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DETERMINATION OF LOCAL MAGNITUDE, M_L, FROM SEISMOSCOPE RECORDS

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ABSTRACT

A method is presented for determining the local magnitude, M_L, from records from seismoscopes and similar instruments. The technique extrapolates the maximum response of the standard Wood-Anderson seismograph, which determines M_L, from the maximum response of the seismoscope. The standard deviation of the steady-state response of an oscillator subjected to white noise excitation is used to derive a relation correcting for the different periods, dampings, and gains of the two instruments. The accuracy of the method is verified by application to data from the San Fernando and Parkfield earthquakes wherein both accelerograph and seismoscope records are available from the same sites. The accelerograms are used to synthesize Wood-Anderson responses whose maxima are compared to those extrapolated from the seismoscope data. In both earthquakes, the average magnitudes and standard deviations determined by the two approaches are very nearly equal.

The method is then applied to the strong-motion data from the Managua, Nicaragua earthquake of December 23, 1972 (M_S = 6.2, m_b = 5.6). A value of M_L = 6.2 is indicated from the seismoscope and accelerograph data. The next application is to the Guatemala earthquake of February 4, 1976 (M_S = 7.5, m_b = 5.8). The only seismic instrumentation available for determining M_L is a seismoscope record from Guatemala City, which indicates M_L = 6.9 when a representative distance of about 35 km is used. As a final example, the records obtained during the 1906 San Francisco earthquake (M_S = $8\frac{1}{4}$) from the Ewing duplex pendulum seismograph at Carson City, Nevada and the simple pendulum at Yountville, California are analyzed. After restoring the Carson City instrument, its period and damping were determined experimentally as were the period and damping of a similar instrument in the London Science Museum. On the basis of the strong-motion records from Carson City and Yountville, it is estimated that the local magnitude of the 1906 earthquake lies in the range $6\frac{3}{4}$ to 7.

The use of seismoscope data further extends the instrumental base from which M_L can be determined and allows the rapid determination of M_L in earthquakes where seismoscope data are available. The applications in this study provide further instrumental evidence for the saturation of M_L in the 7 to $7\frac{1}{2}$ range, with the value of 7.2 for the Kern County earthquake of 1952, the largest so far determined.

INTRODUCTION

The concept of the magnitude of an earthquake was first introduced by C. F. Richter (1935) to measure the size of earthquakes in southern California. The idea has since gained wide acceptance and is now the most commonly used measure of the size of the earthquake, particularly for applications in engineering and for reporting earthquake information to the general public. The original concept has been expanded and modified so that there are now several magnitude scales in use, with the term local magnitude reserved for the original definition. Other common magnitudes include the surface-wave magnitude, M_S, and the body-wave magnitude, m_b. Although the surface-wave and body-wave magnitudes are perhaps used more now in seismology, the local magnitude, M_L, is the most directly relevant of the magnitude scales for engineering applications because it is defined in terms of the

response of an instrument, the Wood-Anderson seismograph, whose period and damping are such that it is sensitive to motions in the frequency range of most interest to engineering. The Wood-Anderson seismograph used in the definition of M_L has a natural period of 0.80 sec, a critical damping fraction of 0.80, and a gain of 2800. In addition, M_L is determined closer to the source of the earthquake than are other magnitude scales so the ground motion at the instrument site resembles more closely, in frequency content and duration, the strong shaking in the epicentral region than do the ground motions which determine other magnitudes.

In a previous paper (Kanamori and Jennings, 1978), the authors presented a method for determining M_L from strong-motion accelerograph records. The method is based on the generation of synthetic seismograms by using the strong-motion accelerograms as acceleration inputs to the equation of motion of the Wood-Anderson seismograph. In the present study, a related method is presented for the calculation of M_L from the response of seismoscopes. The accuracy of the approach is demonstrated by application to the 1971 San Fernando earthquake and the 1966 Parkfield earthquake. It is then applied to the 1972 Managua, Nicaragua earthquake, the Guatemala earthquake of 1976, and the 1906 San Francisco earthquake. In the last two cases, seismoscopes, or similar instruments, provided the only seismological records obtained within several hundred kilometers of the causative fault.

The Seismoscope

The seismoscope is a low-cost passive instrument designed to produce a representative point on the response spectrum. A photograph of the instrument is given in Figure 1; it is basically a conical pendulum. Typically, the seismoscope has a period near 0.75 sec and a nominal damping value of 0.10. The properties and capabilities of the seismoscope are well-documented in the literature (Cloud and Hudson, 1961; Hudson and Cloud, 1967; Morrill, 1971).

The seismoscope record consists of a hodograph of response scratched on a standard $2\frac{1}{2}$-in smoked watch glass, as shown in Figure 2. The dynamic range of the instrument is slightly larger than one order of magnitude, covering the range of shaking from about the threshold of human perceptibility up to nearly the strongest expected motions. (Some seismoscopes have gone off-scale under very strong shaking.)

In standard applications the maximum amplitude of the record is read and converted into a response spectrum ordinate for a specified value of damping, normally 10 per cent of critical (Cloud and Hudson, 1961; Morrill, 1971). The overall maximum can be determined or the maxima of the response can be established in desired directions. The seismoscope does not have a time signal, but in some cases the oscillations of one of the higher modes of the pendulum provides timing information on the record which enables the approximate calculation of the acceleration input to the instrument (Trifunac and Hudson, 1970; Scott, 1973).

