

THE PHYSICAL SIGNIFICANCE OF "SHIFTED" GRADIENTS AND THE
LOCATION OF THE SOLAR WIND TERMINATION SHOCK

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Abstract

We use observations of anomalous cosmic-ray (ACR) oxygen from the cosmic-ray experiments on the Voyager 2 (V2) and Pioneer 10 (P10) spacecraft during 1985-1988 to investigate the physical significance of the "shifted" radial gradient. The "shifted" gradient is computed by comparing fluxes measured at different times at the two spacecraft to "correct" for the propagation delay of solar modulation. We use a simple model of particle propagation in which the flow is inwards along the wavy neutral sheet to suggest that for $q_A < 0$ near solar minimum conditions the "shifted" gradient is a measure of the gradient near the solar wind termination shock. This simple model leads to an estimate of the location of the termination shock at 55_{-5}^{+9} AU, suggesting that the shock could be encountered within this decade.

Introduction. Average radial gradients of cosmic rays in the outer heliosphere are generally computed from the logarithmic ratio of particle fluxes at two spacecraft divided by their separation distance. This radial gradient may be either "instantaneous" or "shifted", depending on whether the particle fluxes being compared were observed at the same or at different times. Both kinds of gradients have been used in the literature in various studies (see, e.g., Lockwood and Webber, 1984). The shifted gradient is often used to "correct" for the propagation of solar modulation outwards from the sun at the solar wind speed.

However, the particles are diffusing much faster through the interplanetary medium than the solar wind speed and therefore their diffusive flow is governed by the instantaneous particle densities. Therefore the physical significance of the "shifted" gradient is not readily apparent. In this paper we explore this question in the context of a simple model in which the radial gradient is a function of the tilt of the current sheet (for $q_A < 0$, appropriate for the 1987 solar minimum time period). We have used this model previously to explain the observed time variation of the radial gradient of ACR oxygen in the outer heliosphere during 1984-89 (Cummings et al., 1990).

Observations. The fluxes of ACR oxygen at V2 and P10 in the energy range 7.1 - 17.1 MeV/nuc are shown in Fig. 1. In Fig. 2 we compare the time variation of the instantaneous radial gradient (Fig. 2c) and the separation distance between the spacecraft with the variation in the average tilt angle of the neutral sheet between V2 and P10. The tilt angle midway between V2 and P10 is shown in Fig. 2a and was estimated from the near-sun neutral sheet tilt value (Hoeksema, 1990), taking into account the delay time corresponding to an outward propagation at 400 km/sec. The separation distance, both radially and along the neutral sheet, is shown in Fig. 2b.

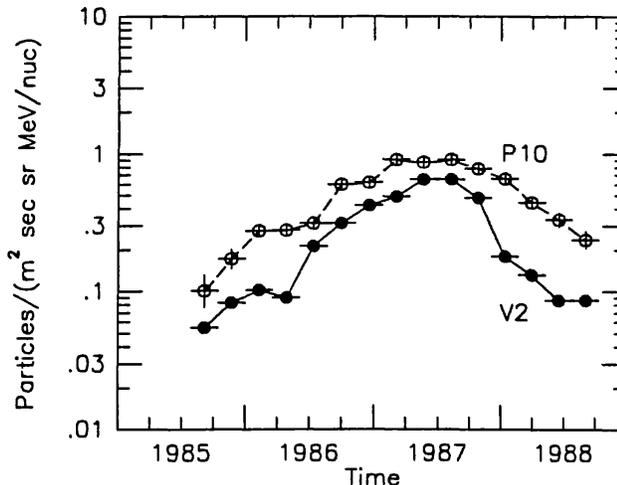


Figure 1. 78-day average fluxes of ACR oxygen at P10 and V2 versus time. The energy interval is 7.1 - 17.1 MeV/nuc.

