagnetic strain seismograph having a galvanometer with period T_g and damping constant ϵ is identical with that of a simple pendulum having the same period T_g and the same damping constant ϵ. Thus, for example, a strain seismograph with a critically damped galvanometer of 12-sec. period has a frequency response characteristic which is identical with that of a Milne-Shaw seismograph. With a critically damped galvanometer of 0.8-sec. period the strain-frequency characteristic is identical with that of the Wood-Anderson short-period torsion seismograph.

The electromagnetic strain seismograph offers several advantages over the equivalent simple pendulum seismograph:

(1) Higher magnification: for example, one of the instruments in routine operation at this Laboratory has a period of 0.2 sec. and an equivalent static

[7] k is positive or negative depending upon the polarity of the galvanometer connections.

magnification of 80,000. The highest magnification available for routine practice with simple pendulum instruments is approximately 3000 (Wood-Anderson torsion seismographs).

(2) Greater flexibility: instruments of a great variety of characteristics are available by merely changing galvanometers or suspensions.

(3) Constructional simplicity: the concentrically balanced galvanometer suspension system with electromagnetic damping is substituted for the eccentric system of all types of pendulum seismographs.

(4) Complete absence of response to earth tilt: the strain seismometer does not respond to tilt, and the transducer-galvanometer system does not respond to such slow movements. As a result of this absence of tilt sensitivity it has been possible to build, for routine operation, an instrument having the characteristics of a pendulum seismograph with a period of 34 sec., critical damping, and an equivalent static magnification of 100. Detailed descriptions of the various galvanometer combinations which have been tested will be given later.

The linear deflection of the galvanometer light spot is

$$z = 2A\theta \tag{21}$$

where A is the distance from the galvanometer lens to the recording drum. If we introduce the value of θ from (21) into equation (20) and set $V = 2Ab$, the resulting differential equation of the electromagnetic strain seismograph is

$$\frac{d^2z}{dt^2} + 2\epsilon\frac{dz}{dt} + \omega_g{}^2 z = -V\frac{\delta^2\xi}{\delta t^2} \tag{22}$$

The quantity $V = 2Ab = \dfrac{2AkgL}{mrc}$ is thus the equivalent pendulum static magnification of the electromagnetic strain seismograph. When $\xi = a \sin \omega t$, equation (22) is written

$$\frac{d^2z}{dt^2} + 2\epsilon\frac{dz}{dt} + \omega_g{}^2 z = Va\omega^2 \sin \omega t \tag{23}$$

Since equation (23) is identical with the well-known pendulum seismograph equation it is unnecessary to discuss the general solution here. The steady state solution is

$$z = \frac{Va\omega^2 \sin(\omega t + \delta)}{\sqrt{(\omega_g{}^2 - \omega^2)^2 + 4\epsilon^2\omega^2}} \tag{24}$$

$$\delta = \tan^{-1}\frac{2\epsilon\omega}{\omega^2 - \omega_g{}^2} \tag{25}$$

If we set

$$P = \frac{\omega^2}{\sqrt{(\omega_g{}^2 - \omega^2) + 4\epsilon^2\omega^2}} \tag{26}$$

equation (24) can be written

$$z = VPa\sin(\omega t+\delta) \tag{27}$$

P is thus the frequency characteristic of the electromagnetic strain seismograph (and of the pendulum seismograph). It may be expressed in another form. Thus if

$$\frac{\omega_g}{\omega} = u = \frac{T}{T_g} \text{ and } \frac{\epsilon}{\omega_g} = h$$

equation (26) may be written

$$P = \frac{1}{\sqrt{(u^2-1)^2+4h^2u^2}} \tag{28}$$

When the damping is critical, $h=1$, and P reduces to the simple form

$$P_1 = \frac{1}{u^2+1} \tag{29}$$

Figure 9 shows a graph of the function P for $h=1$ and $h=\frac{1}{2}\sqrt{2}=0.707$.

In the form

$$V = \frac{2AgkL}{mrc}$$

the expression for the equivalent static magnification contains the galvanometer constants g and m. These are usually unknown or difficult to determine and consequently it is desirable to substitute practical constants for them. Thus if D is the linear deflection in cm. of the galvanometer light spot for unit current (1 amp. or 1 e.m.u., say) then it is easy to show that

$$D = \frac{T_g^2 Ag}{2\pi^2 m} \tag{30}$$

in which $T_g = \frac{2\pi}{\omega_g}$ = the galvanometer period.

