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EFFECT OF FOUNDATION COMPLIANCE ON EARTHQUAKE STRESSES IN MULTISTORY BUILDINGS*

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ABSTRACT

This paper shows the quantitative effect that foundation compliance has on the maximum base shear force and the fundamental period of vibration in typical tall buildings subjected to strong-motion earthquakes. A study was made of five-, ten-, and fifteen-story building models on the Electric Analog Computer, subjecting them to the ground accelerations of actual earthquakes. The base shear forces were measured, the foundation compliance of the models being changed through a very wide range.

The properties specified for the building models are shown to be similar to the properties found in real buildings. The experimental results imply that the maximum base shear forces in typical buildings of five stories and higher during strong-motion earthquakes will be essentially unaffected by any degree of foundation compliance that can be expected in normal building practice. The fundamental period of typical buildings will be increased by about 10 per cent if the foundation compliance is the maximum that can be expected in standard building practice.

Introduction

Engineers, in surveying the extent of structural damage caused by strong-motion earthquakes, have often postulated that foundation materials have a considerable influence on the seismic stresses undergone by buildings. Some reports of earthquake damage have not supported this point of view. On the basis of observations primarily, the Seismic Research Group of Japan has proposed reductions in the lateral force design coefficients for various values of soil-bearing capacity.[1]

A Joint Committee of the San Francisco, California, Section of the American Society of Civil Engineers recently discussed the effect of foundation yielding.[2] They stated that the rotation of a building on its foundation may have a very considerable effect in lengthening the natural periods of vibration of the building, with resulting decrease of dynamic shears and moments throughout the structure. It is clear that as the foundation material becomes increasingly soft there will be a mitigating effect upon the stresses produced in a building. The question to be answered is, What degree of softness of foundation material is required for the achievement of significant reduction in stresses when the building is subjected to earthquake ground motion?

* Manuscript received for publication October 13, 1953.

[1] Seismic Research Group, Architectural Institute of Japan, "Discussion of Lateral Forces of Earthquake and Wind," *Trans. Am. Soc. Civil Engineers*, 117: 755–764 (1952).

[2] "Lateral Forces of Earthquake and Wind," *Trans. Am. Soc. Civil Engineers*, 117: 716–754 (1952).

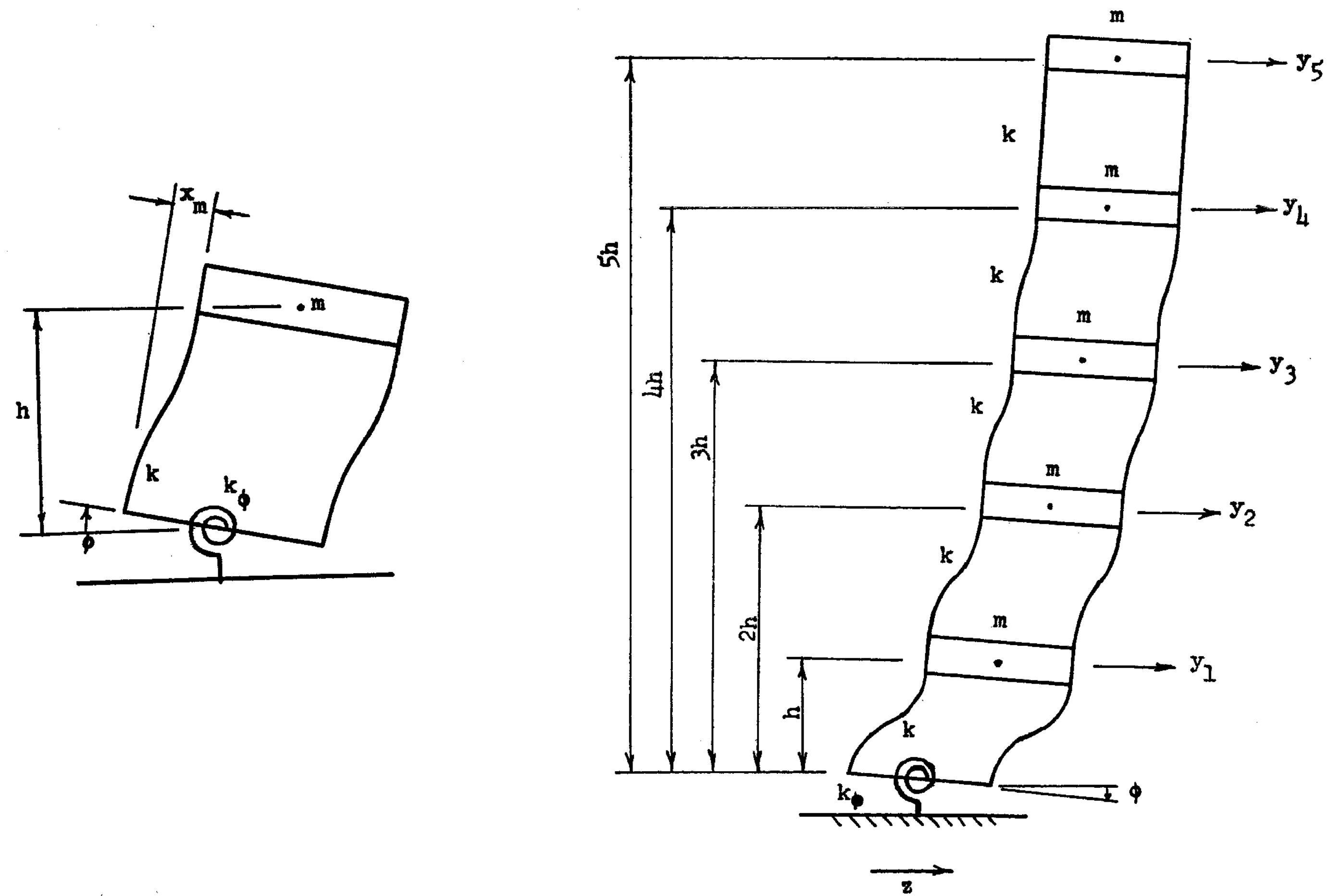

Fig. 1. Models of a one-story and a five-story building.

It is important to know the actual effect that foundation yielding has on the maximum stresses in typical buildings subjected to strong-motion earthquakes, in order that a rational basis may be provided for aseismic design. This paper provides a quantitative answer to the problem, which was approached in the following manner. Specific building models were selected for study and were subjected to the recorded ground accelerations of actual earthquakes. The base shear forces were measured, the foundation yielding (hereafter called "compliance") of the model being varied through a wide range. This was done electrically on the Analog Computer,[3] although it could have been achieved by actually constructing the models and measuring their response mechanically. An estimation was made of the maximum amount of foundation compliance to be expected for real buildings of the type that were modeled. This made possible an evaluation of the beneficial effect of the compliance to be provided.