Analysis

The response of the seismoscope in a given direction can be viewed as that of a single-degree-of-freedom oscillator with a period of about 0.75 sec, a damping of about 0.10, and a static magnification, or gain, which depends on the geometry of the instrument. The Wood-Anderson Seismograph is also a simple oscillator with a period of 0.80 sec, a damping of 0.80, and a gain of 2800. A comparison of the properties of the two instruments suggests that if a correction for the different dampings and the small difference in periods could be developed, the response of

the Wood-Anderson seismograph could be inferred from the response of the seismoscope. The difference in gains requires only a simple multiplicative factor.

The correction factor we apply to account for the different characteristics of the two instruments is based on a result from the theory of random vibrations (Crandall and Mark, 1963). If a single-degree-of-freedom oscillator with unit mass is subjected to a force which is a white noise with mean zero and spectral density D, and the response is allowed to achieve statistical stationarity, the temporal mean value of the response is zero and the mean square is given by

$$\langle x^2 \rangle = \frac{DT_n^{\,3}}{16\zeta\pi^2} \tag{1}$$

in which $\langle x^2 \rangle$ is the mean square of the response and T_n and ζ are the undamped natural period and damping factor, respectively, of the oscillator. The result

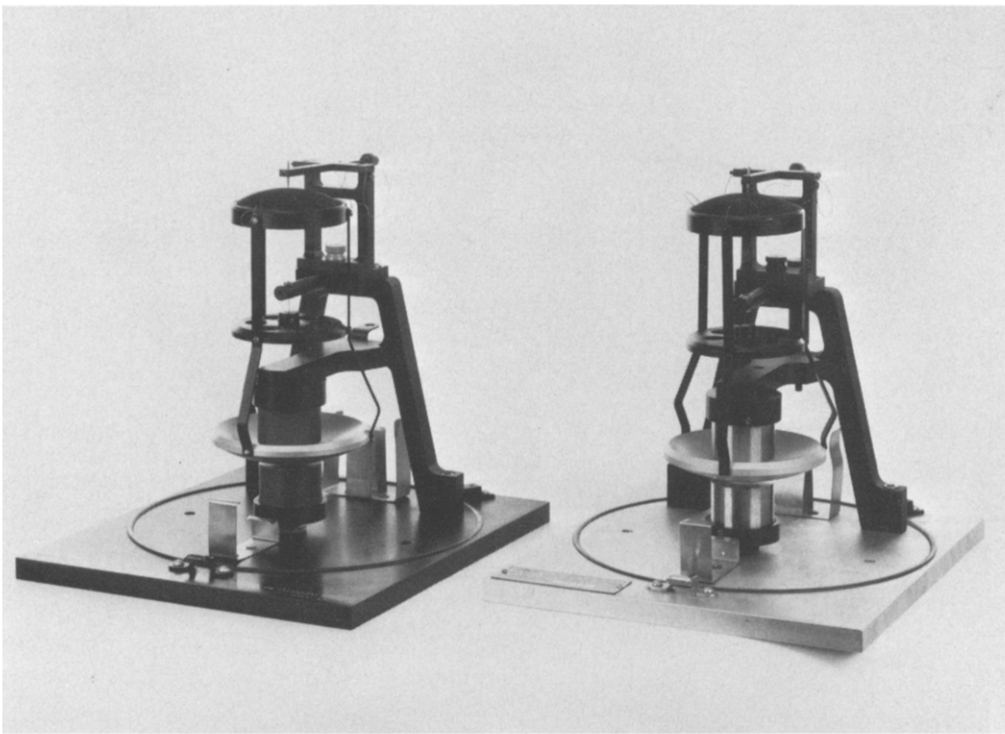

FIG. 1. Two versions of the strong-motion seismoscope. The transducer is a conical pendulum suspended by a fine wire from the horizontal beam. The record is scribed on an inverted smoked watch glass.

holds for all values of T_n and ζ and for an oscillator gain of unity. In addition to describing the statistics of response of a single oscillator subjected to an infinitely long white noise excitation, the results also hold for the ensemble average response of a family of identical oscillators subjected to an ensemble of white noise excitations of spectral density D. Thus, under these conditions, the ensemble mean of the response is zero, and the mean square response is given by equation (1).

Equation (1) can be applied to the response of simple oscillators to strong ground shaking if the excitation has a spectrally smooth, broadband character in the period range of interest, and if the duration is long with respect to the natural periods

