

CORRIGENDUM

CORRIGENDUM TO “COMMUNICATION AND EQUILIBRIUM IN
DISCONTINUOUS GAMES OF INCOMPLETE INFORMATION”
(*Econometrica*, Vol. 70, No. 5, September 2002, 1711–1740)

BY MATTHEW O. JACKSON, LEO K. SIMON, JEROEN M. SWINKELS,
AND WILLIAM R. ZAME

WE THANK ANDREAS BLUME for pointing out an error in a claim that we made in Example 1, and Eric Balder for pointing out that the proof of a lemma contains an assertion that is far from obvious and that two steps of the main proof contain typos and misstatements that make the steps difficult to follow. We offer corrections here.

In Example 1 we claim that no equilibrium exists for any type-independent tie-breaking rule. Andreas Blume has pointed out the following counterexample to that claim. Consider a tie-breaking rule where all ties are broken in favor of bidder 1.¹ Bidder 2 bids 3.5 for all types. Bidder 1 bids 3.5 if his type is .5 or higher, and bids $3.5 - \varepsilon t_1$ when of type $t_1 < .5$. For small $\varepsilon > 0$, this is an equilibrium. The error in our proof of the nonexistence claim comes in the assertion that if there exists an equilibrium, then there exists one in weakly increasing strategies. To justify that assertion, we appealed to Proposition 1 of Maskin and Riley (2002), to which the above is also a counterexample.²

A slight modification of our Example 1 resurrects the claim that there does not exist an equilibrium for any type-independent tie-breaking rule. Consider exactly the same setting except for the following: with probability $1 - \varepsilon$ there is one item available that is awarded to the high bidder at the high price; and with probability $0 < \varepsilon < 1/200$ there are two items available, in which case the auctioneer randomly draws a number x from a uniform distribution on $[0, 6]$ and gives an item to each bidder whose bid exceeds x at a price of 0.³ With this modification, whenever there exists an equilibrium there also exists one in weakly increasing strategies. This follows from the observation that if one type of a bidder weakly prefers a high bid to a low one, then a higher type of the bidder will strictly prefer the higher bid to the lower one, as now any increase in bids increases the probability of obtaining an object. The remaining proof shows that in any equilibrium the bottoms of the supports of the bidding strategies must be identical and must be bid by both bidders with positive probability. A contradiction is then reached by showing that for any choice of this lowest bid that results in a positive probability of ties, at least one bidder would deviate. The (nontrivial) details can be found in “Supplement to Corrigendum to ‘Communication and Equilibrium in Discontinuous Games of Incomplete Information’ (*Econometrica*, Vol. 70, No. 5, September 2002, 1711–1740),” *Econometrica Supplementary Material*,

¹The claim that there are no equilibria is true if the tie-breaking rule gives both bidders a positive probability of winning an object.

²The statement of Proposition 1 in earlier working paper versions of Maskin and Riley (2002) asserts monotonicity only when there is a strictly positive probability of winning, and is correct.

³This has some flavor of an Amsterdam auction (see Goeree and Offerman (2004)) in that its effect is to make bidders care about their bids even in the presence of asymmetries (here, in the tie-breaking).

<http://www.econometricsociety.org/ecta/supmat/4688proofs.pdf>. There are other modified versions of the auction that also have no equilibria with type-independent tie-breaking rules, but the details of formulating such examples are very delicate.

In the proof of Lemma 2 part (i) we state that “we may implicitly define a unique element $\mu(E) = \Omega$ by requiring that $\phi \cdot \mu(E) = \lambda_\phi(E)$ for every $\phi \in \mathcal{E}^*$.” As it is not obvious why this is true, we also provide a fuller proof of part (i) of Lemma 2 in “Supplement to Corrigendum to ‘Communication and Equilibrium in Discontinuous Games of Incomplete Information’ (*Econometrica*, Vol. 70, No. 5, September 2002, 1711–1740),” *Econometrica Supplementary Material*, <http://www.econometricsociety.org/ecta/supmat/4688proofs.pdf>.

The last paragraph on page 1735 contains several typographical errors (t_i^* 's should be a_i^* 's) and a misstatement of (36). Delete from the beginning of the paragraph (“Fix an arbitrary...”) through (37) and substitute the following.

