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A NOTE ON THE CORRELATION OF FREQUENCY-DEPENDENT DURATION OF STRONG EARTHQUAKE GROUND MOTION WITH THE MODIFIED MERCALLI INTENSITY AND THE GEOLOGIC CONDITIONS AT THE RECORDING STATIONS

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ABSTRACT

The frequency-dependent duration of strong earthquake ground motion, based on the mean-square integrals of motion, has been correlated with the reported Modified Mercalli Intensity at the recording site. Simple relations have been presented which describe the overall trends of computed durations for different levels of Modified Mercalli Intensity and for three classes of site geology.

Introduction

The duration of shaking is no doubt one of the most important characteristics of strong earthquake ground motion in determining its destructive capabilities. The response of any linear or nonlinear yielding structure during an earthquake depends directly on the time length of such shaking. From a more general viewpoint, it is useful to examine the frequency dependence of the duration of strong ground motion, because such a study provides better insight into the nature of seismic-wave attenuation and scattering.

The availability of instrumental data for nearby earthquakes is quite limited and of recent origin since the modern strong ground-motion accelerographs have been recording only since the early 1930's. Although many parts of the world are now being equipped with modern strong-motion instruments, it appears that it will take many years before the amount of data comparable to the records obtained in the Western United States is collected elsewhere. In an effort to characterize earthquakes that occurred prior to this time, or in a region where no strong-motion instrumentation is available, a crude and qualitative description of strong ground motion in terms of the Modified Mercalli intensity (MMI) or its equivalent is often used.

The MMI rating of strong ground motion is based on the subjective assessment of shaking and the resulting damage as experienced by persons who witnessed the event and by experts who visit the area after the earthquake. Approximate correlations of the frequency-dependent duration of strong shaking with the MMI would therefore not only provide rough estimates of the duration for "non-instrumented" earthquakes but might also be of use in more detailed characterization of the intensity ratings, whenever some information on the duration of strong shaking is available. Since the MMI scale is also based on the shorthand description of structural damage, correlations with duration would provide additional insight into the response and damage of a structure for such an approximate description of shaking. The purpose of this paper, therefore, is to present the trends and correlations of the frequency-dependent durations with the MMI and the recording site conditions.

Some Definitions, Description of the Strong-Motion Data, and the Principles of Analysis

The integrals of the form $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, $\int_0^T d^2\,dt$, where $a(t)$, $v(t)$, and $d(t)$ stand for the acceleration, velocity, and displacement, respectively, and T is record length, are related to various characteristics of strong earthquake ground motion such as the different measures of instrumental intensity (e.g., Arias, 1970; Housner, 1952), the seismic-wave energy, and the expected maximum value of peaks of $a(t)$, $v(t)$, or $d(t)$ (Trifunac and Brady, 1975a; Trifunac and Westermo, 1976a, b). These integrals generally increase rapidly with the onset of the strong ground motion and then gradually tend to their maxima, $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, $\int_0^T d^2\,dt$. We recently proposed a refined definition of the duration of strong ground motion (Trifunac Westermo, 1976a, b) in terms of the sum of the time intervals during which the contributions made to these integrals makes up 90 per cent of their ultimate amplitudes $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, $\int_0^T d^2,\,dt$. With this definition of duration and again following our previous work (Trifunac and Brady, 1975b; Trifunac and Westermo, 1976a, b) we define the average "rate" at which strong shaking evolves at a point by

$$\text{Rate} = \int_0^T \begin{Bmatrix} a^2 \\ v^2 \\ d^2 \end{Bmatrix} dt/\text{duration of} \begin{Bmatrix} a \\ v \\ d \end{Bmatrix}. \tag{1}$$

This definition corresponds to the average slopes of the functions $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, $\int_0^T d^2,\,dt$ for the time intervals of strong motion and for $\int_0^T a^2\,dt$ is proportional to the power of the same motion.

The six narrow-frequency bands of acceleration, velocity, and displacement with center frequencies, f_c, equal to 0.2, 0.5, 1.1, 2.7, 7.0, and 18.0 Hz used in this analysis are those used in our previous work (Trifunac and Westermo, 1976a, b). The details describing these frequency bands, digital filters used to derive them, and other pertinent characteristics of the data can be found in our previous work and will not be repeated here.