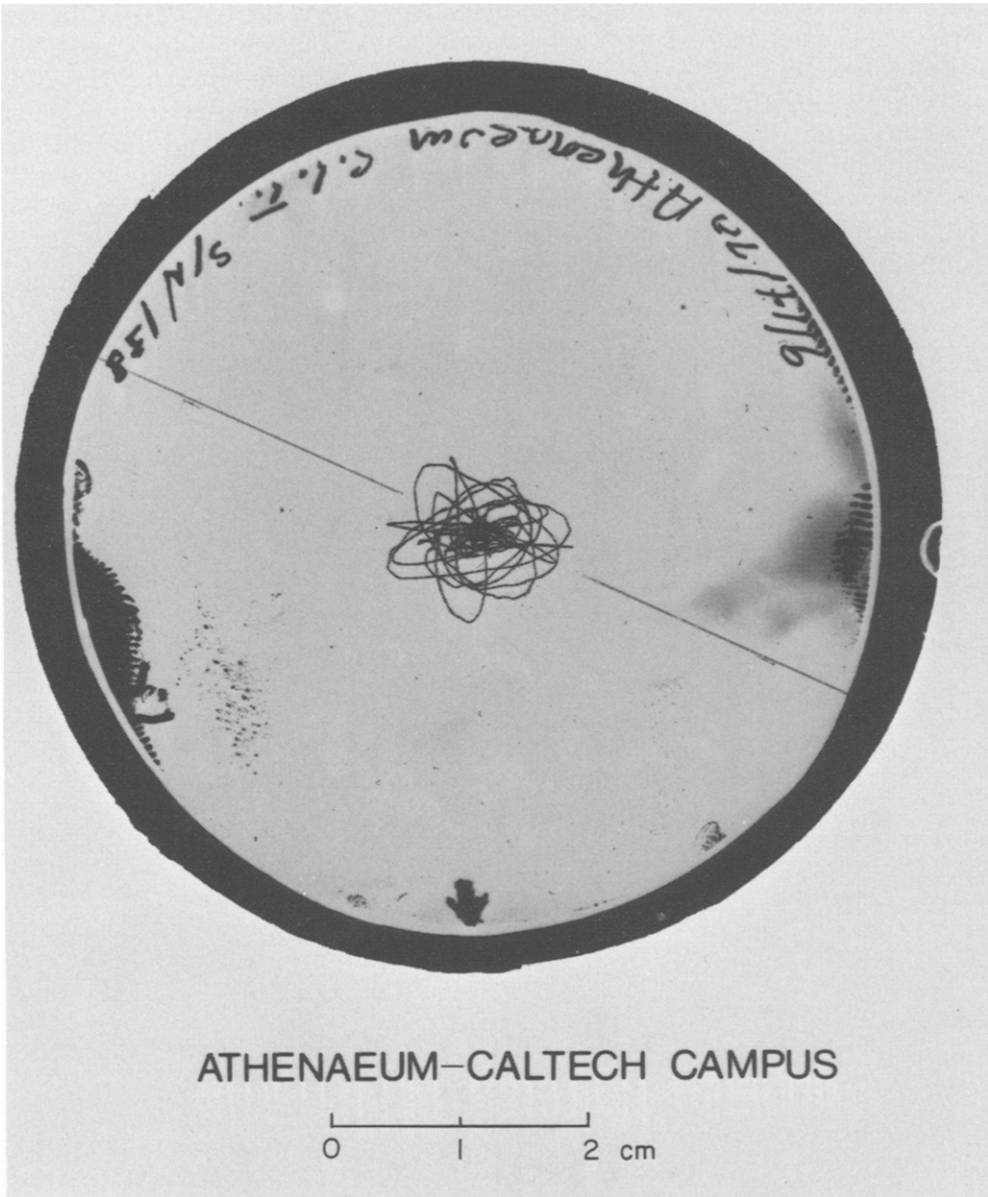

FIG. 2. Sample record from a seismoscope. This particular trace was obtained on the Caltech campus during the San Fernando earthquake of February 9, 1971.

involved. For example, Housner and Jennings (1964) used equation (1) in the development of an approximate formula describing average response spectra.

If equation (1) is applied to the response of a Wood-Anderson seismograph subjected to a strong ground acceleration, the result can be written

$$A_{wa} \propto V_{wa} \sqrt{\frac{T_{wa}^3}{16\pi^2 \zeta_{wa}}} \tag{2}$$

in which V_{wa} is the static magnification of the instrument, T_{wa} is the period, ζ_{wa} the fraction of critical damping, and A_{wa} is the amplitude of response.

The seismoscope is also a simple one-degree-of-freedom oscillator when the response to one component of ground motion is considered. Cloud and Hudson (1961) give the governing equation as

$$\ddot{\phi} + \frac{cb^2}{I_0}\dot{\phi} + \frac{mga}{I_0}\phi = -\frac{ma}{I_0}\ddot{y}(t). \tag{3}$$

In equation (3) ϕ is the angular deflection, $\ddot{y}(t)$ the base acceleration, m the mass of the pendulum, and g the gravitational constant. The remaining parameters are instrumental constants. The equation can be rewritten in the form

$$\ddot{\phi} + 2\omega\zeta\dot{\phi} + \omega^2\phi = -\frac{\omega^2}{g}\ddot{y}(t) \tag{4}$$

in which

$$\omega^2 = \frac{mga}{I_0} \quad \text{and} \quad \zeta = \frac{cb^2}{2\sqrt{mgaI_0}}.$$

The displacement on the seismoscope plate is related to the angular deflection by the sensitivity S

$$x = S\phi \tag{5}$$

(e.g., Morrill, 1971, p. 76). In terms of x, equation (4) becomes

$$\ddot{x} + 2\omega\zeta\dot{x} + \omega^2 x = -\frac{\omega^2 S}{g}\ddot{y}(t). \tag{6}$$

Thus, the gain or static magnification of the instrument is

$$V_{sc} = \frac{\omega^2 S}{g} = \frac{4\pi^2 S}{gT_{sc}^2} \tag{7}$$

in which V_{sc} is the gain of the seismoscope and T_{sc} is the natural period of the instrument.

This development shows that the seismoscope can be considered as a seismometer with known period, damping, and static amplification. In response to the same excitation as the Wood-Anderson seismograph, the equation corresponding to equation (2) is

$$A_{sc} \propto V_{sc}\sqrt{\frac{T_{sc}^3}{16\pi^2\zeta_{sc}}} \tag{8}$$

in which A_{sc} is the maximum value of x. From equations (2) and (8)

$$A_{wa} = \frac{V_{wa}}{V_{sc}}\sqrt{\left(\frac{T_{wa}}{T_{sc}}\right)^3\left(\frac{\zeta_{sc}}{\zeta_{wa}}\right)}A_{sc}. \tag{9}$$

Using the properties of the Wood-Anderson instrument noted above and the properties for the two most common seismoscopes: Wilmot ($T_{sc} = 0.75$, $S = 5.45 \times$

