Bulletin of the Seismological Society of America. Vol. 49, No. 4, pp. 365–368 October, 1959

# A COMMENT ON THE FLATTENING OF THE GROUP VELOCITY CURVE OF MANTLE RAYLEIGH WAVES WITH PERIODS ABOUT 500 SEC.

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ABSTRACT

A scale-ratio consideration and a calculation on statical deformations of the earth by surface loads suggest that the flattening of the group velocity curve of mantle Rayleigh waves with periods about 500 sec. is not due to the existence of the earth's core, as has been suggested.

In a paper on the torsional oscillations of the earth (Takeuchi, 1959) we calculated a theoretical phase velocity dispersion curve for a model earth. The phase velocity $C$ thus calculated has a maximum of about 6.9 km/sec. for a period $T$ of about 1,300 sec., or for a wave length of about 9,000 km. That $C$ becomes smaller for wave lengths longer than this critical value shows the existence of some lower (shear-wave) velocity layer at great depth. This must be the earth's core. The core boundary is at 2,900 km. depth. The scale ratio (wave length/effective depth) in this case is about 3. We have obtained scale ratios of this order of magnitude several times. Thus for the wave number $n = 8$ (or wave length = 4,700 km.) we found $C = 6.48$ km/sec. The depth at which the shear-wave velocity takes this value is about 1,200 km. The scale ratio in this case is about 4. Similarly, for $n = 16$ and 32 we get scale ratios of about 4 and 3, respectively. In another paper (Takeuchi, Press, and Kobayashi, 1959) we also found scale ratios of this order of magnitude. We calculated dispersion curves for two earth models which are identical for depths shallower than 35 km. and deeper than 400 km. The two dispersion curves were almost identical for periods shorter than 25 sec. and longer than 250 sec. The corresponding phase velocities $C = 3.8$ and 5.0 km/sec. give scale ratios of about 3. Now at $T = 400$ sec. in this paper we found $C = 6.06$ km/sec., which gives an effective depth of about 800 km., a value much smaller than the depth to the core (2,900 km.). This suggests that the flattening of the observed group velocity curve for mantle Rayleigh waves around $T = 500$ sec. is not due to the existence of the core as proposed earlier (Ewing and Press, 1954, and Benioff and Press, 1958). In this paper we shall make a further study.

We begin by studying statical deformations of the earth as a whole by normal traction at the surface. Since this is deformation of the dilatational type, and statical deformation penetrates deeper than the corresponding dynamical deformation, this study will be one of the crucial tests for the problem set forth above. Statical deformation of the earth by surface load has already been studied (Takeuchi, 1951; see also Tomashek, 1957). Since the original periodical is not easily accessible, we recapitulate the results obtained there. Let us assume that the surface density $T$ corresponding to the load is expressed by

$$T = T_n a^n S_n = T_n [W_n]_{r=a} \tag{1}$$

Manuscript received for publication February 24, 1959.

where $T_n$ is a constant, $S_n$ and $W_n$ surface and solid spherical harmonics of degree $n$, $r$ the distance from the center of the earth, $a$ the earth's radius. This surface density gives rise to a potential $A$ within the earth

$$A = A_n W_n = \frac{4\pi fa}{2n + 1} T_n W_n \tag{2}$$

The earth is deformed by the potential $A$ and the surface traction

$$R_r = -T_n g[W_n]_{r=a} \tag{3}$$

By elastic deformation of the earth there arises an additional potential. The sum of the potential and $A$ will be denoted by

$$K = K_n W_n \tag{4}$$

In this way the equations for the deformation of the earth by surface loads can be reduced to the same type of equations as we have in the theory of earth tide. The ratio (additional potential at $r = a$ / primary potential at $r = a$) is thus calculated to be $-0.2653$, $-0.1132$, and $-0.0635$ for $n = 2$, 4, and 8, respectively. Adding this ratio to 1, we have the ratio (total potential at $r = a$ / primary potential at $r = a$) equal to 0.7347, 0.8868, and 0.9365 for $n = 2$, 4, and 8, respectively. The result for $n = 2$ in the foregoing is related to the problem of annual latitude variation. The annual variation is considered to be due to mass transportation during a year over the earth's surface. In order to get a theoretical estimation of the variation, we calculate products of inertia due to mass transportation. In doing so, we must take into account a secondary negative mass caused by the earth's deformation. Since the mass distribution in the calculation of the products of inertia is $S_2$ type, this is exactly the problem we have above. In order to take into account the earth's deformation, we must multiply the primary mass by the coefficient 0.7347. According to Jeffreys, the coefficient is somewhere between 0.75 (1929, p. 244) and 0.696 (1952, p. 214). The results given above are also useful in discussing the effect of gravity on the period of the earth's dilatational oscillations. For $n = 2$, for example, the additional density caused by the deformation of the earth is 26.5 per cent of the primary density. Almost the same figure is obtained in the theory of the earth tide and is called the Love number $k$. Since period is proportional to square root of effective density, we may expect a 13 per cent decrease, half the 26.5 per cent, of the period of $n = 2$ type oscillation. Similarly, we may expect about 5.7 and 3.2 per cent decrease of the periods for $n = 4$ and $n = 8$ type oscillations of the earth due to its self-gravitation. Although we have no calculation for $n$ larger than 8, the fairly smooth trend of the correction figures given above will suggest a decrease of the period of $n = 16$ type oscillation, for example, by about 1 per cent, which is almost negligible. This will justify what we have done in a paper already referred to (Takeuchi, Press, and Kobayashi, 1959). By using the same method as in that paper, we also calculate the period of the earth's oscillation of $S_2$ type, neglecting its self-gravitation. The first, second, and third approximations give $T = 48$, 52, and 60 min., respectively. The convergency here is not so good as we had for $n = 16$ previously. Furthermore, attacking the same problem, N. Jobert (1957) gets $T = 66$ min. In the variational calculus method here the longer period is considered to be the better result. Thus the disagreement between our result and Jobert's may be due to a worse choice of our trial functions. Since the same trial functions are used in Pekeris and Jarosch's calculation (1958), some doubt may be cast on the accuracy of their final result $T = 52$ min. for the corresponding self-gravitating earth. Anyway, by the earth's self-gravitation, $T = 60$ min. is decreased to $T = 52$