One hundred and eighty-six strong-motion records were used in the analysis in which we correlate the integrals $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, $\int_0^T d^2\,dt$, the duration and the "rate" of strong ground motion with the MMI reported at the recording site. Since all of these 186 records have been obtained in the Western United States, these data and the correlations in this paper reflect the procedures which are used to arrive at MMI for this part of the United States only. The type of buildings and other man-made structures that were damaged or affected by the shaking characterized by these 186 records as well as the characteristics of the human response to shaking in this part of the United States are also implicitly contained in these correlations. Therefore, before an attempt is made to transfer the experience gathered from the related damage and the corresponding characteristics of these data to other regions of the United States and the world, caution must be exercised to account for all possible biases in the computation of the MMI or its equivalents and the type of construction damaged by the earthquakes studied in this paper, relative to the construction and the related experiences elsewhere (e.g., Trifunac, 1977).

It is useful to emphasize here that the MMI is essentially a crude and short description of observed damage. For such a scale to give reliable and reproducible shorthand descriptions of the effects of strong shaking on man-made structures, it would be necessary, for example, that the same scale be applied all over the world and that all buildings and other structures be of the same type and be constructed in the same manner in all cities. Furthermore, it would be essential that the characteristics of the population responding to questionnaires on the level of damage and shaking as well as the interpretations of the experts who carry out detailed field investigations and report on the damage lead to identical responses for the same inputs all over the world. Since all of these conditions clearly cannot be met even within one metropolitan area, the same reported level on the MMI scale may correspond to significantly different instrumental characteristics of shaking even within a city which is shaken by the same earthquake. Thus, the characteristics of recorded accelerograms studied in this paper necessarily reflect all these uncertainties which result in wide variations and considerable scatter of data.

On the other hand, in most parts of the world the reports on the MMI or its equivalent still represent the important source of historic information on earthquake occurrences and in some cases are all that is available for the analysis of seismic risk. Therefore, it seems worthwhile to present the overall trends and the characteristics of recorded strong ground motion with respect to the Modified Mercalli intensity scale and to show what the uncertainties associated with such correlations are.

In this paper as in a previous related work (Trifunac and Brady, 1975a) we have used only the MMI level reported at the stations which provided the 186 records we analyze here. In some cases these levels result from small nearby earthquakes, while in other cases they correspond to larger and more distant earthquakes. We did not try to distinguish between such cases on purpose because we intend to determine the trends and variations of recorded characteristics of strong shaking with respect to the MMI level at a station when only the MMI level is provided. Considering maximum epicentral intensity and or distance in these correlations would have reduced the scatter in the correlations but would have implicitly introduced the information on amplitude attenuation with distance for the data we employed. This would have further restricted the applicability of the correlations we present here to the Western United States only. Omitting the maximum MMI at the epicenter and the distance increases the scatter of correlations but makes results more applicable to other seismic regions outside the Western United States.

CORRELATION OF $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, AND $\int_0^T d^2\,dt$ WITH THE MODIFIED MERCALLI INTENSITIES

Because of the rough nature of and the uncertainties associated with the classification of strong ground motion in terms of the Modified Mercalli intensity, I_{MM}, we consider only the simplest correlation of the form

$$\log_{10} \int_0^T \begin{Bmatrix} a^2 \\ v^2 \\ d^2 \end{Bmatrix} dt = A + BI_{MM} \pm \sigma. \tag{2}$$

Table 1 of Trifunac and Westermo (1976b) gives the values of the coefficients A, B and the standard deviation σ for the six frequency bands of data and for the vertical and horizontal components of acceleration, velocity, and displacement for the least-squares regression given by equation (2). If it is assumed that the frequency bands used are narrow enough so that the band-pass filtered functions can be characterized by their center frequencies $\omega_c = 2\pi f_c$, then it is seen that

$$\log_{10} \int_0^T d^2\,dt \simeq \log_{10} \int_0^T a^2\,dt - 4 \log_{10} \omega_c$$

$$\log_{10} \int_0^T v^2\,dt \simeq \log_{10} \int_0^T a^2\,dt - 2 \log_{10} \omega_c. \tag{3}$$

Thus B should be the same for a, v and d while A should differ from a, to v to d by the factors shown in equation (3). Figure 1 shows the amplitudes of A and B in equation (2) versus $\log_{10} \omega_c$ for $\int_0^T a^2\, dt$. These curves show the range of values bounded by $\bar{A} \pm \max\,(\bar{A} - A_{\text{accel.}}, \bar{A} - A_{\text{vel.}}, \bar{A} - A_{\text{displ.}})$ and $\bar{B} \pm \max\,(\bar{B} - B_{\text{accel.}}, \bar{B} - B_{\text{vel.}}, \bar{B} - B_{\text{displ.}})$ and were filtered along $\log_{10} \omega_c$ with a three-point running mean filter $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$ to present the smoother trends of A and B versus frequency. The high-frequency displacement and the low-frequency acceleration data were omitted from this averaging process as in our previous paper (Trifunac and Westermo, 1976a) for reasons of low signal-to-noise ratio in the data processing. Figure 1 shows the values of A corrected to represent horizontal and vertical components of acceleration, and equation (3) then gives the amplitudes of the related integrals for velocities and displacements.