10^{-2} m/rad), and Sprengnether ($T_{sc} = 0.78$, $S = 6.00 \times 10^{-2}$ m/rad) given by Morrill (1971), equation (9) reduces to

$$A_{wa} = 8840 \sqrt{\zeta_{sc}}\, A_{sc} \quad \text{(Wilmot)} \tag{10}$$

$$A_{wa} = 8180 \sqrt{\zeta_{sc}}\, A_{sc} \quad \text{(Sprengnether).} \tag{11}$$

Sometimes it is more convenient to work with the seismoscope results after they have been converted into ordinates of response spectra. Because the displacement response spectrum is defined by the maximum absolute value of z in the equation

$$\ddot{z} + 2\omega\zeta\dot{z} + \omega^2 z = -\ddot{y}(t), \tag{12}$$

it follows from comparison of equations (6) and (12) that

$$S_d = \frac{g T_{sc}{}^2 A_{sc}}{4\pi^2 S} \tag{13}$$

in which S_d is the displacement spectrum ordinate for the period and damping of the seismoscope. The seismoscopes have variable damping and the ordinates are usually corrected to 10 per cent damping by the relation suggested by Cloud and Hudson (1961)

$$S_{d_{10}} = \frac{g T_{sc}{}^2 A_{sc}}{4\pi^2 S} \sqrt{\frac{\zeta_{sc}}{0.10}}. \tag{14}$$

Using equations (14) and (7) in equation (9) with the given properties of the Wood-Anderson seismograph,

$$A_{wa} = 708 \frac{S_{d_{10}}}{T_{sc}{}^{3/2}}. \tag{15}$$

After the amplitude of the Wood-Anderson instrument is calculated by use of one of the above equations, the result can be used to determine M_L. For the results reported in this study, the amplitude of the seismoscope response, A_{sc}, was taken as one-half the peak-to-peak response. The readings typically were taken in each of two perpendicular directions (e.g., NS and EW) to estimate two components of Wood-Anderson response. The local magnitude was then found from using a nomographic version of the amplitude attenuation function given by Richter (1958, p. 342).

Applications

San Fernando earthquake. In the San Fernando earthquake of February 9, 1971 there were 16 installations where both accelerographs and seismoscopes gave useable records of the motion. The records at these sites can be used to evaluate the accuracy of determining M_L from seismoscope records by comparing the maximum Wood-Anderson response extrapolated from the seismoscope record to that synthesized from the corresponding accelerogram according to the procedures of our earlier study (Kanamori and Jennings, 1978). The 16 sites used were determined from examination of the results presented by Morrill (1971) and Hudson *et al.* (1969–

TABLE 1

COMPARISON OF M_L FOR THE SAN FERNANDO EARTHQUAKE, FEBRUARY 9, 1971 CALCULATED FROM SEISMOSCOPE RESPONSE AND FROM ACCELEROGRAPH RECORDS

Station	Accelerograph Ref.*	Seismoscope Ref.†	Component	Δ‡ (km)	From Accelerograph Records§		From Seismoscope Response§	
					PP/2 (m)	M_L	PP/2 (m)	M_L
Arcadia								
Santa Anita Reservoir	P221	565	N03E	38.5	4.92	6.05	8.6	6.25
			N87W		5.74	6.10	7.5	6.20
Lake Hughes								
Station 4	J142	2891	S21W	35.2	15.8	6.4	11.6	6.30
			S69E		10.0	6.25	16.2	6.45
Station 9	J143	2892	N21E	34.0	6.74	6.10	9.1	6.20
			N69W		6.40	6.10	9.8	6.20
Station 12	J144	2893	N21E	30.8	15.4	6.3	11.2	6.15
			N69W		12.2	6.2	18.6	6.40
Long Beach								
Utilities Bldg.	0204	147	N/S	65.2	5.98	6.5	8.0	6.65
			E/W		5.64	6.5	7.2	6.60
Terminal Island	0205	149	N21W	65.0	5.09	6.45	7.8	6.65
			S69W		6.60	6.55	7.8	6.65
Los Angeles								
Hollywood Storage	D058	146	N/S	28.0	16.3	6.30	22.4	6.40
			E/W		30.4	6.50	27.4	6.50
UCLA	F105	137	N/S	30.2	6.65	5.9	10.5	6.15
			E/W		8.68	6.05	12.6	6.20
Vernon	F086	148	S07W	40.9	14.3	6.6	15.0	6.55
			N83W		17.4	6.65	19.0	6.70
Pasadena								
Athenaeum	G107	138	N/S	33.0	13.3	6.40	19.6	6.50
			E/W		18.9	6.50	20.6	6.50
Millikan	G108	166	N/S	32.6	15.4	6.4	25.1	6.55
			E/W		25.7	6.6	25.5	6.55
Seismology Laboratory	G106	152	N/S	29.0	7.93	6.00	9.5	6.05
			E/W		19.1	6.40	24.1	6.45
Pearblossom								
Pumping Plant	F103	2847	N/S	48.0	5.88	6.30	8.1	6.45
			E/W		6.28	6.40	8.3	6.45
Piru								
Santa Felicia Dam	E081	588	S08E	36.0	11.4	6.40	11.9	6.30
(outlet works)			S82W		7.35	6.15	11.8	6.30
San Dimas								
Puddingstone Reservoir	P223	521	N55E	59.5	5.58	6.4	11.8	6.70
			N35W		6.09	6.45	11.8	6.70
Santa Ana								
Orange County	F087	159	S04E	80.7	3.68	6.45	6.4	6.70
Engineering Bldg.			S86W		4.11	6.4	6.4	6.70
			M_L average		6.34 ± 0.19		6.44 ± 0.20	

* Accelerograph Ref. denotes the reference number of the accelerograms in the EERL Reports (Hudson *et al.* 1969–76).

† Seismoscope Ref. denotes the instrument number in Morrill (1971).