The strong correlation of the separation distance along the neutral sheet with the radial gradient (for tilt angles $\Theta \lesssim 30^\circ$) led Cummings et al. (1990) to adopt a model in which the differential gradient is specified along the neutral sheet. In this model positively charged particles drift and diffuse inwards along the neutral sheet during the $qA < 0$ portion of the solar cycle (Jokipii et al., 1977). The time variation of the tilt angle causes a corresponding variation in the distance between the two spacecraft along the neutral sheet, leading to a variation in the P10/V2 intensity ratio, and hence to a variation in the measured radial gradient given by $G_r = \ln(f_{P10}/f_{V2})/(r_{P10}-r_{V2})$, where f is flux and r is heliocentric radial position. In this model, the differential gradient along the neutral sheet is assumed to be given by $\frac{1}{f} \frac{\partial f}{\partial s} = \frac{C}{r}$, where C is a constant and s is distance along the neutral sheet. This model for a wavy neutral sheet is a consequence of the relationship derived by Levy (1978) for a flat neutral sheet, assuming $\kappa_1 \propto r$ and the latitudinal gradient is independent of r . The solid line in Fig. 2c shows the radial gradient computed from

$$G_r = \frac{\int_{r_{V2}}^{r_{P10}} \frac{1}{f} \frac{\partial f}{\partial r} dr}{(r_{P10} - r_{V2})} = \frac{\int_{r_{V2}}^{r_{P10}} \frac{1}{f} \frac{\partial f}{\partial s} ds}{(r_{P10} - r_{V2})} = \frac{\int_{r_{V2}}^{r_{P10}} \frac{C}{r} ds}{(r_{P10} - r_{V2})} \quad (1)$$

(See Cummings et al., 1990, for a method of evaluating this integral.) The parameter C was chosen to be 0.21, corresponding to a gradient of 21%/AU at 1 AU, and the wavelength of the neutral sheet $\lambda = 6.3$ AU. Note that from Fig. 2b the radial separation distance $r_{P10} - r_{V2}$ (hereinafter ΔR) is ~ 18 AU throughout the period.

We used 78 day intervals in Fig. 2c to facilitate the calculation of "shifted" radial gradients, since 78 days is approximately the propagation time of the solar wind between V2 and P10 during this time period. In Fig. 3 we show the measured instantaneous and shifted radial gradients.

Discussion. Since in our model the particles are propagating inwards along the neutral sheet from the solar wind termination shock, the "shifted" radial gradient (G_r) may be expressed in the following way:

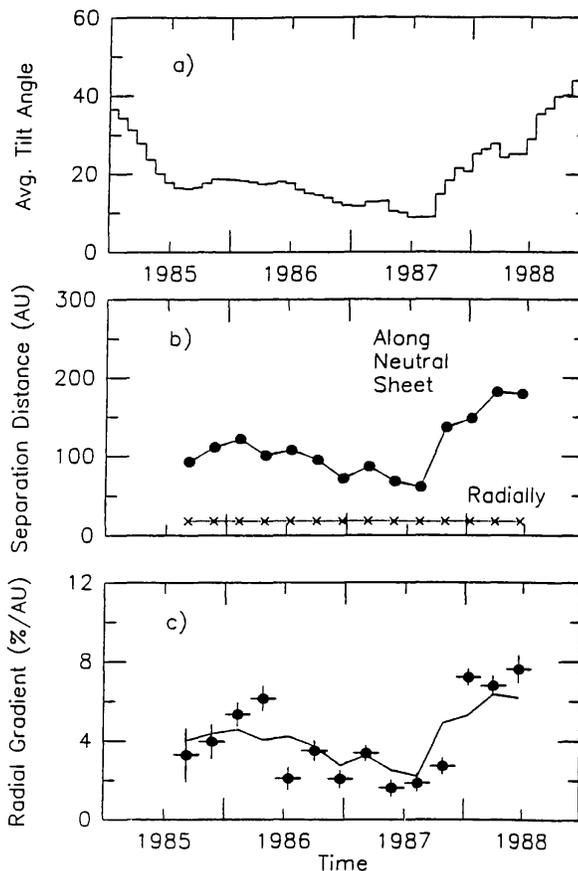


Figure 2. (a) Estimated tilt of neutral sheet between V2 and P10 versus time. (b) V2 - P10 separation distance both radially (\times) and along the neutral sheet (\bullet). (c) Instantaneous radial gradient between V2 and P10 of ACR oxygen in the energy interval 7.1 - 17.1 MeV/nuc (filled circles). The solid line is calculated from a model as described in the text.