Hence

$$V = \frac{4\pi^2 kLD}{T_g^2 rc} \tag{31}$$

The constants D, T_g, and r may be obtained directly from manufacturer's specifications or from easy measurements. The response of the seismograph with any desired galvanometer can thus be readily calculated. It should be remembered that r is the sum of the galvanometer resistance and the transducer resistance. The value of the latter is chosen to conform with the desired damping constant h. If D is given in cm. deflection per amp., r should be expressed in ohms.

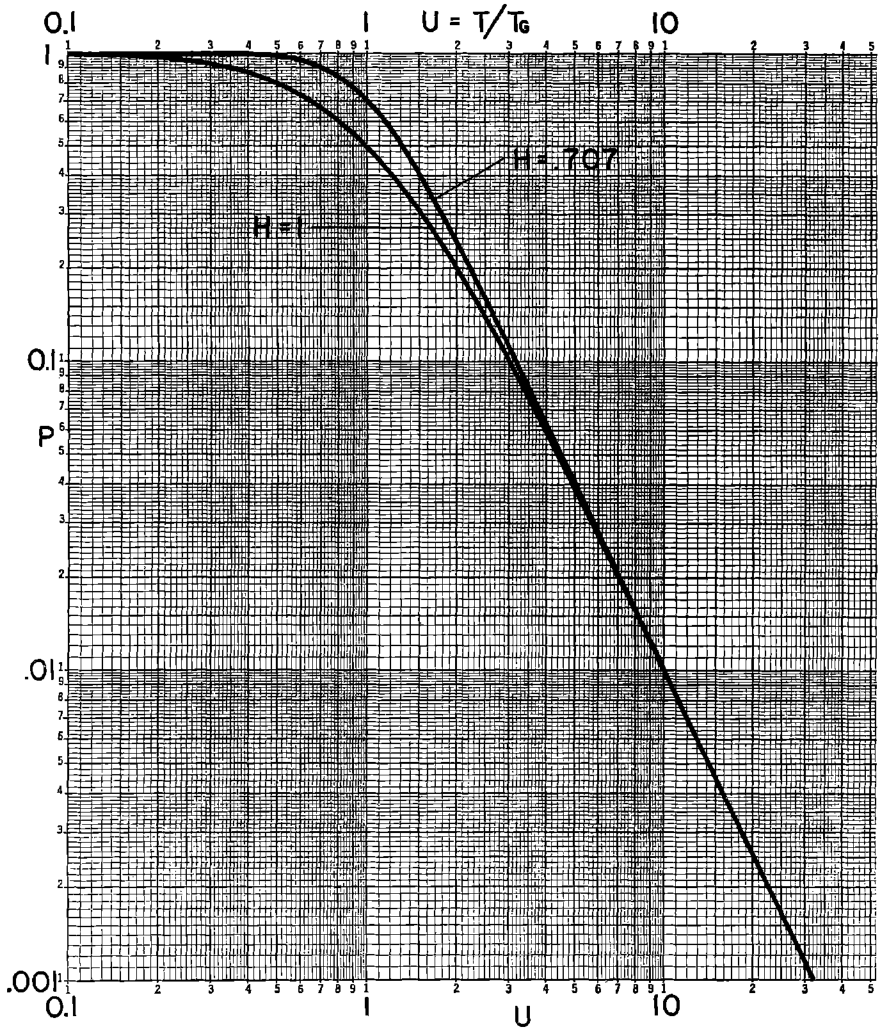


Fig. 9. Frequency response characteristics for electromagnetic linear strain seismograph with damping constants $h=1$ and $h=0.707$. These are also frequency response characteristics of a simple pendulum seismometer.

A rather wide variety of galvanometric combinations have been assembled for experimental study of the instrument's characteristics. Only three of the more important assemblies will be described, since the observed characteristics of all the combinations conformed closely with theoretical predictions. Usually two galvanometers of different characteristics are operated simultaneously. A combination which has been found very satisfactory for local earthquakes uses a galvanometer having a period of 0.25 sec. The galvanometer and its associated