‡ Δ is calculated from the center of faulting, inferred to be at Pacoima Dam (34°20.04′; 118°23.29′).

§ PP/2 denotes one-half of the maximum peak-to-peak amplitude (in meters) of the Wood-Anderson seismograph.

1976). The results of the analysis of this data are given in Table 1 which presents the features of the records and instrument sites, values of Wood-Anderson response (peak-to-peak divided by two), and the local magnitudes determined from the two types of instruments.

It is seen from the results that the average value of M_L found from the accelerographs is 6.34 ± 0.19, whereas the average value of M_L determined from the seismoscopes is 6.44 ± 0.20. The two average values differ only by one-half a standard deviation which is not considered a significant amount. Also, the standard deviations are about the same size which suggests that the approximation introduced to determine M_L from the seismoscope response does not add much to the scatter in values of M_L caused by the source mechanism, travel paths, and local conditions. For comparison, in our previous study, 14 accelerograph records selected on the

TABLE 2

COMPARISON OF M_L FOR THE PARKFIELD EARTHQUAKE, JUNE 27, 1966 CALCULATED FROM SEISMOSCOPE RESPONSE AND FROM ACCELEROGRAPH RECORDS

Station	Accelerograph Ref.*	Component	Δ (km)	From Accelerograph Records		From Seismoscope Response	
				PP/2 (m)	M_L	PP/2 (m)	M_L
Cholame							
Array 2†	B033	N65E	0.08	92.2	6.35	>77.3	>6.3
		—		—	—	—	—
Array 5	B034	N05W	5.5	30.9	5.95	27.9	5.85
		N85E		27.7	5.9	20.6	5.75
Array 8	B035	N50E	9.7	15.0	5.7	15.1	5.65
		N40W		16.6	5.70	17.5	5.7
Array 12	B036	N50E	15.4	5.97	5.35	6.27	5.4
		N40W		8.56	5.55	6.27	5.4
Temblor—"APP"‡	B037	N65W	10.7	16.5	5.7	13.2	5.6
		S25W	11.2§	30.4	6.00	20.9	5.85
		M_L Average		5.73 ± 0.22		5.65 ± 0.18	

* Seismoscope Ref. denotes instrument number in Hudson and Cloud (1967). See Table 1 for explanation of other entries.

† Not used in computing overages, A_{sc} exceeded plate radius of 1.25 in.

‡ Seismoscope location 0.5 miles from accelerograph site.

§ To seismoscope.

basis of their locations produced an average M_L of 6.35 ± 0.26, and the value of M_L determined from four seismographic stations is 6.3 (Kanamori and Jennings, 1978).

Parkfield earthquake. A similar analysis was performed for the four installations where both accelerograms and seismoscope records are available from the Parkfield, California earthquake of June 27, 1966 (Hudson and Cloud, 1967). The results are shown in Table 2, along with partial results from the Cholame site 2 where the seismoscope went off scale. Again it is seen that the average values of M_L determined from the two sets of data are in good agreement and the standard deviations are about the same size.

From the results for the Parkfield and San Fernando data, it appears that the seismoscope response can be used to calculate the local magnitude reliably. Although the estimation of the Wood-Anderson response is approximate, as can be seen by comparing individual results in Tables 1 and 2, the approximation does not significantly affect the mean magnitude, nor does it appear to add significantly to the

standard deviation of the M_L values, in comparison to the results using the recorded acceleration.

The character of the extrapolation of the Wood-Anderson response from the seismoscope records is illustrated in Figure 3 which is a plot of the maximum ($\frac{1}{2}$ peak-to-peak) Wood-Anderson response calculated from the accelerograms and the same response approximated from the seismoscope records. If the extrapolation