CORRELATIONS OF THE FREQUENCY-DEPENDENT DURATION OF $a(t)$, $v(t)$, AND $d(t)$ WITH THE MODIFIED MERCALLI INTENSITY

A correlation of the form

$$\text{Duration of} \begin{Bmatrix} a \\ v \\ d \end{Bmatrix} = A + B\mathrm{I}_{\mathrm{MM}} \pm \sigma \tag{4}$$

is used here where A and B are the coefficients for vertical and horizontal components of acceleration, velocity, and displacement for the six frequency bands, and σ is the standard deviation with respect to the regression equation $A + B\mathrm{I}_{\mathrm{MM}}$. Table IV of Trifunac and Westermo (1976b) gives the values of A, B, and σ. If we again assume that the frequency bands are narrow enough to approximate the characteristics of $a(t)$, $v(t)$, and $d(t)$ by a center frequency, ω_c, then it follows that

$$\text{duration of } \{v(t)\} \simeq \text{duration of } \{a(t)\}$$

$$\text{duration of } \{d(t)\} \simeq \text{duration of } \{a(t)\}. \tag{5}$$

Figure 2 shows the plots of A and B for horizontal and vertical components of motion versus $\log_{10} \omega_c$. These curves were drawn in the same manner as the coefficients A and B in the previous section.

The duration of strong shaking does not appear to differ much for the horizontal and vertical components of motion as shown in Figure 2. For all frequencies the duration decreases with an increasing intensity. For low frequencies ($f_c = 0.2$ Hz) the duration decreases by about 5 sec for each level of intensity while for high frequencies ($f_c = 18.0$, 7.0, and 2.7 Hz) it decreases by only about 2 to 3 sec.

Figure 3 shows the mean value and standard deviations of the duration of acceleration for all the frequency bands, grouped by intensity. The duration tends to be long for low frequencies ($f_c = 0.2$, 0.5 Hz) and short for high frequencies ($f_c = 7.0$, 18.0 Hz). The difference between the durations for high frequencies and low frequencies is about 10 to 20 sec, and is close to 20 sec for low intensities.

To examine the effects of the recording site conditions all the data were divided into the three site classification groups $s = 0$, 1, and 2. Figure 4 shows the means and standard deviations of the durations for each site classification within each intensity level. Typically the durations are found to be shorter for hard sites ($s = 2$) than for