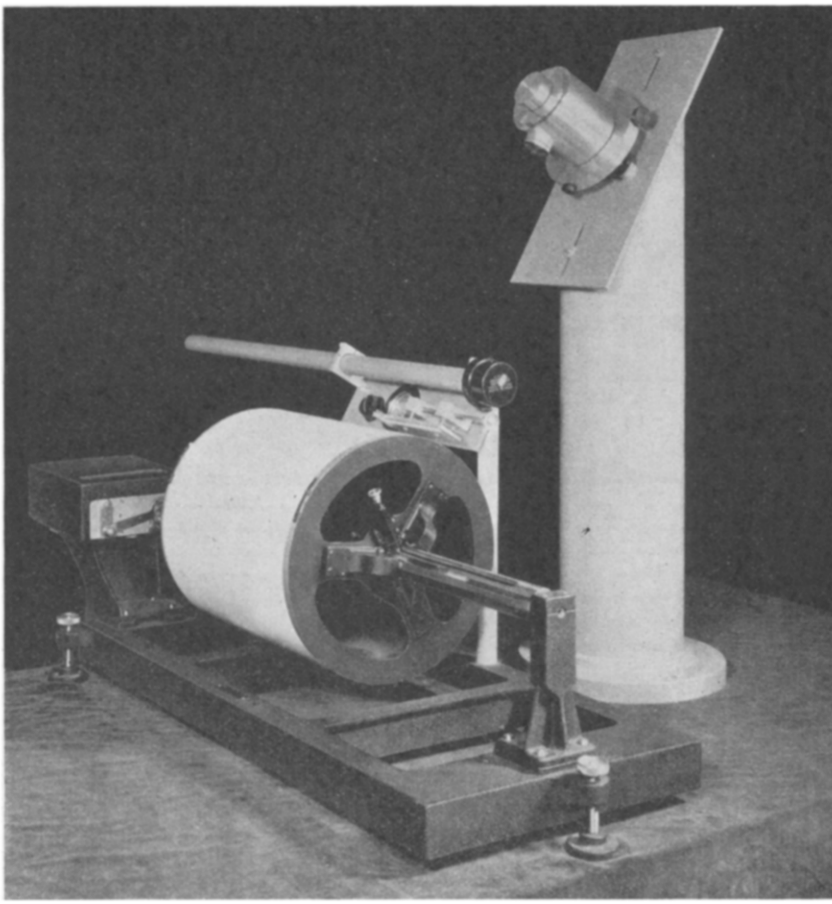


Fig. 10. Short-period galvanometer recording assembly.

recording apparatus are shown in figure 10.[8] The galvanometer pedestal is made of 6-in. iron pipe and is cut off at an angle of 30° so as to permit the recording beam to fall conveniently on the upper surface of the recording drum. This arrangement makes for easy inspection and adjustment of the light spot during operation. The galvanometer is held to the mounting plate by the tension of a stiff helical spring which hooks into the base of the galvanometer. This type of galvanometer mounting is sufficiently strong to withstand severe earthquakes, yet permits delicate adjustment of the leveling screws. The cylindrical lens near the drum is focused by a rack and pinion. The galvanometer lens (also a cylinder) is focused by sliding the mounting plate along the cut surface of the pedestal. An automotive type tail lamp mounted at the end of the long cylin-

[8] This galvanometer was designed jointly by Wm. Miller and the writer.

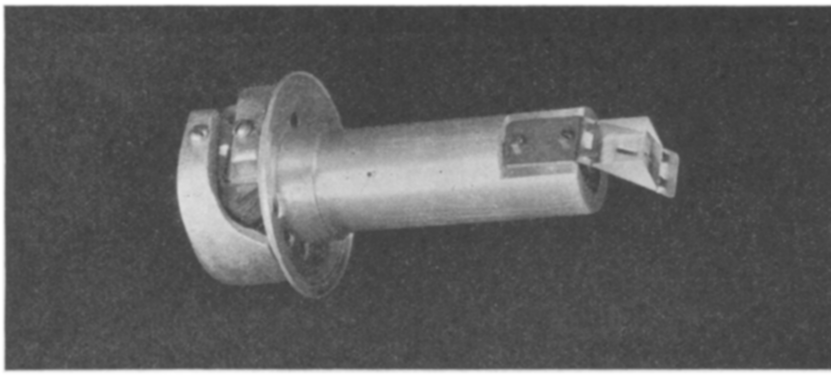


Fig. 11. Time-marker assembly.