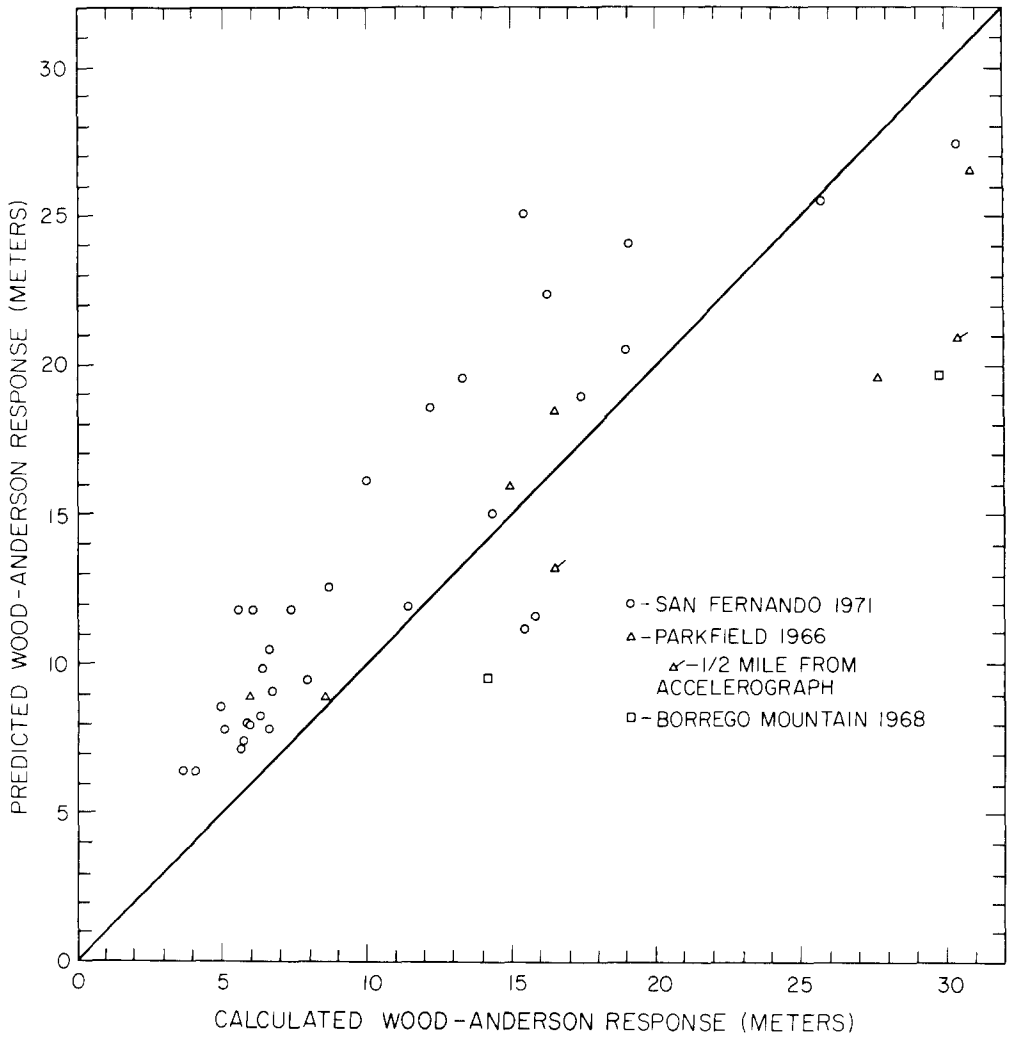

FIG. 3. Comparative plot of Wood-Anderson response as calculated from accelerograph records and estimated from seismoscope responses obtained at the same location. Data are from the San Fernando, Parkfield, and Borrego Mountain earthquakes.

were exact, all the points would fall on the line in the figure. The data in Figure 3 include that from Tables 1 and 2, plus two points from the Borrego Mountain earthquake of April 9, (1830 PST April 8) 1968. As implied by the average results in Tables 1 and 2, the data tend to cluster about the line. There is, however, a consistent tendency for the seismoscope analysis to overestimate the Wood-Anderson response for amplitudes less than 10 m. One possible cause may be an incorrect assessment of the values of seismoscope damping, ζ_{sc}, appropriate to very small response. (A Wood-Anderson amplitude of 10 m corresponds to a seismoscope

response of less than 4 mm.) No empirical modifications of the results presented here were made on the basis of Figure 3, but as more data accumulate it may be possible to make empirical adjustments to the predicted Wood-Anderson response.

It is also seen from Figure 3 that nearly all of the points lie within 0.6 to 1.4 (±40 per cent) of the values calculated from the accelerograms, which are considered correct. This corresponds to local magnitude differences of −0.22 and +0.15. Differences of this size often occur between different stations.

Managua, Nicaragua earthquake of December 23, 1972. The approach is applied next to the Managua, Nicaragua earthquake of December 23, 1972. This earthquake has been assigned a body-wave magnitude of 5.6 and a surface-wave magnitude of 6.2. The strong-motion instrumentation and other features of this very destructive earthquake have been reported in the proceedings of a special conference (Earthquake Engineering Research Institute, 1973). As reported by C. F. Knudson and

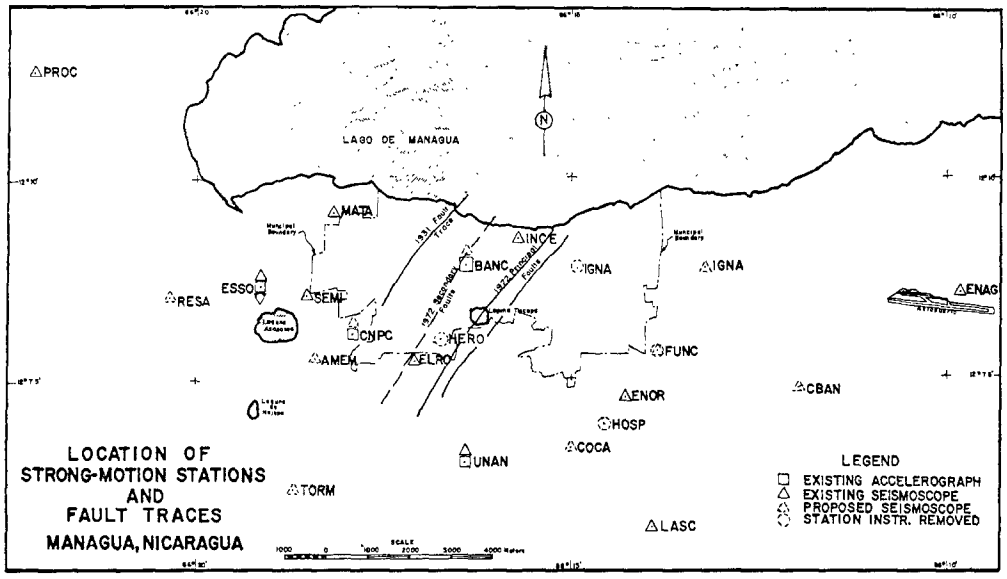

FIG. 4. Sketch map of Managua, Nicaragua showing fault traces and locations of strong-motion instrumentation at the time of the December 23, 1972 earthquake (Earthquake Engineering Research Institute, 1973).

F. Hansen A. in these proceedings, the strong-motion records included one accelerogram at the Esso refinery, and several seismoscope records in the area of Managua. Figure 4, which shows these sites, is taken from their paper. This earthquake provides an opportunity to determine M_L from both accelerograms and seismoscope records. In addition, the strong motion was recorded on seismoscopes of significantly different periods, allowing additional insight into the extrapolation to the Wood-Anderson response. It should be noted also that the strong-motion records provide the only means for determining M_L in this earthquake.