Fix an arbitrary $(s_i^*, a_i^*) \in H$. Continuity of u_i (in outcomes and types) and continuity of Ψ on T_i^k guarantees that there is a compact neighborhood L of $h(s_i^*, a_i^*)$ in T_i^k such that if $(s_i, a_i) \in S_i \times A_i$ and $t_i, t_i' \in T_i^k$, then

$$(36) \quad |Eu_i(s_i, a_i | \sigma_{-i}, t_i, \theta) - Eu_i(s_i, a_i | \sigma_{-i}, t_i', \theta)| < \frac{1}{4j}.$$

Continuity of h on H means that we can choose a compact neighborhood K of (s_i^*, a_i^*) in H such that $h(K) \subset L$. Applying (36), then (35), and then (36) again yields that

$$(37) \quad \begin{aligned} Eu_i(s_i, a_i | \sigma_{-i}, t_i, \theta) &> Eu_i(s_i, a_i | \sigma_{-i}, h(s_i, a_i), \theta) - \frac{1}{4j} \\ &> Eu_i(\sigma_i | \sigma_{-i}, h(s_i, a_i), \theta) + \frac{3}{4j} \\ &> Eu_i(\sigma_i | \sigma_{-i}, t_i, \theta) + \frac{1}{2j}. \end{aligned}$$

The β 's and B 's in Step 5 are missing subscripts that make the argument hard to follow. On page 1737, delete the two sentences starting “Define B .” and “For each r , define $\beta^r \dots$ ” and substitute the following:

For each r and each type/action pair $(\bar{s}_i, \bar{a}_i) \in S_i^r \times A_i^r$, define $\beta_{(\bar{s}_i, \bar{a}_i)}^r : \Delta_{-i} \rightarrow \Omega$ by $\beta_{(\bar{s}_i, \bar{a}_i)}^r(s_{-i}, a_{-i}, s_{-i}) = \theta^r(\bar{s}_i, \bar{a}_i, s_{-i}, a_{-i})$ and $B_{(\bar{s}_i, \bar{a}_i)} : \Delta_{-i} \rightarrow \Omega$ by

$$B_{(\bar{s}_i, \bar{a}_i)}(s_{-i}, a_{-i}, s_{-i}) = \Theta(\bar{s}_i, \bar{a}_i, s_{-i}, a_{-i}).$$

Note that $\beta_{(\bar{s}_i, \bar{a}_i)}^r$ is a selection from $B_{(\bar{s}_i, \bar{a}_i)}$.

Correspondingly, two paragraphs later replace “ $\beta^r \bar{\sigma}_{-i}^r \rightarrow \xi$ ” with “ $\beta_{(s_i^r, a_i^r)}^r \bar{\sigma}_{-i}^r \rightarrow \xi$.” Finally, in the last paragraph on page 1737 replace “there is a selection β from B ” with “there is a selection β from $B_{(s_i, a_i)}$.”

The Division of the Humanities and Social Sciences, 228-77, California Institute of Technology, Pasadena, CA 91125, U.S.A.; jacksonm@hss.caltech.edu,

Dept. of Agricultural Economics, University of California, Berkeley, CA 94720, U.S.A.; simon@are.berkeley.edu,

Olin School of Business, Washington University in St. Louis, St. Louis, MO 63130, U.S.A.; swinkels@olin.wustl.edu,

and

Dept. of Economics, University of California, Los Angeles, CA 90095, U.S.A.; zame@econ.ucla.edu.

Manuscript received June, 2003; final revision received March, 2004.

REFERENCES

- GOEREE, J., AND T. OFFERMAN (2004): "The Amsterdam Auction," *Econometrica*, 72, 281–294.
- JACKSON, M. O., L. K. SIMON, J. M. SWINKELS, AND W. R. ZAME (2004): "Supplement to Corrigendum to 'Communication and Equilibrium in Discontinuous Games of Incomplete Information' (*Econometrica*, Vol. 70, No. 5, September 2002, 1711–1740)," *Econometrica Supplementary Material*, <http://www.econometricsociety.org/ecta/supmat/4688proofs.pdf>.
- MASKIN, E., AND J. RILEY (2002): "Equilibrium in Sealed High Bid Auctions," *Review of Economic Studies*, 67, 439–452.