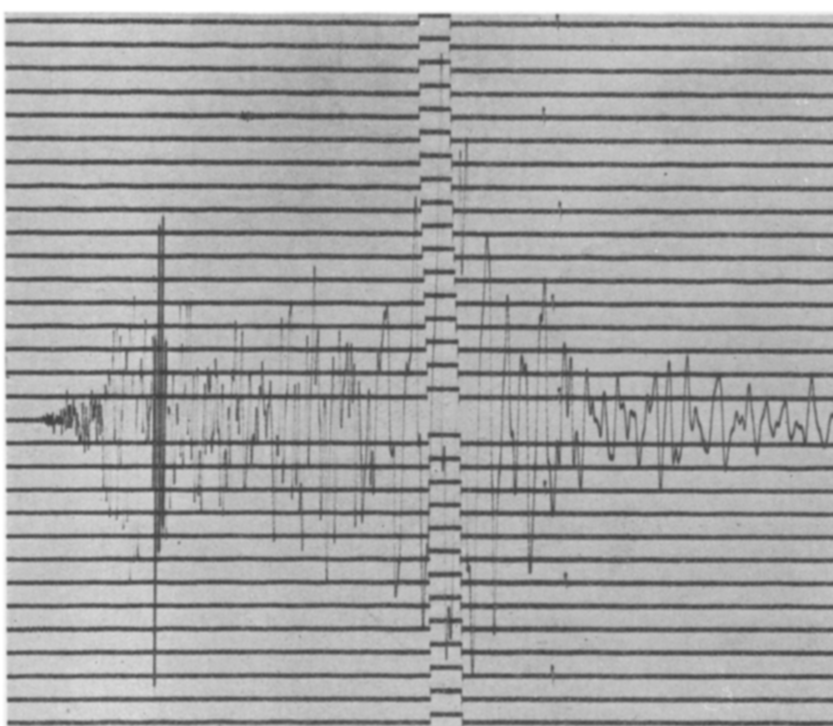


Fig. 12. Part of strain seismogram of local earthquake, $\Delta = 35$ km. approximately.

drical tube serves as light source. The light beam from the lamp is deflected at right angles toward the galvanometer mirror by a prism which is mounted within the tube. Time marks consist of 1-mm. deflections of the recording light spot and are produced by the slight bending of the flat spring mounting of the prism. The bending force is transmitted by a phosphor-bronze ribbon from the armature of a modified Baldwin telephone receiver. The period of the time

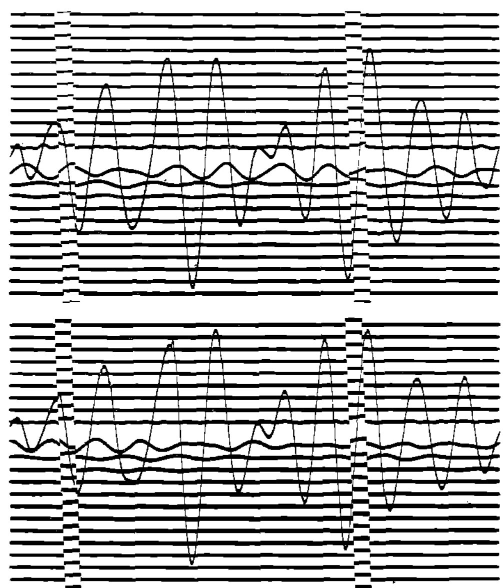


Fig. 13. Part of Rayleigh wave group of Baffin Bay earthquake of November 20, 1933 ($\Delta = 5100$ km.). A was recorded on N-S torsion seismograph, $T_o = 0.8$ sec.; B was recorded on N-S strain seismograph, $T_g = 0.2$ sec.

marker is approximately 1/50 sec. The operating current is 5 ma. at 5 volts. Besides its low power consumption and high precision, this type of time-marking mechanism has the further advantage of requiring no readjustment when the recording lamp is changed. Figure 11 shows a photograph of the time marker. Figure 12 is a copy of a portion of a seismogram written by this combination with an equivalent static magnification of 80,000.