Formulation of the Foundation Problem

Some insight can be gained into the effect of foundation compliance on the response of a building to earthquake ground motion by a study of a simplified model of a one-story building. Foundation compliance can be of two types, horizontal and rotary. The rotary or "rocking" compliance depends in part on the vertical deformation constant of the soil, which is considered a perfectly elastic medium. Therefore, small rocking deformations can account for some motion in the top of the model. The problem can be simplified by neglecting the effect of horizontal compliance. This is justified by a study of the earthquake records themselves. Since most accelerometers and displacement meters for measuring seismic motion are housed in the basements of buildings, any horizontal oscillations of the basement independent of the seismic ground motion would be recorded. Analysis of the records has not revealed any significant horizontal yielding. On the other hand, rotary motion of the foundation would not appear on records of horizontal ground motion.

The simplified model of a one-story building is shown in figure 1. Conditions are necessarily imposed in order to simplify the analysis: all springs are considered weightless, damping is neglected, and motion is limited to small oscillations so that gravity effects can be neglected.

k = stiffness of the building (lb/ft.)
k_ϕ = stiffness of the foundation in rocking (lb-ft/radian)
m = lumped mass of the building (lb-sec^2/ft.)
h = height of mass m (ft.)

The generalized coördinates are

x_m = displacement of the mass from the equilibrium position of the vertical springs
$h\phi$ = displacement caused by rocking compliance

[3] G. W. Housner and G. D. McCann, "The Analysis of Strong-Motion Earthquake Records with the Electric Analog Computor," *Bull. Seism. Soc. Am.*, 39: 47–56 (1949).

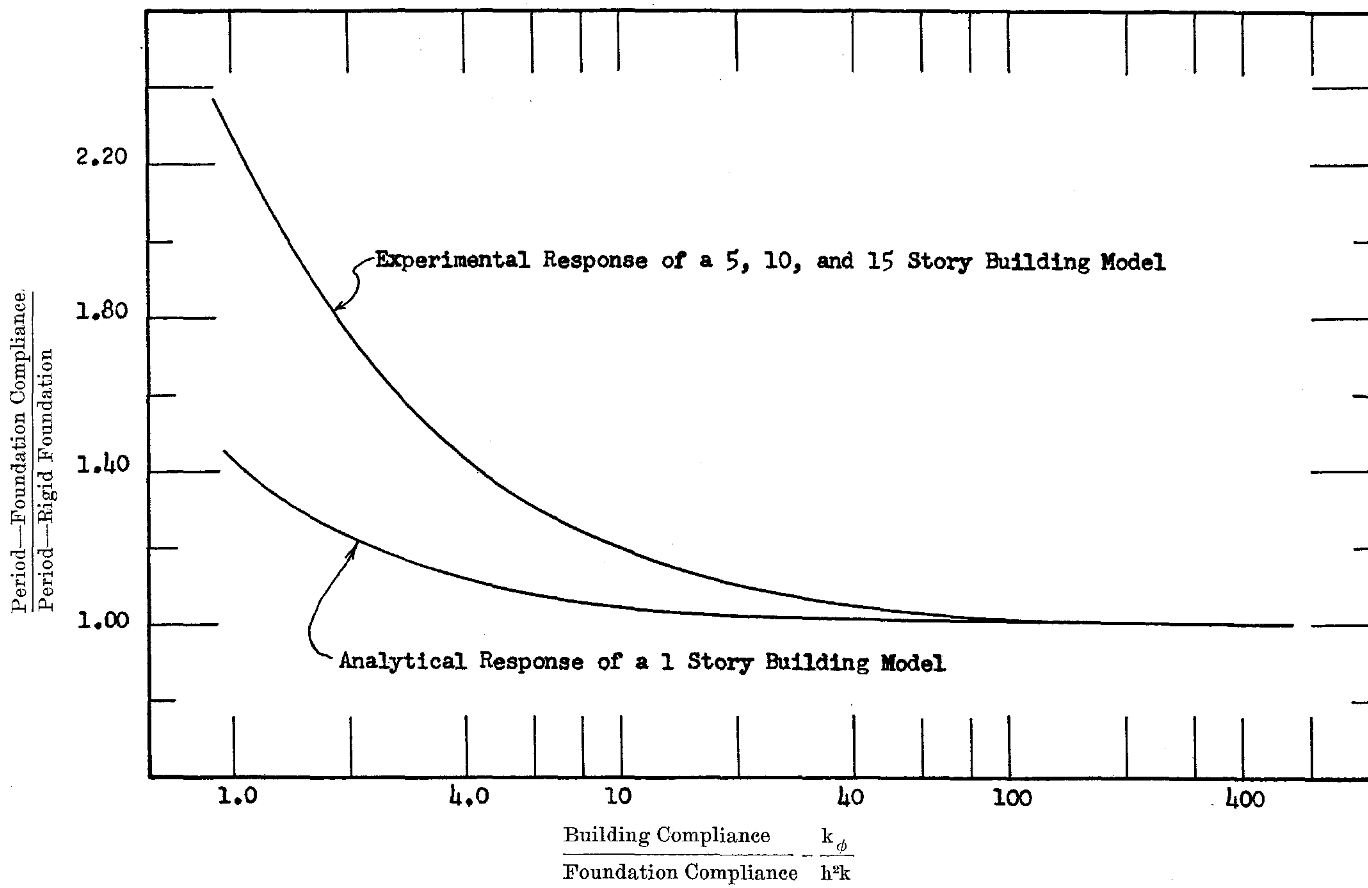

Fig. 2. Effect of foundation compliance on the fundamental period of vibration of a one-, five-, ten-, and fifteen-story building model.

The kinetic and potential energy of the system is

$$
\begin{aligned}
T &= \tfrac{1}{2}m[\dot{x}_m^2 + h^2\dot{\phi}^2 + 2\dot{x}_m h\dot{\phi}] \\
V &= \tfrac{1}{2}[kx_m^2 + k_\phi \phi^2]
\end{aligned}
\tag{1}
$$

Using Lagrange's equation, the equations of motion reduce to

$$
\begin{aligned}
\ddot{x}_m + h\ddot{\phi} + \frac{k}{m} x_m &= 0 \\
\ddot{x}_m + h\ddot{\phi} + \frac{k_\phi}{hm} \phi &= 0
\end{aligned}
\tag{2}
$$

Assuming harmonic oscillations, the equation for the period of free vibration is obtained.

$$
\tau = \left[1 + \frac{h^2 k}{k_\phi}\right]^{1/2} \cdot 2\pi \sqrt{\frac{m}{k}} \tag{3}
$$

The building compliance (reciprocal of the stiffness) is $1/k$ and the rocking foundation compliance is h^2/k_ϕ. The ratio of building compliance to foundation compliance is the dimensionless parameter k_ϕ/h^2k. Equation (3) is plotted in figure 2 and shows how the period of free vibration is affected by various degrees of foundation compliance.

Multistory Building Models

It was decided to model five-, ten-, and fifteen-story buildings in this investigation because these heights are representative of many buildings in the West Coast states. The building models were given the following characteristics.