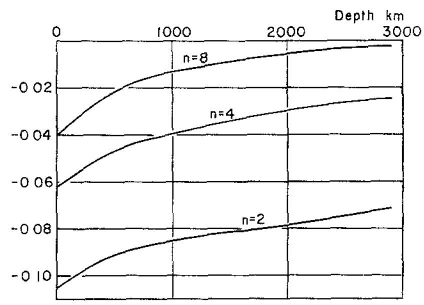


Fig. 1. Radial displacement distributions caused by surface loads of $S_n$ type.

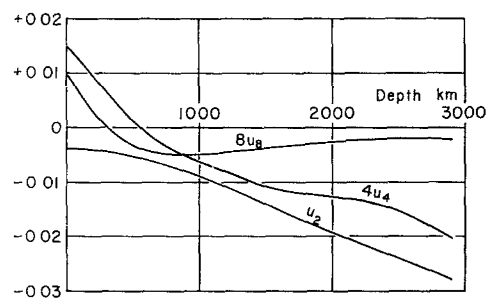


Fig. 2. Lateral displacement distributions caused by surface loads of $S_n$ type.

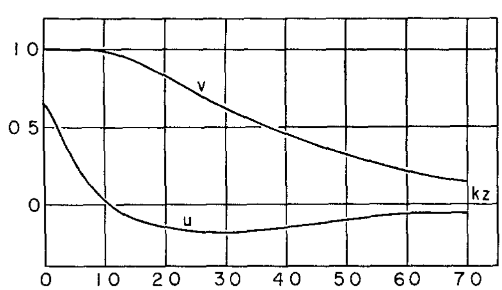


Fig. 3. Horizontal ($u$) and vertical ($v$) displacement distributions corresponding to Rayleigh waves in a homogeneous half-space.

min., about 13 per cent decrease as predicted above. The 13 per cent decrease of Jobert's period = 66 min. will give $T = 57.4$ min., which agrees well with the period 58 min. found by Benioff (1954) in a seismogram of the Kamchatka earthquake in 1952.

In figure 1 are shown radial displacement distributions within the earth caused by surface densities equal to $S_2$, $S_4$, and $S_8$, $S_n$ being a surface spherical harmonic of degree $n$. The unit of radial displacement $u_r$ in figure 1 is $S_n$. Figure 1 shows that the smaller the wave number $n$, that is, the longer the wave length, the deeper the depth to which the displacement penetrates. For $n = 8$, that is, for the wave length 4,700 km., the displacement at the core boundary is about 1/20 of that at the surface, and the depth where the displacement becomes 1/2 of that at the surface is about 500 km. Thus we see that the mantle Rayleigh wave of $T = 500$, or wave length of 3,000 km., is of too small a scale to be influenced by the existence of the earth's core. In figure 2 are shown the corresponding distributions of lateral displacement $u_\theta$ and $u_\phi$ taking $\partial S_n/\partial\theta$ and $\partial S_n/(\sin\theta\ \partial\phi)$ as units. Lateral displacements for larger $n$ or shorter wave lengths change signs at some intermediate depths. That this change of sign is not due to any computational error may be understood by comparing figures 1 and 2 with figure 3, which shows distributions of vertical ($v$) and horizontal ($u$) displacement corresponding to the Rayleigh wave in a uniform medium bounded by a surface. The abscissa in figure 3 is $kz$, that is, the depth from the free surface divided by $L/2\pi$, $L$ being the wave length.

## Acknowledgments

This study was carried out at the Seismological Laboratory, California Institute of Technology. The author wishes to thank Dr. F. Press for the opportunity to visit there and the Alfred P. Sloan Foundation for financial support.

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