Part of the Rayleigh wave group of the Baffin Bay earthquake of November 20, 1933, is shown in figure 13. Section A was written by the N-S short-period torsion seismograph ($T_o = 1$ sec. approximately) and B was written by the N-S strain seismograph with the galvanometer of 0.2 sec. period. Since the periods of the seismic waves are long in comparison to the instrument periods, the theoretical responses of the two instruments are proportional to the ground acceleration and consequently the two records should be identical. The fact

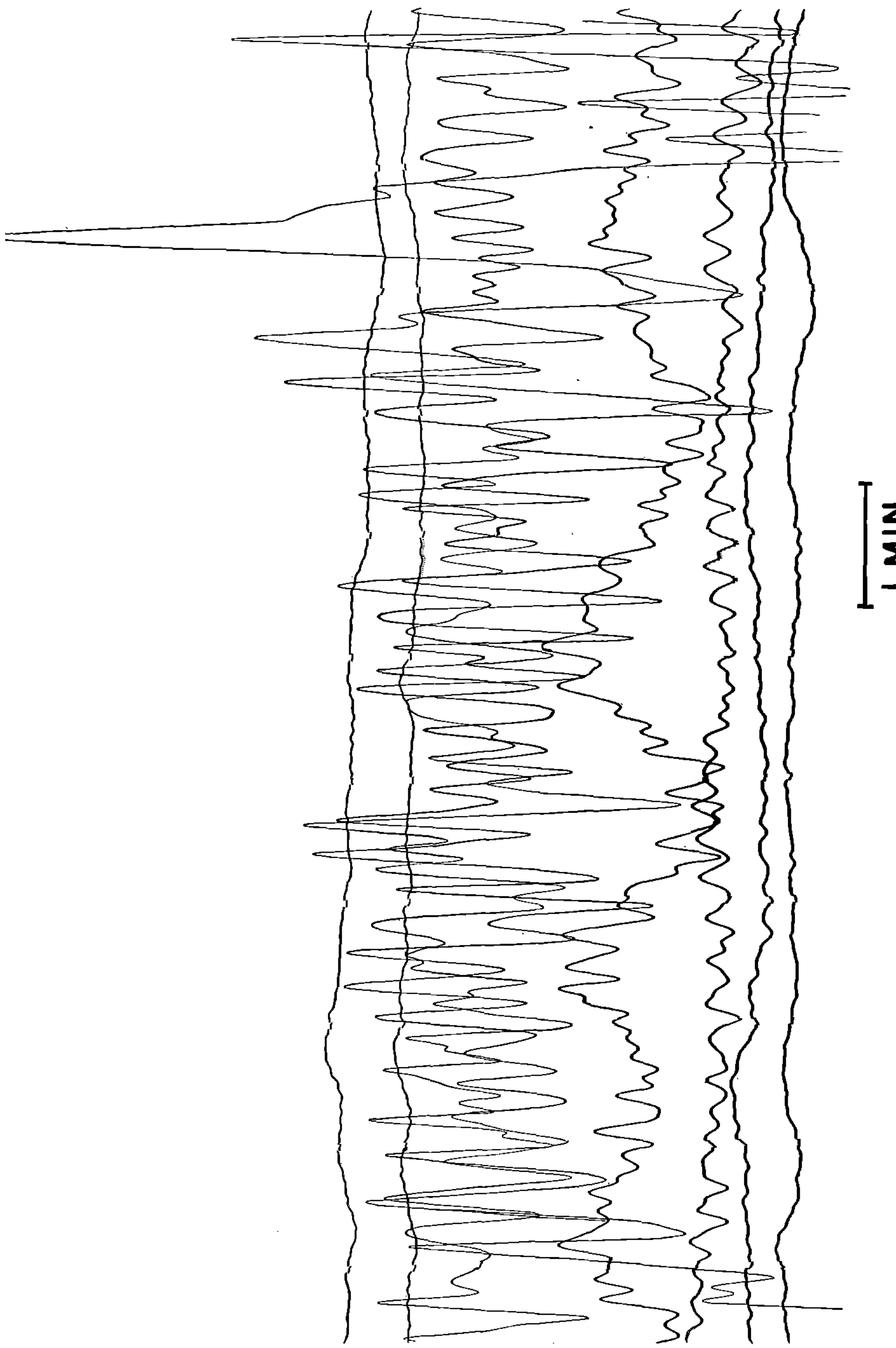


Fig. 14. Part of seismogram of Japanese earthquake of March 2, 1933 (Δ=8300 km.), recorded by long-period galvanometer (T_g=35 sec.) of strain seismograph. Note large S wave and very long surface waves.