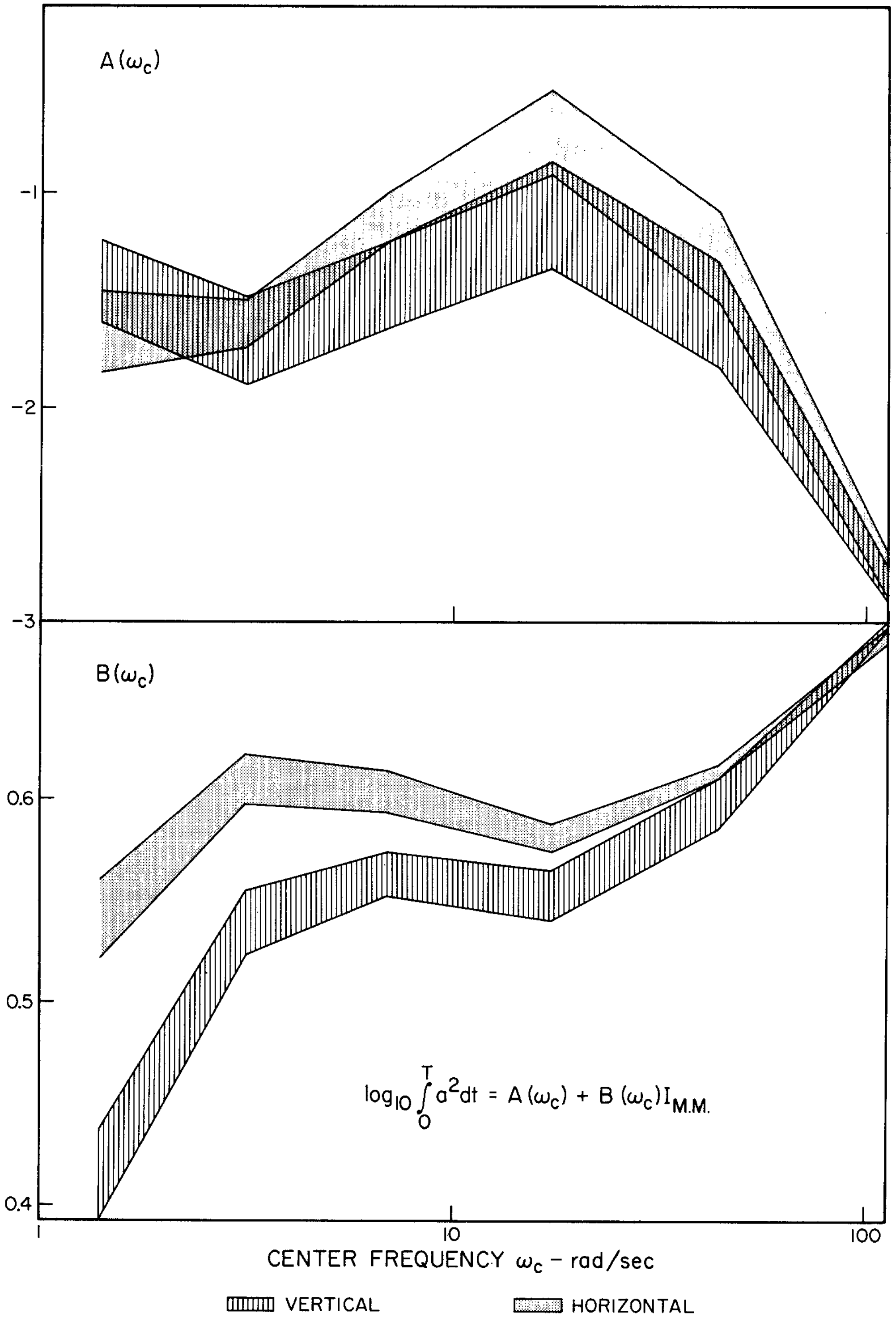

FIG. 1. Amplitudes of $A(\omega_c)$ and $B(\omega_c)$ in $\log_{10} \int_0^T a^2\,dt, \int_0^T v^2\,dt, \int_0^T d^2\,dt) = A(\omega_c) + B(\omega_c)\mathrm{I}_{\mathrm{MM}}$ for horizontal and vertical components of strong earthquake ground motion plotted versus ω_c.