The results of the analysis of strong-motion records are summarized in Table 3. In the case of the accelerogram, M_L was determined from a synthesized record (Kanamori and Jennings, 1978). The seismoscope results were found by use of equations (7) and (9), with instrument sensitivities provided by personal communication from C. F. Knudson for two instruments whose sensitivities were not available in the aforementioned proceedings.

It is seen from the results that the three instruments at the Esso refinery, including

the seismoscope with the 0.50-sec period, all indicate M_L = 6.0 to 6.1. This same value results from the PROC seismoscope, which is also west of the epicentral area of the earthquake. It is more difficult to interpret the other three seismoscope records, because they either skipped or went off-scale. It does seem clear, however, that these three instruments indicate a larger M_L, perhaps 6.2 to 6.3. Considering all the records, M_L = 6.2 is recommended as an appropriate value of local magnitude for the Managua earthquake.

Guatemala earthquake of February 4, 1976. The Guatemala earthquake of February 4, 1976 occurred on the Motagua fault in the central and eastern part of the country. The shock was very destructive, particularly near the western end of the fault rupture, north of Guatemala City. A seismoscope in Guatemala City produced a strong-motion record and was the only seismic recording obtained near the fault, although a very small accelerogram was obtained in San Salvador (U.S.

TABLE 3

DETERMINATION OF M_L FOR THE MANAGUA, NICARAGUA EARTHQUAKE OF DECEMBER 23, 1972 FROM SEISMOSCOPE AND ACCELEROGRAPH RECORDS

Station	Instrument*	Component	Wood-Anderson Displ. PP/2	Δ†	M_L
ESSO	AR-240 Accelero-	NS	40.8	5.5	6.05
	graph	EW	35.8	5.5	6.0
ESSO	Seismoscope	NS	37.5	5.5	6.0
	#671, T = 0.75	EW	45.0	5.5	6.1
ESSO	Seismoscope	NS	49.0	5.5	6.1
	#673, T = 0.50	EW	36.7	5.5	6.0
PROC	Seismoscope	NS	32.0	12.0	6.05
	#672, T = 0.75	EW	33.2	12.0	6.05
MATA	Seismoscope	NS	68.2‡	4.5	6.25
	#561, T = 0.75	EW	48.6	4.5	6.1
BANC	Seismoscope	NS	80.7‡	1.5	6.3
	#558, T = 0.75	EW	80.7‡	1.5	6.3
ENAG	Seismoscope	NS	49.3§	11.5	6.2
	#670, T = 0.75	EW	54.2	11.5	6.25

* Instrument numbers are from Knudson and Hansen (Earthquake Engineering Research Institute, 1973). See Table 1 for explanation of other entries.

† Measured from center of faulting.

‡ Seismoscope off-scale, radius used as maximum displacement.

§ Stylus appears to have snagged during maximum excursion.

Geological Survey, Seismic Engineering Branch, 1976). The seismoscope record obtained from the instrument located on the ground floor of the University Administration building is shown in Figure 5. The earthquake has been assigned a surface-wave magnitude of M_S = 7.5 (NIS) and a body-wave magnitude of m_b = 5.8. More discussion of the seismological features of the earthquake are available in the publications of Espinoza (1976) and Kanamori and Stewart (1978).

The Guatemalan Seismoscope has a period of 0.78 sec, a sensitivity of 5.5 cm/rad, and for the amplitude shown in the record, a damping of 10 per cent. These values and the known ×2.97 enlargement of the record in the published photograph of the record (C. F. Knudson, personal communication) allow the calculation of $S_{d_{10}}$ by equation (14) for the NS and EW directions. These values, 4.41 and 4.85 cm, respectively, were then converted to Wood-Anderson responses by use of equation (15). The resulting values are A_{wa} = 45.3 m (NS) and A_{wa} = 49.9 m (EW).