The multistory building model was to deflect in shear. Thus the floors of the building moved parallel to each other. The mass of the building was lumped at the floor levels, and the stiffness of any story, that is, the force required to deflect one floor relative to an adjacent floor a distance of 1 foot, was taken to be the same for every story. The five-story building, on a rigid foundation, was given a fundamental period of 0.5 sec.; the ten-story building, 1.0 sec.; the fifteen-story building, 1.5 sec. It was not necessary to impose any restrictions on the dimensions of the model or its type of construction since the model was actually only a mathematical formulation and not a physical structure. The foundation rocking compliance was simulated by a torsion spring at the base where the stiffness of the spring, as in the case of the one-story building model, was k_ϕ, that is, the moment in pound-feet required to rotate the base through an angle of one radian. The problem was further particularized by letting the mass of each story and the height of each story be the same. The rotary moment of inertia of the floors about their centers of gravity was disregarded in the investigation. This excluded extraordinarily wide buildings from the study. Figure 1 shows a schematic drawing of a five-story building model.

y_i = absolute displacement of the i-th floor
ϕ = rotation of the base
z = absolute displacement of the ground
m = lumped mass of one story
h = story height

The equations of motion are

$$\begin{aligned}
&\ddot{y}_5 + \frac{k}{m}(y_5 - y_4 - h\phi) = 0 \\
&\ddot{y}_4 + \frac{k}{m}(2y_4 - y_5 - y_3 - 2h\phi) = 0 \\
&\ddot{y}_3 + \frac{k}{m}(2y_3 - y_4 - y_2 - 2h\phi) = 0 \\
&\ddot{y}_2 + \frac{k}{m}(2y_2 - y_3 - y_1 - 2h\phi) = 0 \\
&\ddot{y}_1 + \frac{k}{m}(2y_1 - y_2 - z - 2h\phi) = 0 \\
&\sum_{i=1}^{5} i\ddot{y}_i + \frac{k_\phi}{mh}\phi = 0
\end{aligned} \tag{4}$$

The shear force in the first story, and the base moment, are

$$\begin{aligned}
F &= k(y_1 - z - h\phi) \\
M_\phi &= k_\phi \phi
\end{aligned} \tag{5}$$

Damping in the Building Models

The response of a multiple degree of freedom system without damping, such as the five-story model building, can be described in terms of the configurations taken by the system when it is vibrating at one of its natural frequencies. These configurations are called the "normal modes" of the system. Rayleigh[4] suggests that it is possible to derive the normal modes of a damped system provided the damping be of a restricted mathematical form that is not generally useful for practical problems.

If the energy dissipation in the structure is caused by viscous damping, the formulation of the problem is greatly simplified. This assumption is justified if the total energy dissipation is small.[5] Two types of viscous damping can be treated easily, absolute and interfloor. Both are equivalent to having dashpots attached to the

[4] Lord Rayleigh, *Theory of Sound*, p. 130 (1937).
[5] L. S. Jacobsen, "Steady Forced Vibration as Influenced by Damping," *Trans. Am. Soc. Mech. Engineers*, 52: 169 (1930).

floors of the model, but in the first the dashpot is actuated by the absolute displacement of the floor whereas in the second the dashpot is actuated by the relative displacement of the floors. Physical reasoning shows that interfloor damping is a more reasonable way to dissipate energy in a building than absolute. But it is of interest to examine both types. It has been shown[6] that absolute damping, interfloor damping, or a linear combination of both, falls into the restricted mathematical form discussed by Rayleigh. The normal modes of the system can be derived, and it is possible to examine the damping as though each mode were a one degree of freedom system. If interfloor damping is used, the percentage of critical damping is proportional to the undamped natural frequency of the modes. Thus, in a five degrees of freedom system, if the first mode is damped 0.10 of critical, the fifth mode is damped 0.67 of critical. If absolute damping is used, the percentage of critical damping is inversely proportional to the undamped natural frequency of the modes. Thus in the five degrees of freedom example, if the first mode is damped 0.10 of critical, the fifth mode is only damped 0.015 of critical. If a linear combination of both types is used, it is possible to select the amount of each so that the damping in any two modes is arbitrarily specified. In this investigation, the damping in the first two modes of vibration of the models was specified to be 0.10 of critical damping.

Earthquake Ground Motion

The U. S. Coast and Geodetic Survey accelerograms of fifteen strong-motion earthquakes were available for use in this study. The stronger acceleration components of the three most intense ground motions[7] were selected for actual investigation. These included the N-S acceleration component of the earthquake recorded at El Centro, California, on May 18, 1940, the S 80° W component of the earthquake recorded at Olympia, Washington, on April 13, 1949, and the N-S component of the earthquake recorded at El Centro, California, on December 30, 1934. Also included, because of its immediate interest, was the recent strong-motion earthquake of July 21, 1952, which had its epicentral location about 20 miles from Tehachapi, California.[8] The recording station nearest the center of the shock was at Taft, California, some 40 miles away. The S 69° E component of this record was used and the accelerogram appears in figure 3.

Analog Computer Technique

The measurement of base shear forces in building models subjected to earthquake ground motions on mechanical shaking tables would be a formidable task. However, the use of the Electric Analog Computer offers a fast, reliable method of analyzing damped multiple degree of freedom structures when subjected to the recorded seismic accelerograms. The shear force in any floor can be read directly, and the effect of foundation compliance on it can be observed as a single dial is turned.

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[6] In an unpublished paper on damping in multiple degree of freedom systems, by J. L. Alford.

[7] J. L. Alford, G. W. Housner, and R. R. Martel, "Spectrum Analyses of Strong-Motion Earthquakes," First Technical Report, ONR Contract N6-onr-244, Task Order 25, California Institute of Technology, Pasadena, Calif., 1951.

[8] G. W. Housner, "The Tehachapi Earthquake of July 21, 1952," *Proceedings* of the Symposium on Earthquake and Blast Effects on Structures, Los Angeles, Calif., June, 1952, pp. 226–232.