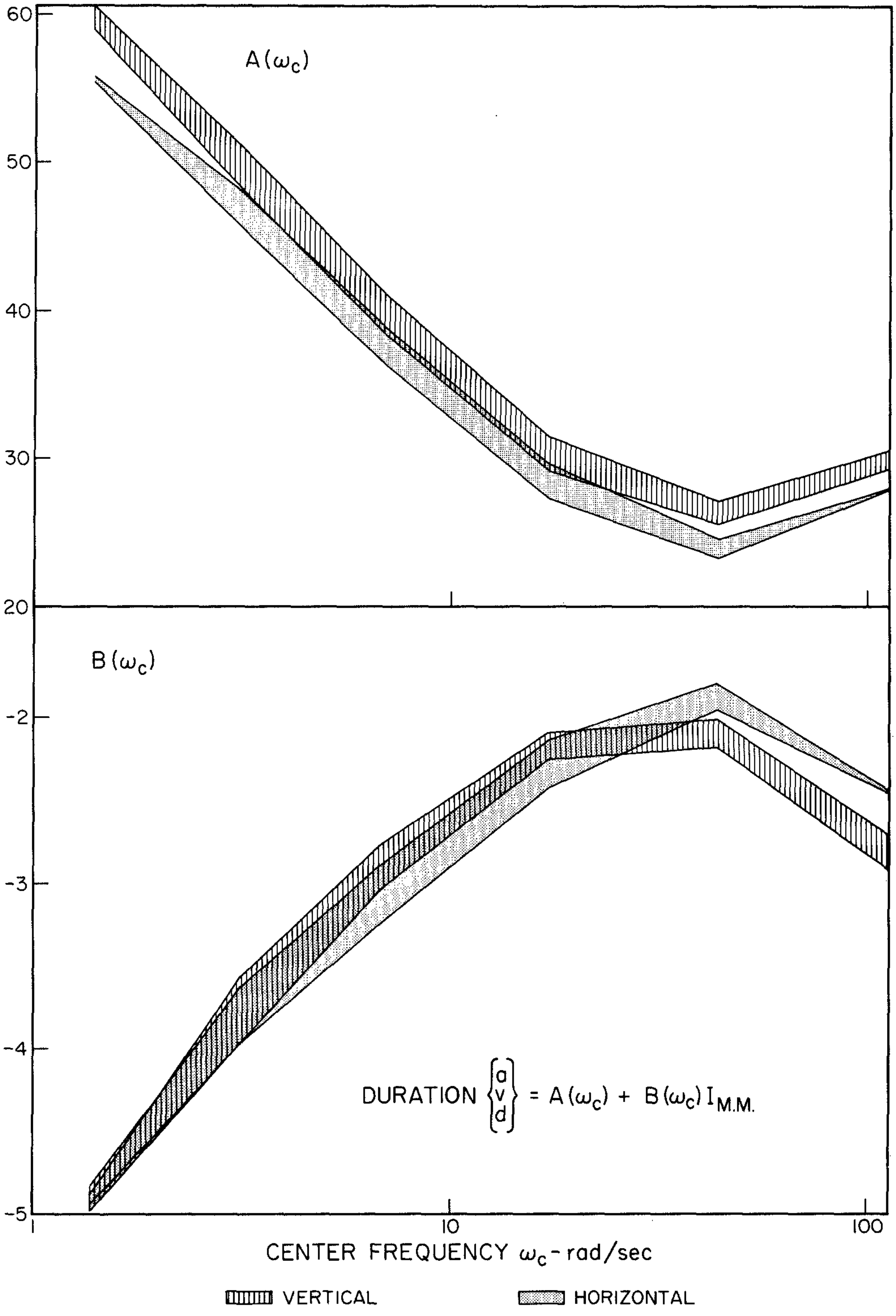

FIG. 2. Amplitudes of $A(\omega_c)$ and $B(\omega_c)$ in duration $(a, v, d) = A(\omega_c) + B(\omega_c)\mathrm{I}_{\mathrm{MM}}$ for horizontal and vertical components of strong earthquake ground motion plotted versus ω_c.

"softer" sites ($s = 0, 1$) for all intensities in IV $\leqq I_{\text{MM}} \leqq$ VIII. For a hard site ($s = 2$) the duration does not vary with the frequency by more than about 7 sec, while for soft sites this variation with frequency is about 20 sec. This trend might be due to the greater attenuation of waves and the predominance of inhomogeneities underneath the stations which are classified under $s = 0$. For all three site classifications the durations of low-frequency waves tend to be longer than the durations of the high-frequency waves. The standard deviations of the durations are also larger for soft sites ($s = 0$) than for hard sites ($s = 2$) for all intensities. This is probably caused by repeated reflections inside the lower velocity layers beneath $s = 0$ sites, and by late arrivals of scattered wave energy which traveled along longer paths before arriving at the recording stations (e.g., Trifunac, 1971; Wong and Trifunac, 1974). However, the fact that many more observations are available for $s = 0$ sites than for $s = 2$ sites may also have an effect on these differences in the observed standard deviations.