$$G_r^* \cdot \Delta R = \int_{r_{V2}}^{R_S - \Delta R} \frac{C ds}{r} \Big|_{t_0} + \int_{R_S - \Delta R}^{R_S} \frac{C ds}{r} \Big|_{t_0} - \int_{r_{P10}}^{R_S} \frac{C ds}{r} \Big|_{t_0 + 78 \text{ days}} \quad (2)$$

where the integral is along the neutral sheet and R_S is the heliocentric radial position of the solar wind termination shock (the assumed source of the ACR nuclei). The principal quantity that varies with time is the pathlength along the neutral sheet between the two spacecraft due to the time dependence of the tilt (see Fig. 2a and b).

In the outer heliosphere, where $r \gg 1$ AU, the distance along the neutral sheet between two points at radial distances R and $R + \Delta R$ scales linearly with R . Thus for a

given tilt and neutral sheet wavelength λ , $\frac{\int_R^{R+\Delta R} \frac{ds}{r}}{\Delta R}$ is independent of R . (The integral does depend on the tilt angle Θ and can be approximated by $k \cdot \Theta + m$ for $\Theta \gtrsim 5^\circ$, where k and m are constants.) The effect of this is that the first and third integrals in Eq. 2 cancel because they are over the same part of the neutral sheet.

This is illustrated in Fig. 4 where we show two hypothetical "snapshots" of the neutral sheet. In this example we assume that R_S is at 100 AU and is fixed in time. The lower curve shows the neutral sheet 78 days after the upper curve. The heavy solid curves represent the same part of the neutral sheet, taking into account its propagation at the solar wind speed. The first integral of Eq. 2 is over the heavy solid upper curve, which is equal to the third integral which is over the heavy solid lower curve. Thus the result of Eq. 2 is the second integral, which is over the dashed portion of the upper curve in Fig. 4:

$$G_r^*(\text{AU}^{-1}) = \frac{\int_{R_S - \Delta R}^{R_S} \frac{C ds}{r} \Big|_{t_0}}{\Delta R} \approx C(1.05 \cdot \Theta^* + 0.94) \text{ for } 5^\circ \lesssim \Theta^* \lesssim 30^\circ \quad (3)$$

where Θ^* is the average tilt angle over $R_S - \Delta R$ to R_S , i.e., over the last 18 AU inside the termination shock. The last step in Eq. 3 was determined empirically for $\lambda = 6.3$ AU. Thus within the context of our simple model, the shifted gradient is the gradient in the vicinity of the termination shock at $t = t_0$.

Using Eq. 3 with $C = 0.21$ and the observed shifted gradients, we have inferred the tilt angle near the termination shock, Θ^* , for each epoch of 78 days. In deriving the appropriate errors, we added an estimated 5.4° systematic uncertainty in quadrature to the statistical error. This value was estimated by using Eq. 3 (with G_r for G_r^* and Θ for Θ^*) to estimate the tilt angle, Θ , between V2 and P10 for each epoch and comparing it to the "observed" values (Fig. 2a).

The inferred tilt angles near the termination shock for 14 epochs are shown in Fig. 5, together with a curve which represents the Hoeksema tilt angle data shifted to 46 AU, the best-fit value. Since the data points

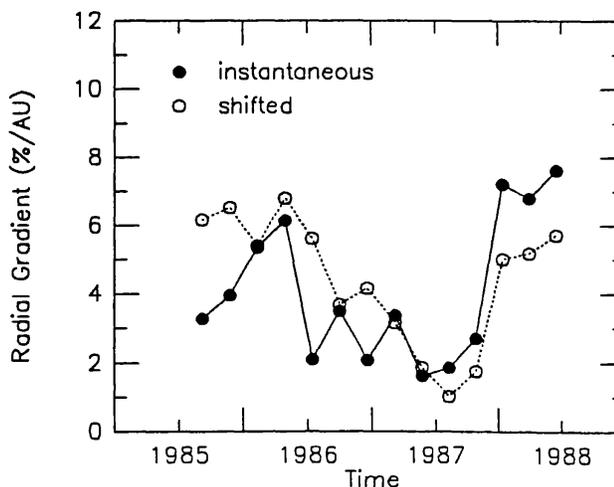


Figure 3. Instantaneous and shifted radial gradients of ACR oxygen between V2 and P10 versus time.