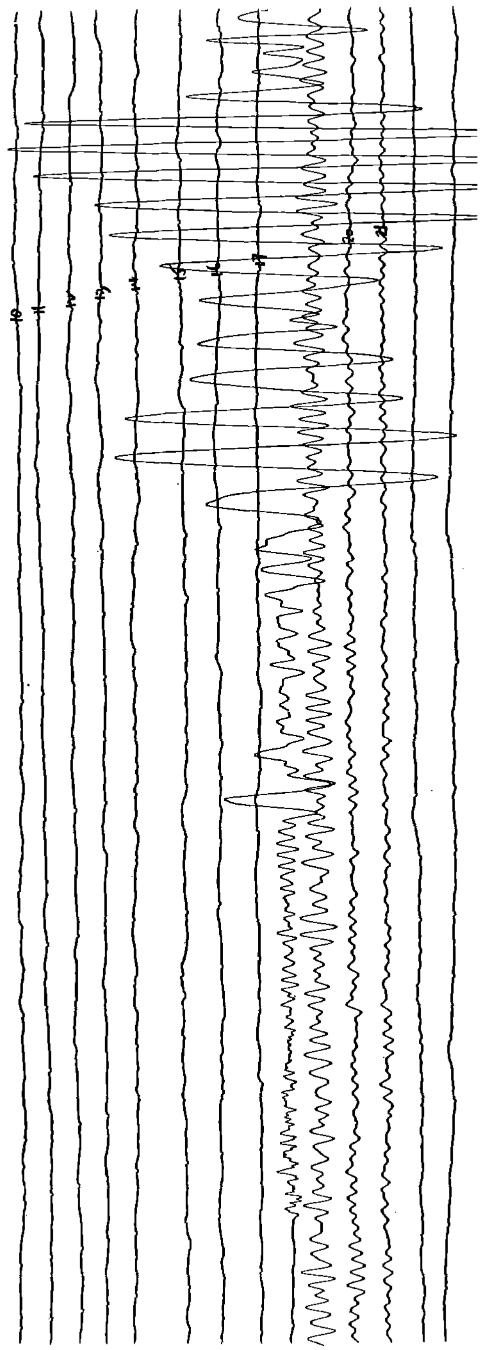


Fig. 15. Part of seismogram of Panama earthquake of July 18, 1934 ($\Delta = 4900$ km.), recorded by long-period galvanometer ($T_g = 35$ sec.) of strain seismograph.

that they are so very nearly the same is convincing evidence of the correctness of the assumptions concerning the behavior of the strain seismograph.[9]

Another useful combination is made with a galvanometer having a period of 1.3 sec. The frequency response characteristic of this combination is very nearly the same as that of the short-period torsion seismograph. For routine work it has been operated with an equivalent static magnification of 12,000.

A combination using a galvanometer with a period of 35 sec. has been found very satisfactory. With this galvanometer an equivalent static magnification of 100 is readily maintained in very stable form.[10] This combination is well suited for registration of teleseisms, especially those with very long waves. Figures 14 and 15 are copies of parts of teleseisms recorded with this instrument. The surface waves in figure 14 have a period of approximately 200 sec.

Vertical Component Strain Seismograph

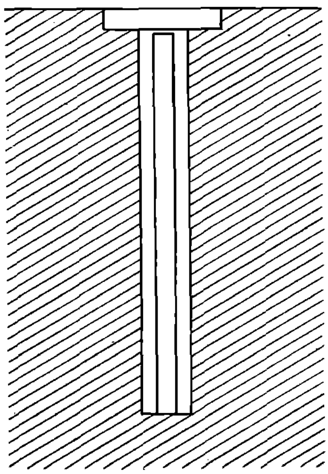
Although a vertical component strain seismograph has not yet been constructed, an outline of its theory will be given in addition to a description of a proposed structure. A possible form of vertical strain seismometer is shown in the diagram of figure 16. However, this type is not very satisfactory because it has a zero response for S waves traveling horizontally and for P waves traveling vertically. The S-wave response is zero, as a result of the fact that the motions of A and B are equal in phase and amplitude. The P-wave response is zero because the free surface of the earth is a node of strain for vertically incident waves.

By tilting the seismometer of figure 16 some 45°, as shown in figure 17, a partial response to vertical components is obtained for all waves except those of normal incidence. Such an instrument would be expensive to construct and is therefore not recommended.

Another possible instrument is shown in figure 18. If the rod is sufficiently rigid, the difference in vertical motion of the two piers may be utilized for the response. When the transverse vibration periods of the rod are short in comparison with the shortest significant seismic periods, the system behaves substantially as though the rod were infinitely rigid. Division of the rod into two sections as shown in figure 19 makes the rod periods much shorter for a given amount of structural material. An instrument of this type is to be built in the near future.