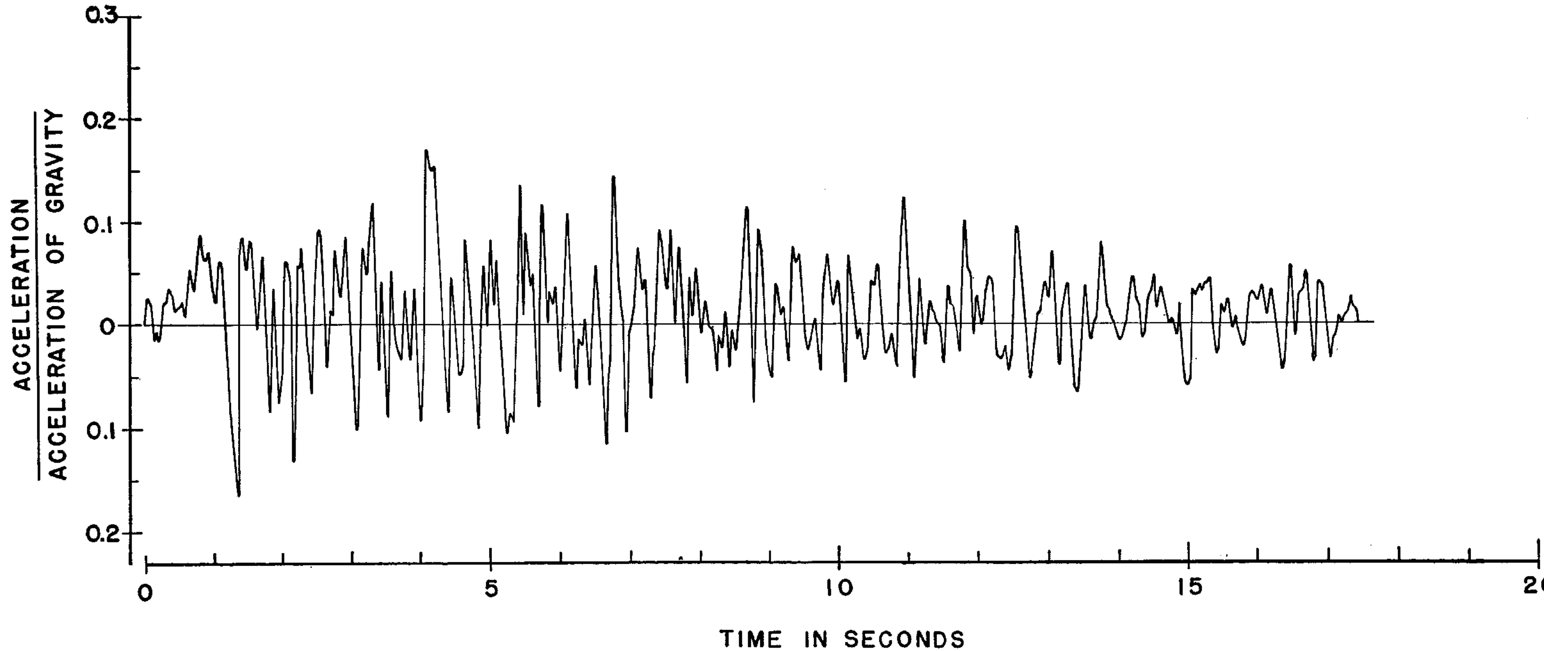

Fig. 3. Accelerogram for Taft, California; earthquake of July 21, 1952. Component S 69 E.

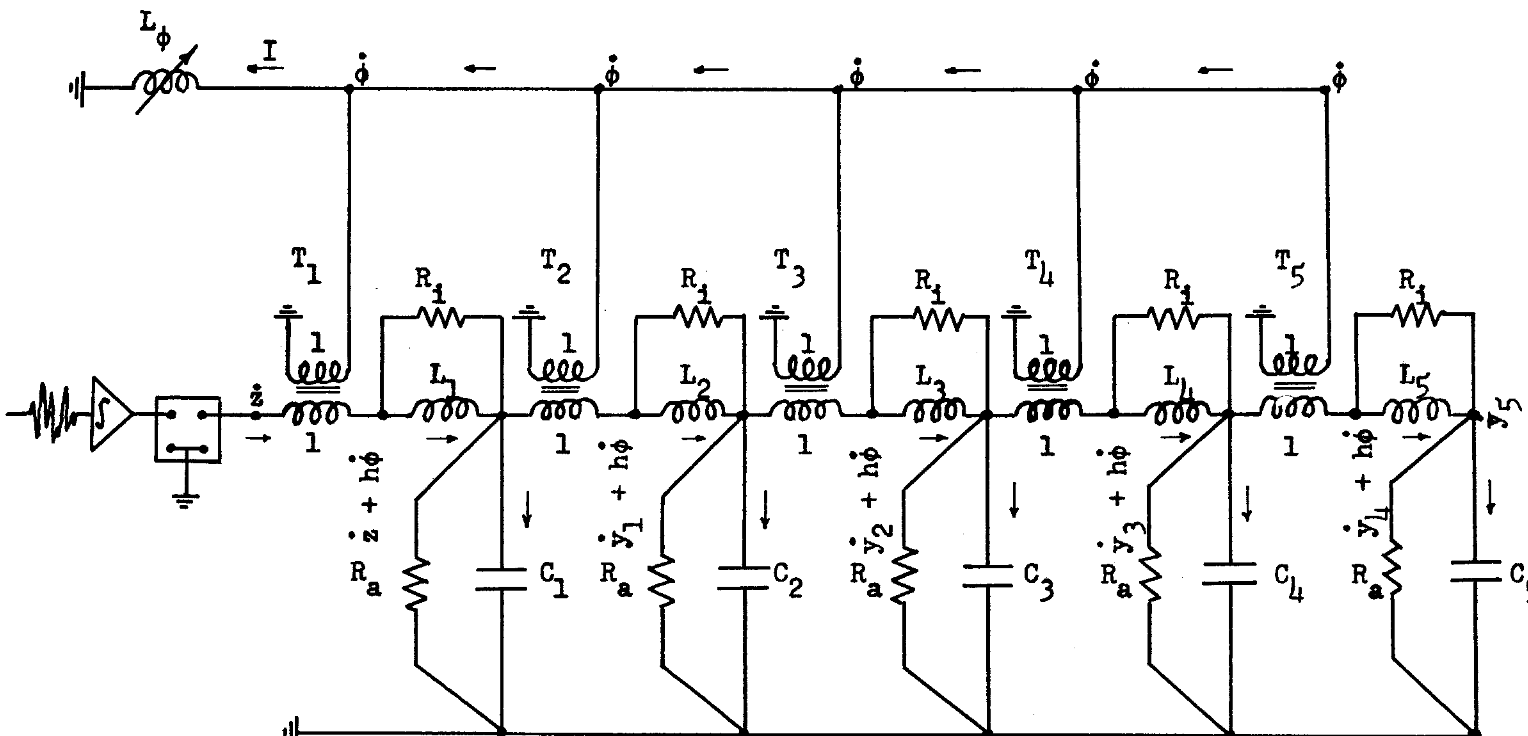

Fig. 4. Nodal analogy circuit for a five-story building model with foundation compliance.

Since the electric analog is an electrical circuit the components of which are analogous to the components of the mechanical system, the equations that govern the electrical circuit are of the same form as the equations of motion of the mechanical system. The properties of the system can be rapidly changed, and the current and voltage can be read at any point in the circuit. Using a set of predetermined proportionality factors, the corresponding forces and velocities can be computed rapidly. More detailed descriptions of the analog computer and its application to earthquake problems appear in the literature.[9]

The Nodal Analogy

The nodal analogy was used for this investigation, and the electrical circuit for a five-story building model is shown in figure 4. In this analogy, voltage is proportional to velocity, and current is proportional to force. Other proportional quantities are capacitance and mass, reciprocal inductance and spring stiffness, and reciprocal resistance and damping coefficient.

