ON THE SIZE OF THE STRAINED REGION PRIOR TO AN EXTREME EARTHQUAKE*

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ABSTRACT

A previous formula of the author's has been adapted to give the volume V of the region in which the material is near breaking-point, just prior to an extreme earthquake, in terms of the released energy and breaking-strength near the focus. In the light of recent earthquake energy-magnitude data of Gutenberg, Richter, and Benioff, it is inferred that V is at least of the order of the volume of a sphere of radius 25 km. Comparison with other estimates of the size of strained regions in earthquakes suggests that V may possibly reach the order of the volume of a sphere of radius 50 km. It is suggested that a factor in the occurrence of extreme earthquakes is the existence, in the zone of strain, of strength rather greater than in the majority of major earthquakes of magnitude one unit or more less than the extreme value.

1. Let E be the energy released in seismic waves in an earthquake, and let qE be the distortional strain energy in the focal region just prior to the occurrence of the earthquake. Let V denote the volume of the strained region, assuming the material to have been at breaking-point throughout the whole volume. Then if S and μ are the Mises strength and the rigidity assumed constant throughout V, it follows, from work in a previous paper,[1] that

$$12 \, q\mu E = S^2 V . \tag{1}$$

On plausible assumptions, the formula (1) sets an upper bound to the value of E/S^2 in the greatest earthquakes. In paper 1, this result was considered in relation to the tentative earthquake energy-magnitude formula of Gutenberg and Richter,[2] namely,

$$\log_{10} E = 12 + 1.8M, \tag{2}$$

which gives an energy of about 10^{27} ergs for the strongest earthquakes. The broad conclusion was drawn that either the energy E as given by (2) is appreciably too high in large earthquakes, or that the fracture-resisting strength of the material in the focal region of the greatest deep-focus earthquakes is appreciably greater than some writers have hitherto assumed.

Professors Gutenberg, Richter, and Benioff have since kindly informed me of recent work in course of publication, according to which the formula

$$\log_{10} E = 11 + 1.6M \tag{3}$$

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[1] K. E. Bullen, "On Strain Energy and Strength in the Earth's Upper Mantle," *Trans. Am. Geophys. Union*, 34:107–109 (1953). This paper will be referred to as paper 1. (In paper 1, the value of the strength at breaking-point was taken as $\sqrt{(\Sigma\Sigma P^2_{ij})}$, where P_{ij} denotes the deviatoric stress. This was sufficiently accurate for the broad deductions made in paper 1; but for the purposes of the present paper it is desirable to use the Mises function, equivalent to $\sqrt{(\Sigma\Sigma \, 3P^2_{ij})}$, and hence the numerical factor on the left side of (1) is 12 instead of 4.)

[2] B. Gutenberg and C. F. Richter, *Seismicity of the Earth and Associated Phenomena* (Princeton University Press, 1951), p. 10.

just before the earthquake. Thus the agreement between (4) and the final estimate of the last paragraph is as good as can be expected.

4. It is next of interest to compare the result (4) further with an estimate by Benioff (in course of publication and kindly made available by Professor Benioff to the author) of the volume of the zone of appreciable strain for the Kern County earthquake of July 21, 1952. Professor Benioff estimates the surface area of the zone to be 10^{13} cm.^{2}, and, assuming the zone to extend vertically downward from the outside surface to the Mohorovičić discontinuity at a depth of 35 km., gives, as a rough approximation to its volume, 7×10^{19} cm.^{3}

This figure is remarkably close to the result (4). But there are two factors that have to be taken into account in the comparison: (i) it is to be expected that the material would have been rather short of breaking-point before the Kern County earthquake in a large fraction of the volume found by Benioff; (ii) the magnitude of this earthquake was one full unit short of the extreme value of 8.6. The effect of (i) would be that V as defined in section 2 above would be perhaps only about one-quarter or so of Benioff's volume. The effect of (ii) would be in the opposite direction, in that increasing M by unity would by (3) entail increasing E by a factor of 40. Since V is proportional to E in (1), the net effect of (i) and (ii) would be a discrepancy of the order of a factor of 10 between Benioff's result and (4).