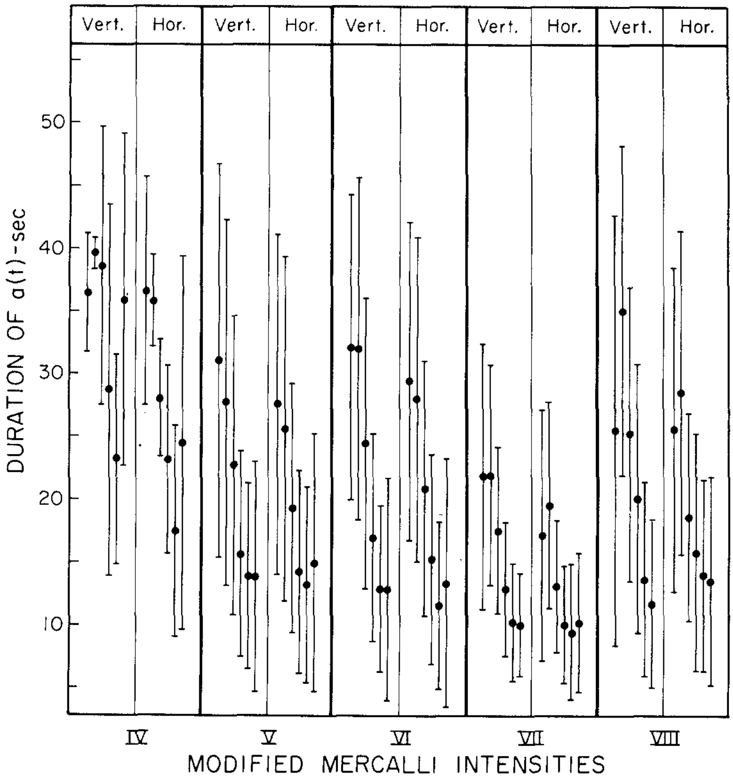

FIG. 3. Mean values and standard deviations of duration of horizontal and vertical accelerations for six frequency bands (f_c = 0.2, 0.5, 1.1, 2.7, 7.0 and 18.0, from *left* to *right*) and for Modified Mercalli intensities IV through VIII.

CORRELATION OF THE RATE OF STRONG GROUND MOTION WITH THE MODIFIED MERCALLI INTENSITY

To avoid the cumulative error in using the correlations for duration and the integrals already presented to calculate the correlation for the average time rate at which the strong shaking evolves with respect to MMI we develop the correlations directly from

$$\log_{10}\left[\int_0^T \left\{\begin{matrix} a^2 \\ v^2 \\ d^2 \end{matrix}\right\} dt/\text{duration of} \left\{\begin{matrix} a \\ v \\ d \end{matrix}\right\}\right] = A + B\mathrm{I}_{\mathrm{MM}} \pm \sigma. \tag{6}$$

Table VII of Trifunac and Westermo (1976b) presents the values of A, B, and the standard deviation, σ, for horizontal and vertical components of acceleration, velocity, and displacement and for the six frequency bands. Making the same assumptions about the narrow-frequency bands of data as in the previous sections yields

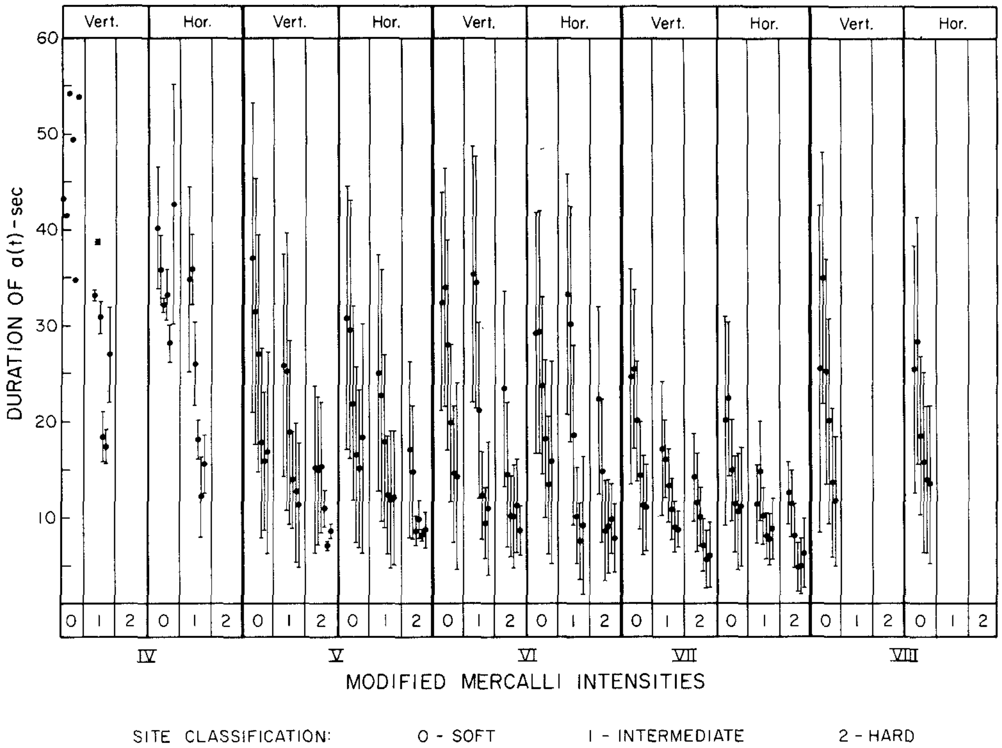

FIG. 4. Mean values and standard deviations of duration of horizontal and vertical accelerations for six frequency bands (f_c = 0.2, 0.5, 1.1, 2.7, 7.0 and 18.0, from *left* to *right*), different site classifications and for Modified Mercalli intensities IV through VIII.