As was stated earlier, a linear combination of absolute and interfloor viscous damping enables the arbitrary specification of the amount of damping in the first two modes of vibration of the building. In the nodal analogy a resistance in parallel across a capacitance corresponds to absolute viscous damping, while a resistance in parallel across an inductance corresponds to interfloor viscous damping. The resistances in figure 4 were adjusted by trial and error until the damping in the first two modes was 0.10 of critical damping. The experimental determination of the damping in the third mode was 0.29–0.33 of critical damping, while the damping in higher modes was so large as to make its measurement impossible.

The circuit excitation was accomplished by means of an "arbitrary function generator." Film records were made from carefully scaled drawings of the earthquake accelerograms[10] which metered the light in a photocell circuit to provide a variable voltage of the same form as the accelerogram. Electrical integration of this voltage provided the proper excitation, $\dot{z}$, to the base of the building model. The base shear force in the structure was the measured current in L_1, figure 4.

The most important properties of the building models are the frequencies (or periods) of vibration. The frequencies for the five-story building are given in table 1. It is interesting to compare these with the frequencies of the first five modes of vibration of a uniform shear beam of the same height and total mass. (In table 1, k and m for the uniform shear beam are stiffness and mass per unit length.) Since contributions to base shear stresses from modes higher than the third are very small, the use of the uniform shear beam would yield essentially the same results as the five-story building model. This applies to the ten- and fifteen-story models as well. As stated earlier, the fundamental periods and thus the frequencies were prescribed for the building models. The inductances and capacitances in the electrical circuit were then adjusted so that the experimental resonant frequencies corresponded to those calculated.

[9] Alford, Housner, and Martel, *op. cit.*; Housner and McCann, *op. cit.*
[10] Alford, Housner, and Martel, *op. cit.*

TABLE 1

FREQUENCIES OF VIBRATION OF FIVE-STORY BUILDING

Mode	Frequency of five-story model	Frequency of uniform shear beam
First	$0.285\sqrt{k/m}$	$0.285\sqrt{k/m}$
Second	0.831	0.854
Third	1.309	1.423
Fourth	1.683	1.992
Fifth	1.919	2.561

EXPERIMENTAL RESULTS

The effect of foundation compliance on the maximum base shear force in a typical five-story building model is plotted in figure 5. When the five-story structure is

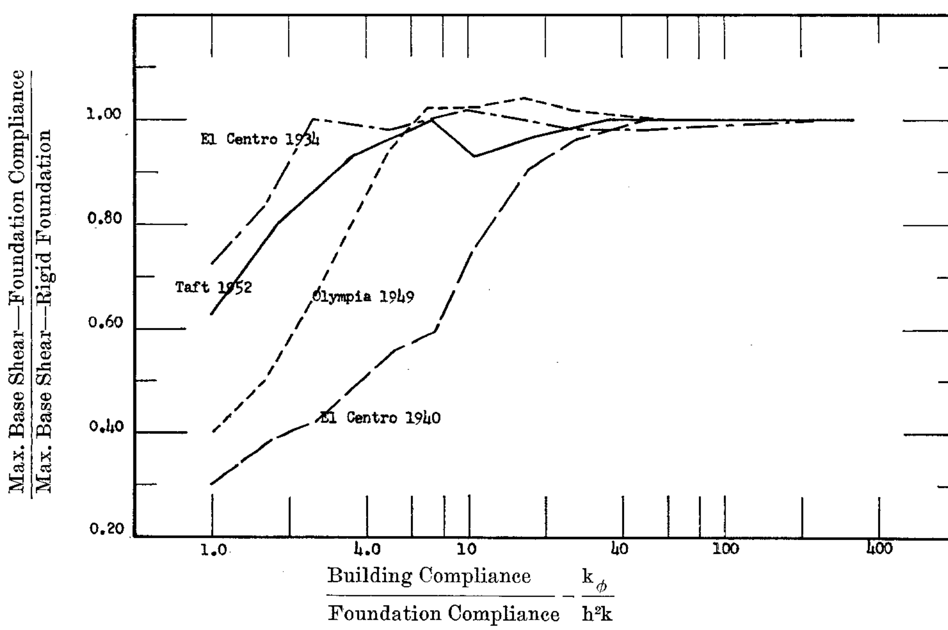

Fig. 5. Effect of foundation compliance on the maximum base shear force in a five-story building model.

subjected to the 1940 El Centro shock, foundation compliance has a considerable effect in reducing the shear if the compliance ratio, k_ϕ/h^2k, is less than 12. In what follows, a significant stress reduction will be taken to be one in which the stress is reduced below 0.80 of the magnitude of the shear in a building on a rigid foundation. On the other hand, when the same five-story building is subjected to the 1934 El Centro earthquake, the shear stress is still unchanged at a compliance ratio of 2.5, and there is no significant reduction until the ratio becomes 1.5. The 1952 Taft shock also keeps the magnitude of the shear force high through a wide range of the compliance ratio, while the Olympia earthquake magnifies the shear force slightly through part of this range.

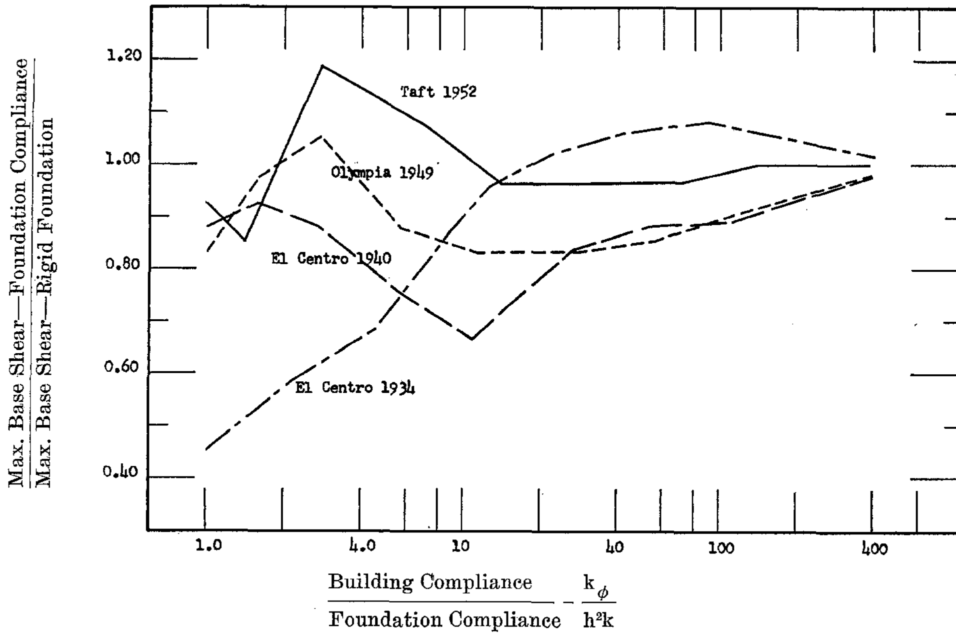

Fig. 6. Effect of foundation compliance on the maximum base shear force in a ten-story building model.