In all the circumstances, the discrepancy must still be regarded as quite small. Even so, there is a fairly ready way of reducing it. This is to assume in the zone of strain of the Kern County earthquake a lower value of the strength S than 10^9 dyn/cm.^{2} In relation to the formula (1), a reduction by a factor of one-third would be formally sufficient. Another section of Benioff's investigation independently indicates that the average strength in the zone of strain was less than 10^9 dyn/cm.^{2} (Benioff in fact would put the strength as appreciably less than one-third of this.)

5. Further comparison may be made with an estimate of Jeffreys[5] of the size of an earthquake focus for the case in which the energy is released by slipping along a plane fault face. Taking the linear dimensions of the area in which slipping occurs at a single stage to be l km., Jeffreys estimates that, for an ordinary fault in which pseudotachylyte is not formed, the released energy would be roughly equal to Kl^2 ergs, where $K = 4 \times 10^{19}$, so that for an earthquake in which $E = 10^{22}$ ergs, l would be 16 km.

Direct comparison of (4) with the method of Jeffreys is probably not permissible since in extreme earthquakes there would presumably be quantities of pseudotachylyte formed, in which case the requisite value of K would be appreciably larger. (The value of K is proportional to θ^2, where θ is the rise in temperature at the fault during the slip; the value $K = 4 \times 10^{19}$ assumes $\theta = 1,000°$.) It is therefore necessary, in attempting a comparison, to keep to the value $E = 10^{22}$ ergs used by Jeffreys. Using this value and other data as in section 2 would give V equal to the volume of a cube of side-length 5 km.

The discrepancy between the two indicated linear dimensions of 16 km. and 5 km. can be partly accounted for by assuming that the dimension of the originally strained zone at right angles to the fault face would be appreciably less than 16 km. But there is the suggestion of a residual discrepancy, which, in keeping with

[5] H. Jeffreys, *The Earth*, 3rd ed. (Cambridge University Press, 1952), p. 341.

Benioff's result, suggests that for most earthquakes which are one magnitude or more short of the extreme value a less value of S than 10^9 dyn/cm.^{2} is relevant.

6. The discrepancies between the results of using (1) and the results in sections 3, 4, and 5 above are not great considering the tentative character of parts of the data used. The fact that several very distinct approaches to estimating the size of the strained region should agree within a factor of order about 10 is in itself significant, and gives good support to (4) as providing a lower limit to the volume of the heavily strained zone in an extreme earthquake.

A question remains, namely, whether on account of the indications of the work of Jeffreys and Benioff the estimate in (4) should be raised by using in (1) a value of S less than 10^9 dyn/cm.^{2} The investigations of Jeffreys[6] on the stress differences needed to support the earth's mountain systems imply that the strength exceeds 10^9 dyn/cm.^{2} at least in certain parts of the crust. In the majority of earthquakes of magnitude say $7\frac{1}{2}$ or less, and in particular the Kern County shock, it is likely that earlier seismic activity or other cause in the affected region has produced weaknesses which make the mean strength appreciably less than this figure. In the case of extreme earthquakes, on the other hand, the great strain energy accumulated may well be a consequence of the fact that the strength in the focal region is greater than in many other seismically active regions; a strength less than 10^9 dyn/cm.^{2} may well force the occurrence of earthquakes with magnitudes appreciably less than the extreme figure of 8.6. For example, it may be significant that the two great Assam earthquakes of 1897 and 1950 occurred in the vicinity of the Himalayas where Jeffreys computes the strength to be 1.6×10^9 dyn/cm.^{2} In these circumstances it would appear best not to raise the minimum estimate of the volume of the strained region provided by (4).

Some doubt remains concerning the best value to take for q, but the uncertainty in q is not likely to affect the calculated linear dimensions of the strained region by more than about 20 per cent.

The essential conclusion of the present paper is, then, that the volume of the zone in which the material is near breaking-point in an extreme earthquake is at least of the order of the volume of a sphere of radius 25 km. Allowing for uncertainties in the assumed values of q, E, μ and S, and taking into account the indications from other evidence, it is possible that the volume may reach that of a sphere of radius 50 km. It is probably legitimate to apply (4) to the case of less extreme earthquakes, but it would appear that the value taken for S should then be usually less than 10^9 dyn/cm.^{2}

[6] H. Jeffreys, *The Earth*, 3rd ed. (Cambridge University Press, 1952), p. 196.

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