[9] During this test the equivalent pendulum magnification of the strain seismograph was reduced to approximately 50,000.

[10] In rainstorms, water leaks into the seismograph tunnel and renders the instrument inoperative.



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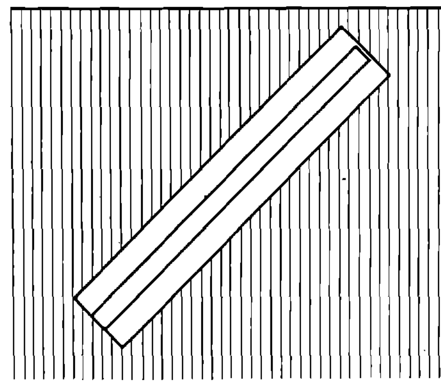

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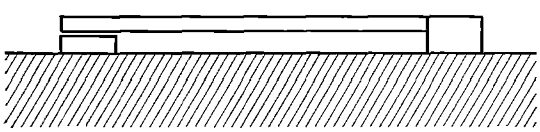

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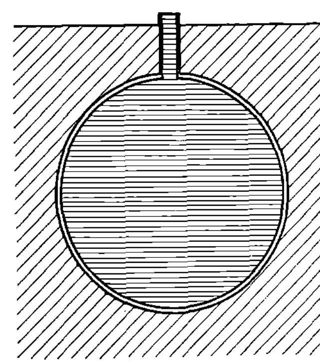

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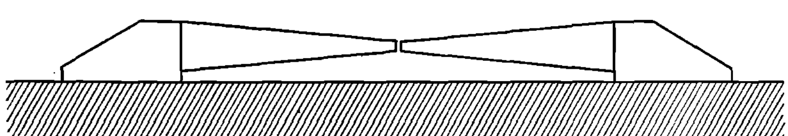

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Figs. 16–19. Schematic representations of vertical strain seismometers.

Fig. 16. Seismometer vertical. Fig. 17. Seismometer tilted. Fig. 18. Vertical seismometer with transversely rigid rod. Fig. 19. Vertical seismometer with two transversely rigid rods.

Fig. 20. Schematic representation of a volume strain or dilatation seismograph.

Theory of the Vertical Strain Seismograph

Let ζ be the vertical displacement of the ground at the point whose horizontal coördinate in the direction parallel to the rods is x. The difference in vertical displacement of two neighboring points of the ground whose coördinates are x and $x+dx$ is

$$\frac{\delta\zeta}{\delta x}dx$$

and consequently the difference in vertical elevation of the two piers distant L from each other is

$$y=\int_0^L \frac{\delta\zeta}{\delta x}dx \tag{32}$$

If the seismic waves are plane, and if L is small in comparison with the shortest significant seismic wave length, $\frac{\delta\zeta}{\delta x}$ may be considered constant and equation (32) becomes

$$y=L\frac{\delta\zeta}{\delta x} \tag{33}$$

It is clear that the instrument responds to the vertical transverse component of the apparent surface waves. Transverse apparent waves are generated by both P and S body waves which are incident at the surface. If r is the coördinate along the line of propagation of an apparent surface wave and a is the angle between r and the direction of L, we may write

$$\frac{\delta\zeta}{\delta x}=\frac{\delta\zeta}{\delta r}\cos a$$

Hence

$$y=L\cos a\frac{\delta\zeta}{\delta r} \tag{34}$$

If ζ is expressed as a plane wave of the form $\phi\left(t-\frac{r}{c}\right)$ we have the relation

$$\frac{\delta\zeta}{\delta r}=-\frac{1}{c}\frac{\delta\zeta}{\delta t}$$

and the response of the vertical strain seismometer is therefore

$$y=-\frac{L}{c}\cos a\frac{\delta\zeta}{\delta t} \tag{35}$$

The output e.m.f. from the transducer is

$$k\frac{\delta y}{\delta t}=-\frac{Lk}{c}\cos a\frac{\delta^2\zeta}{\delta t^2} \tag{36}$$

It will be noted that for this form of vertical seismometer the response varies with the azimuth of the incoming waves. Furthermore, its response to P waves is quite small because those which have large vertical components arrive at the surface with steep angles of incidence and thus produce apparent surface waves of high velocity.