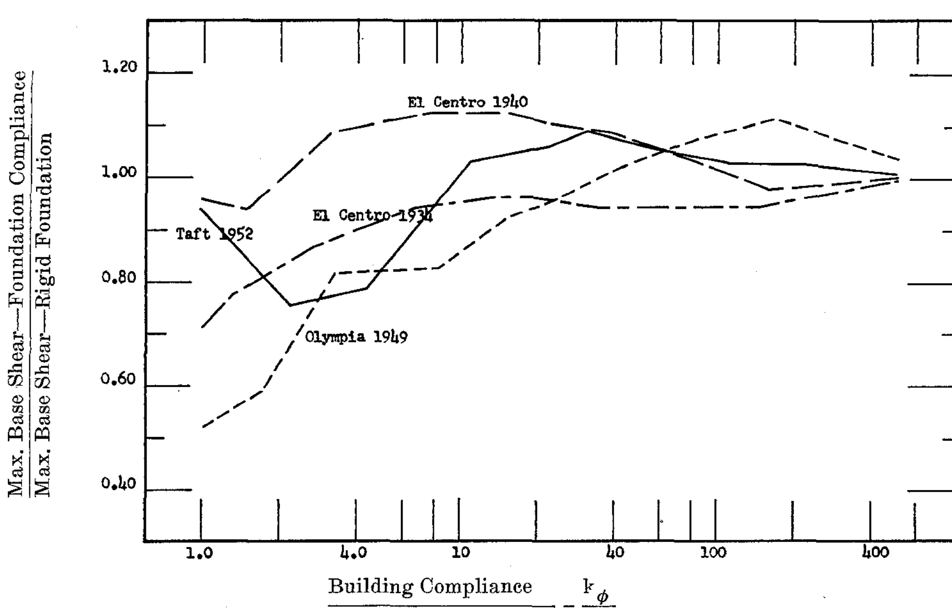

Fig. 7. Effect of foundation compliance on the maximum base shear force in a fifteen-story building model.

The effect of foundation compliance on the maximum base shear force in a typical ten-story building model is plotted in figure 6. For this building, only the 1934 El Centro earthquake reduces the base shear significantly over the range of compliance ratio investigated. It is interesting to note the magnification that occurs for the other three earthquakes when the compliance ratio is approximately 2–3. The ten-story building with a compliance ratio of 3.0 actually experiences a base shear almost 1.2 times what it experiences on a rigid foundation if subjected to the Taft earthquake. The 1940 El Centro shock has a significantly reduced effect over part of the compliance ratio range, but at 1.0 the base shear is not significantly reduced.

The effect of foundation compliance on the maximum base shear force in a typical fifteen-story building model is plotted in figure 7. Three of the four earthquakes used in the investigation maintain significant base shear stresses throughout the range of compliance ratio studied.

It is of interest to note that the same earthquake has different effects on buildings of different heights. The 1934 El Centro shock permits a significant reduction in stress at very low values of the compliance ratio in the ten-story building, but maintains a high stress level in the five- and fifteen-story buildings. The 1940 El Centro earthquake maintains a high stress level in the ten- and fifteen-story buildings, but permits a significant reduction in stress in the five-story building when the compliance ratio drops below 12. The 1949 Olympia earthquake maintains a high stress level in the ten-story building, but at low compliance-ratio values it drops below the 0.80 level in the five- and fifteen-story buildings. The 1952 Taft shock maintains a very high stress level over the entire compliance-ratio range in the ten-story building.

Figure 2 shows the effect of foundation compliance on the fundamental period of vibration of the typical five-, ten-, and fifteen-story buildings. This curve was obtained experimentally by driving the top of the model with an oscillator and measuring the fundamental resonant frequency over a wide range of the ratio of building compliance to foundation compliance. All the experimental points fell on the smooth curve. It is seen that the fundamental period of a multiple degree of freedom system is more sensitive to the compliance ratio than the single degree of freedom system studied analytically.

Figures 8, 9, and 10 show the effect of increasing the fundamental period on the maximum base shear force in a typical five-, ten-, and fifteen-story building model, respectively. These curves were derived from the relationship between the fundamental period and the compliance ratio plotted in figure 2. Essentially, they show the same information that is plotted in figures 5–7.

Properties of Real Buildings

The models used in this investigation were given certain specified properties, and it is necessary to compare these properties with those found in actual buildings. Observations have shown that real buildings tend to deflect in shear, that is, the floors tend to move parallel to each other. With shear deflections, the fundamental periods of buildings tend to increase proportionately with the height. If buildings deflected in bending, the fundamental periods would vary as the square of the

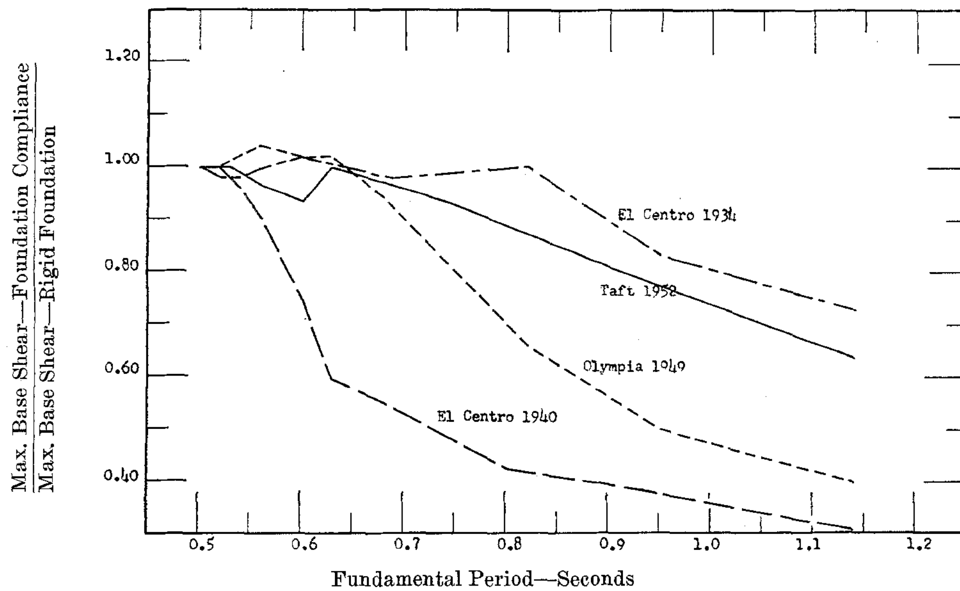

Fig. 8. Effect of increasing the fundamental period on the maximum base shear force in a five-story building model.