$$\log_{10}\left[\int_0^T d^2\, dt/\text{duration}\right] \simeq \log_{10}\left[\int_0^T a^2\, dt/\text{duration}\right] - 4\log_{10}\omega_c$$

$$\log_{10}\left[\int_0^T v^2\, dt/\text{duration}\right] \simeq \log_{10}\left[\int_0^T a^2\, dt/\text{duration}\right] - 2\log_{10}\omega_c. \tag{7}$$

Figure 5 shows the coefficients A and B versus $\log_{10}\omega_c$, smoothed and drawn as mentioned before. It is seen from this figure that the vertical component of motion is characterized by a larger variation of the values of B than the horizontal component.

For the high frequencies (f_c = 18.0 Hz), the vertical and horizontal components of rate increase by about 5 to 6 times for each level of MMI. At the low frequencies (f_c = 0.2 Hz) the horizontal and vertical components increase by about 4 and 3 times, respectively, for each intensity level.

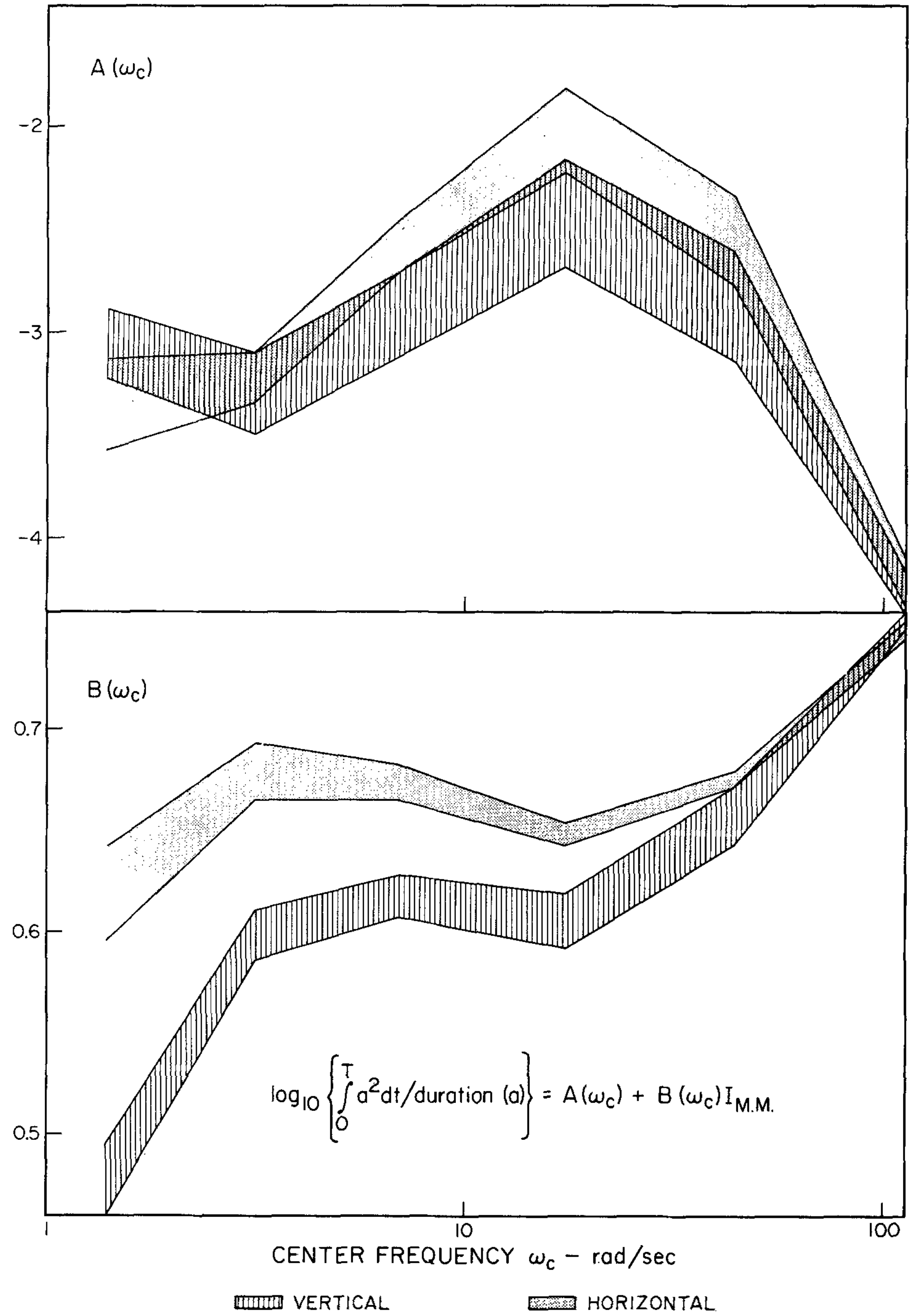

FIG. 5. Amplitudes of $A(\omega_c)$ and $B(\omega_c)$ in $\log_{10} \{ \int_0^T a^2\, dt/\text{duration (a)} \} = A(\omega_c) + B(\omega_c)\mathrm{I}_{\mathrm{MM}}$ for horizontal and vertical components of strong earthquake ground motion plotted versus ω_c.

CONCLUSIONS

The characteristics of the duration of strong earthquake ground motion and the properties of the related integrals of squared acceleration, velocity, and displacement are too detailed and complex to be summarized in a few concluding remarks. Instead we present here only some of the more important findings and invite the reader to examine the numerous figures presented by Trifunac and Westermo (1976b) while reading this last section in order to fill in the details not mentioned in the following text.

The amplitudes of the integrals $\int_0^T a^2\,dt$, $\int_0^T v^2\,dt$, $\int_0^T d^2\,dt$ for the horizontal components of strong shaking may be as much as one order of magnitude larger than the corresponding amplitudes for vertical components of motion and for the Modified Mercalli intensity VIII. For I_{MM} = IV this factor reduces to about 3. The effect of site condition on $\int_0^T a^2\,dt$ is such that at the hard sites ($s = 2$) the amplitudes of this integral tend to have maxima at the higher frequencies ($f_c = 7.0$ Hz) than for the soft sites ($s = 0$).