Three-Component Strain Seismometer

With a single instrument of the transverse rigid bar type shown in figure 19, all three components of the ground motion may be derived by means of three properly arranged transducers. The horizontal component parallel to the rods is derived from the relative longitudinal motion of the rods. The horizontal component perpendicular to the rods is derived from the relative horizontal transverse motion of the rods. The vertical component is derived from the relative transverse vertical motion of the rods as described in the preceding paragraph.

Dilatation Seismograph

Another type of instrument which is suggested by the linear strain seismograph is a device which may be designated a volume strain or dilatation seismograph. In the form which the writer proposes to build, a container of any convenient shape is buried preferably in rock in such a manner that firm contact with the surrounding medium is maintained (see fig. 20). The container is filled with a liquid and sealed with a diaphragm or silphon. A side tube of capillary dimensions equalizes slow-pressure variations brought about by temperature or barometric changes or other causes. Volume strains of the ground change the capacity of the container and thus produce movements of the diaphragm which are recorded by means of an electromagnetic transducer and galvanometer.

Theory of the Dilatation Seismograph

Let the condensation be

$$\sigma = \frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}$$

in which u, v, and w are the ground displacements parallel to the coördinate axes, x, y, and z, respectively. Consider an elementary rectangular parallelopiped with sides dx, dy, and dz. Its quiescent volume is

$$dV = dx\, dy\, dz$$

When strained, its volume is $dV(1+\sigma)$. Thus the volume increment resulting from strain is σdV. If V_o is the volume of the unstrained container, its total volume increment is

$$S = \int_0^{V_o} \sigma dV \tag{37}$$

When the seismic disturbance consists of plane waves which are long in comparison with the dimensions of the container, σ is constant, and equation (37) is written

$$S = \sigma V_o$$

Thus a strain σ forces a quantity of liquid S into the outlet pipe and displaces the diaphragm through a distance

$$s = \frac{S}{A}$$

A is the effective area of the diaphragm. Hence,

$$s = \frac{V_o \sigma}{A} \tag{38}$$

The e.m.f. induced in the transducer coils is

$$k\frac{\delta s}{\delta t} = \frac{kV_o}{A} \frac{\delta \sigma}{\delta t} \tag{39}$$

If there be considered a plane condensational wave propagating horizontally with the form

$$u = \phi\left(t - \frac{x}{c}\right)$$

it can be shown that

$$\sigma = \frac{\delta u}{\delta x}$$

and

$$\frac{\delta u}{\delta x} = -\frac{1}{c} \frac{\delta u}{\delta t}$$

Consequently

$$s = -\frac{V_o}{Ac} \frac{\delta u}{\delta t}$$

For this case, therefore, the response of the dilatation seismometer is identical in type with that of the linear strain seismometer except that it is independent of the azimuth of the incoming waves. In shear waves σ is zero and consequently they produce no response. However, when they are incident on the surface with angles between $\pi/2$ and zero, shear waves produce reflected longitudinal waves and the instrument responds to these in the same manner as to the primary longitudinal waves. Thus it is impossible, completely, to separate longitudinal and shear waves by means of instruments of this type.

APPENDIX

Theory of the Undamped Rod

The well-known differential equation for longitudinal waves in a rod is

$$\frac{\delta^2 u}{\delta t^2}=c^2\frac{\delta^2 u}{\delta x^2}$$

when u is the displacement, x the coördinate parallel to the rod, and c^2 a constant.

Assume a solution of the form

$$u=\left[A\cos\frac{\omega x}{c}+B\sin\frac{\omega x}{c}\right]\sin\omega t$$

For a rod clamped at one end and free at the other end the boundary conditions are $\frac{\delta u}{\delta x}=0$ at the free end where $x=L$, and $u=a\sin\omega t$ at the clamped end, where the ground displacement is $a\sin\omega t$.

From these boundary conditions we find that

$$A=a$$

and

$$B=a\tan\frac{\omega L}{c}$$

Hence

$$u=a\cos\frac{\omega}{c}(L-x)\sec\frac{\omega L}{c}\sin\omega t$$

The displacement at the free end where $x=L$ is therefore

$$u_f=a\sin\omega t\sec\frac{\omega L}{c}$$

With respect to the seismometer rod $\frac{L}{c}$ is small (*ca.* 1/100), so that for long waves $\sec\frac{\omega L}{c}$ is approximately equal to 1. For waves having a period of 1 sec. $\sec\frac{\omega L}{c}$ is approximately 1.0003. It is clear therefore that for long waves the displacement of the free end of the rod is substantially the same as that of the clamped end.