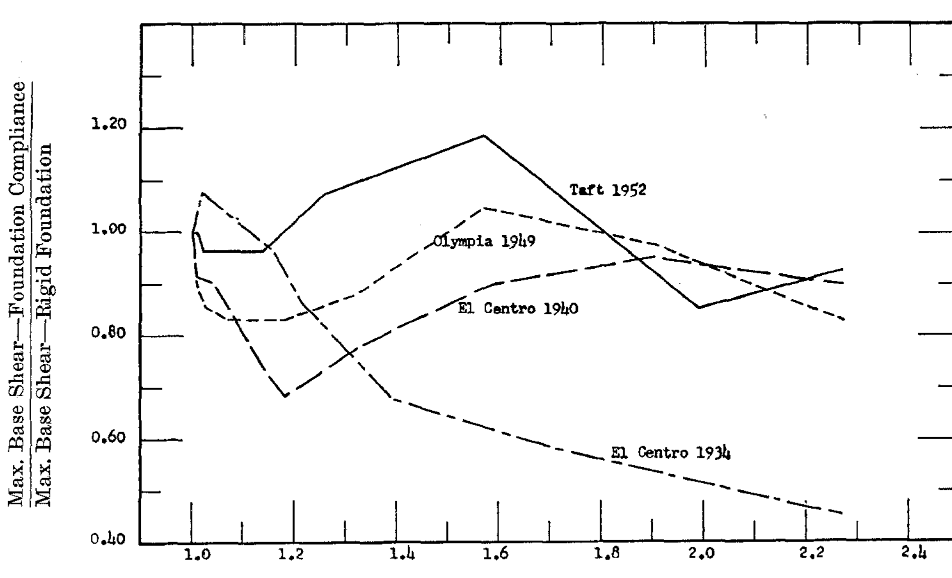

Fig. 9. Effect of increasing the fundamental period on the maximum base shear force in a ten-story building model.

height. It is concluded by the U. S. Coast and Geodetic Survey[11] after the measurement of the periods of 212 buildings in California that shear distortions are predominant in most Pacific Coast buildings. This study also shows that the fundamental period of vibration of the average building is approximately 0.1 sec. multiplied by the number of stories. Thus the 0.5-sec. fundamental period of the five-story model, the 1.0-sec. fundamental period of the ten-story model, and the 1.5-sec. fundamental period of the fifteen-story model are representative of the actual periods of real buildings.

In real buildings, the mass of the structure is not lumped at the floors, nor is it distributed as in the case of the uniform shear beam. The real mass distribution lies

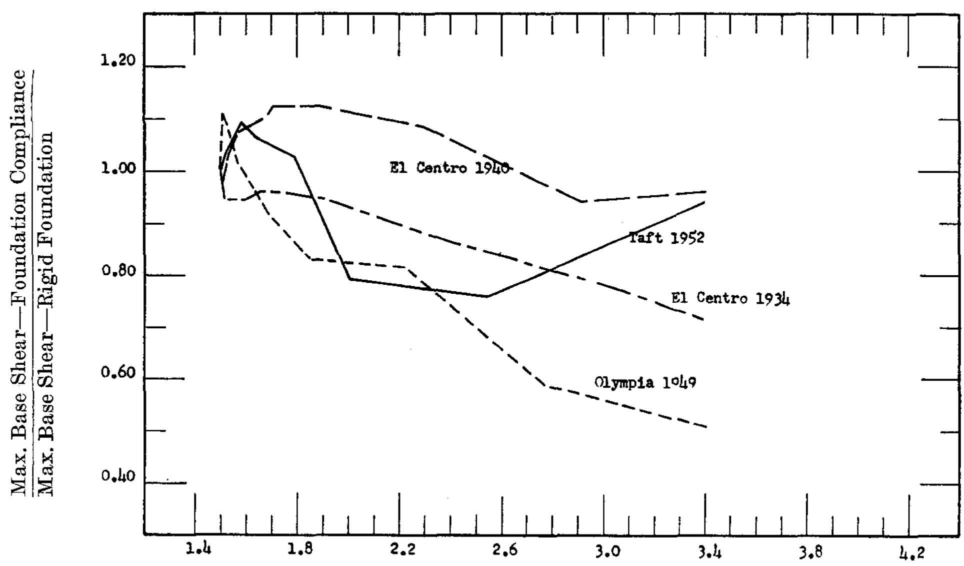

Fig. 10. Effect of increasing the fundamental period on the maximum base shear force in a fifteen-story building model.

somewhere in between. Since the frequencies of the lumped mass and uniform system differ by only a small amount (table 1), it follows that the mode shapes and frequencies of a real building must agree closely with those of the models used. The story heights, the mass of each story, and the stiffness of each story will generally vary in a real building; hence the results of this study pertain only to such structures as have approximately uniform properties from story to story.

Damping has a significant effect on the response of a building to earthquake ground motion.[12] Damping in real buildings has been found to range from 0.07 to 0.40 of critical damping.[13] A dynamic test on a rigid four-story reinforced concrete

[11] "Earthquake Investigations in California, 1934–1935," *Spec. Publ.* No. 201, p. 51.
[12] Alford, Housner, and Martel, *op. cit.*
[13] G. W. Housner, "Intensity of Ground Motion During Strong Earthquakes," Second Technical Report, ONR Contract N6-onr-244, Task Order 25, California Institute of Technology, Pasadena, Calif., 1952, p. 6.

warehouse[14] revealed damping to be 0.07 of critical damping. The damping was found to be independent of the frequency in the first two modes of vibration, the only two that were measurable. Thus damping in real buildings is neither interfloor viscous damping nor absolute viscous damping. In general, the more monolithic the construction of a building, the less will be the damping. Therefore, the 0.07 of critical damping in the warehouse is probably as small an amount of damping as can be anticipated, and average buildings will have somewhat more. The damping used in this investigation, 0.10 of critical in both the first and second modes, reproduces the features of the actual damping observed in buildings, though it is probably somewhat less than would be observed in an actual building when subjected to a strong-motion earthquake.

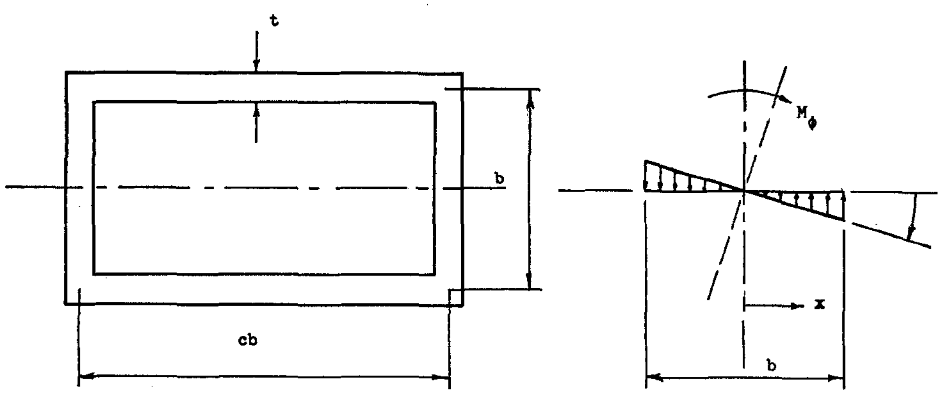

Fig. 11. Foundation reaction due to rocking of a building with continuous wall footings.