The computed durations of strong ground motion tend to decrease with an increase in intensity. For a unit increment on the MMI scale the durations at low frequencies ($f_c = 0.2$ Hz) decrease by about 5 sec, while the durations for higher frequencies (f_c = 18.0, 7.0, and 2.7 Hz) decrease by about 2 to 3 sec. For the Modified Mercalli intensity of VII the computed durations do not vary with frequency by more than about 5 to 10 sec, while at I_{MM} = V, this variation with frequency may be as much as 20 sec. The observed durations at hard sites ($s = 2$) are typically 10 to 15 sec shorter than the durations for soft sites ($s = 0$). The standard deviations of the durations tend to be smaller for hard sites than for soft sites.

The average rate of growth of the integrals of the squared acceleration, velocity, and displacement (also referred to briefly as "power") tends to increase by about six times at high frequencies ($f_c = 18.0$ Hz) for each unit of MMI. For the low frequencies ($f_c = 0.2$ Hz) this rate increases by about three times for vertical and four times for horizontal components of motion for each level of the intensity. For strong shaking recorded at hard sites ($s = 2$), the rate can be as much as one order of magnitude larger than the corresponding motion at soft sites ($s = 0$), for high frequencies ($f_c = 18.0$ Hz) and for low Modified Mercalli intensities considered here. This trend is completely reversed for low-frequency motions ($f_c = 0.2$ Hz) and higher Modified Mercalli intensities.

The large standard deviations accompanying the proposed regression models appear to result mainly from three sources: (1) the purposely neglected dependence of the observed characteristics of shaking on the source-to-station distance, (2) the low signal-to-noise ratio in processing certain frequency bands of data, and (3) the imprecision in characterizing the level of shaking at a recording site by the MMI. While examining the trends displayed in different figures, one should also examine the tables in Trifunac and Westermo (1976b) to note that for many of the intensities and site classifications the number of available data points is far from adequate to provide the detailed and reliable picture of all and complete characteristics of the strong ground motion which have been studied in this paper. Therefore, the numerous tables and the corresponding correlations can only be interpreted to represent an interim and preliminary picture which must be modified and updated when more strong-motion records become available.

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