represent the tilt angle ~ 9 AU inside the shock, the inferred shock position is ~ 55 AU. The range of shock positions corresponding to the formal uncertainty (1σ) in the least-squares fit is 50 - 64 AU. Fig. 6 shows χ^2 as a function of shock distance. P10 is already at ~ 53 AU and both V1 and P10 reach 64 AU in 1996. V2 is ~ 10 AU behind V1 and is traveling a little slower so that it reaches 64 AU in 2001. Although additional uncertainty is inherent in the application of such a simple model, it appears reasonable that the termination shock is close enough to be encountered before the turn of the century when P10 and V1 will be at ~ 76 AU.

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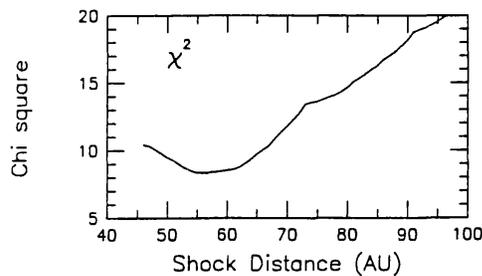


Figure 6. χ^2 computed from the data in Fig. 5.

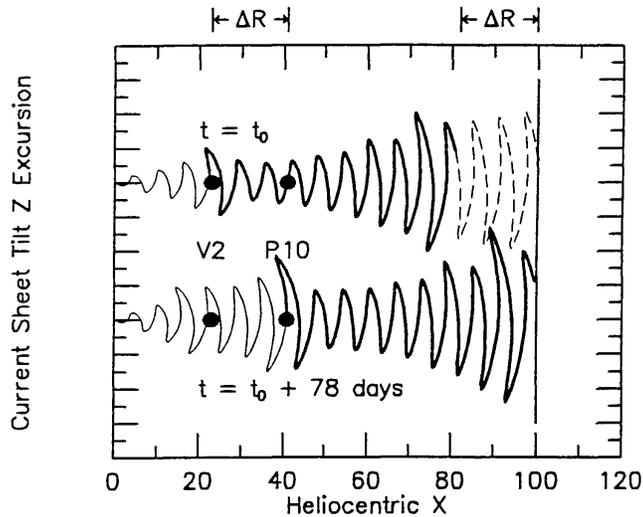


Figure 4. Schematic cross-section of wavy neutral sheet at two different times. The heavy lines represent the same portion of the sheet, taking into account propagation. The vertical line is the assumed shock position. The integral $\int ds/r$ over a portion of the upper curve is equal to the integral over the corresponding portion of the lower curve, e.g., over the heavy portions of the two curves. See text for discussion.

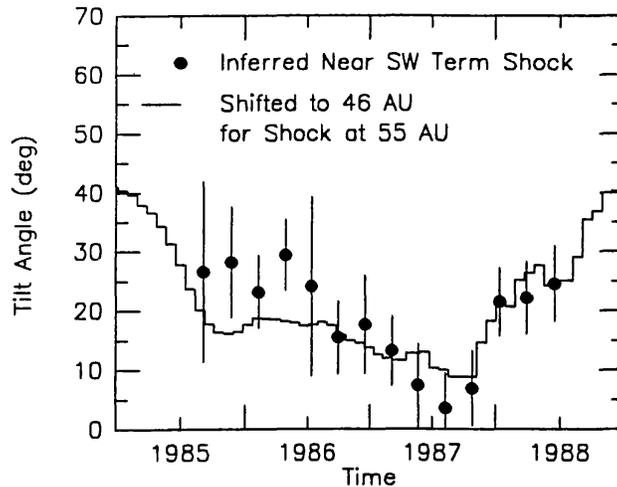


Figure 5. Tilt angles inferred near the solar wind termination shock (filled circles) and compared with the Hoeksema data shifted to 46 AU at 400 km/sec (histogram). The inferred tilt angles correspond to a position ~ 9 AU inside the shock; thus the inferred shock position is ~ 55 AU.