Estimation of the Compliance Ratio for Real Buildings

Having established the relationship between the maximum base shear force in typical buildings having foundations with a wide range of compliance, it is necessary to estimate what the lowest actual value of the compliance ratio (ratio of building compliance to foundation compliance) might be. The compliance ratio, k_ϕ/h^2k, has three independent variables, each having an effect on its value. An average story height, h, for typical buildings is 12 feet. On the other hand, to lower the ratio, a typical k must be as large as possible and a typical k_ϕ must be as small as possible.

The problem of determining the stiffness of real buildings is greatly simplified by noting that the fundamental period, τ_1, is related to the stiffness and story mass, m. A general expression involving τ_1 may be written in the form

$$\frac{2\pi}{\tau_1} = 2\sqrt{\frac{k}{m}} \cdot \sin\left(\frac{\pi}{4n + 2}\right) \tag{6}$$

where n is the number of stories. Therefore,

$$k = \frac{\pi^2}{\sin^2\left(\frac{\pi}{4n + 2}\right)} \cdot \frac{m}{\tau_1^2} \tag{7}$$

[14] J. L. Alford and G. W. Housner, "A Dynamic Test of a Four-Story Reinforced Concrete Building," *Bull. Seism. Soc. Am.*, 43: 7–16 (1953).

The foundation reaction due to the rocking of a building is shown in figure 11.

A = total footing area (ft^2.)
B = allowable bearing pressure of foundation material (lb/ft^2.)
b = width of building (ft.)
c = length to width ratio of building
δ = deflection of the foundation material (ft.)
M_ϕ = base moment (lb/ft.)
ϕ = angular rotation (radians)
p_v = pressure corresponding to unit deflection of the ground (lb-ft^3.)
n = number of stories
t = width of footing (ft.)

The reacting moment of an increment of footing area at a distance x from the center of rocking is

$$dM_x = x\delta p_v dA \tag{8}$$

For small displacements, $\phi = \delta/x$, so that

$$dM_x = \phi x^2 p_v dA \tag{9}$$

The total base moment, M_ϕ, is the integral of equation (9). By definition, k_ϕ is the moment per radian of rotation, so the foundation stiffness is

$$k_\phi = p_v \int_{-b/2}^{b/2} x^2 dA \tag{10}$$

The integral in equation (10) is the moment of inertia of the area of the foundation about its rocking axis, I_{xx}.

If the foundation material is linearly elastic in rocking, then the pressure corresponding to unit deflection, p_v, is the modulus of elasticity or the "elastic constant" of the soil. Essentially, it is the slope of the "elastic rebound" portion of the load-settlement diagram for a soil. A medium soft clay might have a p_v as low as 5.2×10^5 lb/ft^3. It is true that much softer foundation materials are encountered in the field, but the allowable bearing pressure for such materials would be so low that it would rule out the construction of tall buildings with spread footings on it.

The area of the continuous wall footing shown in figure 11 is dependent on the total weight of the building (*mgn*) and the allowable bearing pressure of the foundation material, that is,

$$A = 2bt(1 + c) = mgn/B \tag{11}$$

The moment of inertia of the foundation area may be taken to be

$$I_{xx} = \frac{b^3 t}{6} (1 + 3c) \tag{12}$$

Therefore,

$$I_{xx} = \frac{mgnb^2}{12B} \cdot \frac{(1 + 3c)}{(1 + c)}$$

and

$$k_\phi = \frac{p_v mgnb^2}{12B} \cdot \frac{(1 + 3c)}{(1 + c)} \tag{13}$$

The general expression for the compliance ratio, k_ϕ/h^2k, follows directly from equations (7) and (13).

$$\frac{k_\phi}{h^2k} = \frac{p_v gnb^2\tau_1^2}{12Bh^2} \cdot \frac{(1 + 3c)}{(1 + c)} \cdot \frac{\sin^2\left(\frac{\pi}{4n + 2}\right)}{\pi^2} \tag{14}$$

If n is five or more stories high, the sine of $\pi/(4n + 2)$ can be approximated by $\pi/4n$ and τ_1 by $0.1n$. If the length to width ratio, c, is made equal to 1.0 and the numerical values for h and g are substituted, the compliance ratio is reduced to

$$\frac{k_\phi}{h^2k} = 2.33 \times 10^{-5} \frac{p_v nb^2}{B} \tag{15}$$

The numerical coefficient in this expression has the units of $(\text{ft.})^{-1}$. It is seen that the compliance ratio is, in part, dependent on the ratio of the elastic soil constant to the allowable bearing pressure. The softer the soil, the lower the allowable bearing pressure. Thus the ratio of the two does not change significantly in value. On the other hand, the compliance ratio is proportional to the total number of stories and the square of the width of the building. If similar buildings differing only in height are compared, the compliance ratio will always increase with height and will never decrease. Therefore, the taller the building, the greater will be the compliance ratio and the less effective will be foundation yielding in mitigating earthquake stresses.

The compliance ratio can be calculated for a critical case. Take as an example a five-story building with a $40' \times 40'$ plan to be built on a medium soft clay ($p_v = 5.2 \times 10^5$ lb/ft^3.) that has an allowable bearing pressure of 4000 lb/ft^2. Certainly a tall building with any smaller plan would not be a typical building. For this example, the compliance ratio would be equal to 24 and the yielding of the soil would not give any appreciable reduction of stresses. Taller buildings would have even larger compliance ratios. It thus appears that only very exceptionally would foundation yielding have a beneficial effect on earthquake stresses. To achieve this would require an exceptionally narrow building with a floating foundation on very soft soil.

Conclusions

It has been shown that the building models used in this investigation are representative of typical real buildings. This implies that the results of this study which are plotted in figures 5–9 are also applicable to real structures. The conclusions drawn from this study are subject to the restrictions already discussed in detail. In particular they apply to typical buildings of standard construction that undergo shear distortions, that are approximately uniform from story to story, and that are five stories and more in height. The conclusions drawn are as follows:

1. The maximum base shear force in a typical tall building subjected to earthquake ground motion will be essentially unaffected by any degree of foundation compliance that can be expected in standard practice.

2. The ratio of building compliance to foundation compliance (the compliance ratio) is proportional to the "elastic constant," p_v, of the soil, the number of stories, and the square of the width of the building, and is inversely proportional to the allowable bearing pressure of the soil. The width of the building has the most critical effect on the compliance ratio. The lower bound for a typical five-story building of standard construction is a compliance ratio of approximately 24.

3. The fundamental period of typical buildings is increased less than 10 per cent when the ratio of building compliance to foundation compliance reaches the lower bound of 24.

4. Any specific earthquake will have different effects on buildings of different heights. It is not improbable that two buildings of unequal height but with equal compliance ratios and equal design strengths can suffer different degrees of damage during a particular strong-motion earthquake.

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