To find the local magnitude from these values requires the determination of the

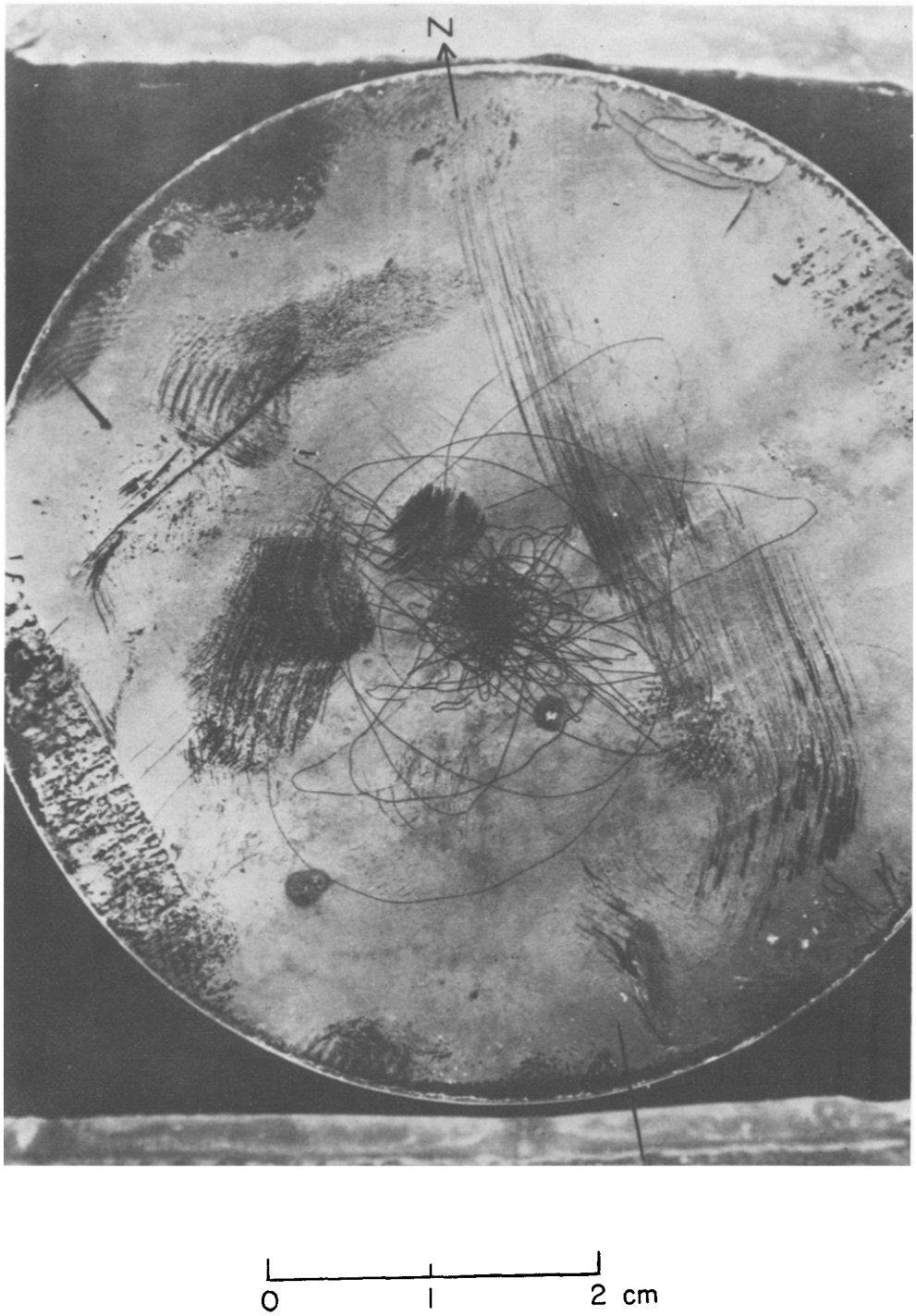

FIG. 5. Seismoscope record obtained on the ground floor of the University administration building in Guatemala City, Guatemala earthquake of February 4, 1976 (U.S. Geological Survey Seismic Engineering Branch, 1976).

distance from the instrument to a representative point on the causative fault. The nature of the problem of determining the appropriate distance is illustrated in Figure 6 in which it is seen that the epicentral distance is about 160 km, while the closest approach to the fault is only about 25 to 30 km. Although the faulting extended over a distance of 200 to 250 km (Plafker, 1976), the energy release does not seem uniform over the entire length of the fault. Kanamori and Stewart (1978) suggested, on the basis of teleseismic body-wave analysis, that the earthquake is a complex multiple shock consisting of at least 10 smaller events. The seismic moments of the individual events increase toward the western end of the fault. The reports of damage in the earthquake (Espinosa *et al.*, 1976) also suggest that a major part of the seismic energy was radiated from the western portion of the fault. Thus, it is most likely that the western part of the fault is responsible for the strong ground motion that affected Guatemala City.

If the 40 km from Guatemala City to El Progreso is taken as Δ, the two Wood-Anderson responses calculated above give $M_L = 7.0$ and 7.05, respectively. If a somewhat less conservative distance of 30 km is used, essentially the distance to the nearest point on the fault, the indicated values of M_L reduce to 6.8 and 6.75. Considering the uncertainties involved in the distances and the fact that only one record is available, M_L cannot be determined accurately in this case, of course, but the seismoscope record indicates that $M_L = 6.9$ is an appropriate value.

It is interesting to note that the seismic moment of the largest event of the multiple-shock sequence determined by Kanamori and Stewart (1978) is 5.3×10^{26} dyne-cm which would correspond to $M_S = 7.1$ if the standard M_S versus seismic moment relation is used. This value is very close to that of M_L determined above.

The San Francisco, California earthquake of 1906. This historic earthquake produced the first recognized evidence for the association of faulting with earthquakes and was the first great earthquake in the United States which was recorded on scientific instruments. The earthquake also provided the first impetus for earthquake resistant design in this country, and serves as a prototype of the potential of a great earthquake for causing a disaster in the United States. A reoccurrence of a similar earthquake on the San Andreas fault or elsewhere is often the controlling event in the design of major projects in California. Fortunately for later investigators, the earthquake was thoroughly reported by Andrew Lawson (1908) in what has become a classic work in seismology and earthquake engineering.

The earthquake is generally assigned a surface-wave magnitude of $8\frac{1}{4}$ (Gutenberg and Richter, 1949). In addition to the distant recordings which were used to determine the magnitude, a variety of instruments recorded the motion within the area of perceptible shaking (Lawson, 1908). For example, the three-component seismograph at the Lick Observatory produced a record which, although partially off-scale, has been analyzed successfully by Boore (1977). The recordings of interest in the present context are those made on the Ewing Duplex Pendulum seismographs, and on the simple pendulum at Yountville [see Reid (1910) pp. 60–65; Lawson (1908) Atlas sheet 3, p. 29]. These instruments are similar in their essential features to the seismoscope, and the records can be analyzed to estimate M_L by the method presented above. The records chosen for analysis are those from Yountville and Carson City. Records from Ewing Duplex Pendulum Seismographs were also obtained from Mt. Hamilton, Alameda, San Jose, Oakland, and Berkeley, but the motion at these sites was too great to produce directly useable records.

The Yountville record, Figure 7, was produced by a pendant mass. As described by Reid (1910) the instrument "... was a simple pendulum about a meter long, the

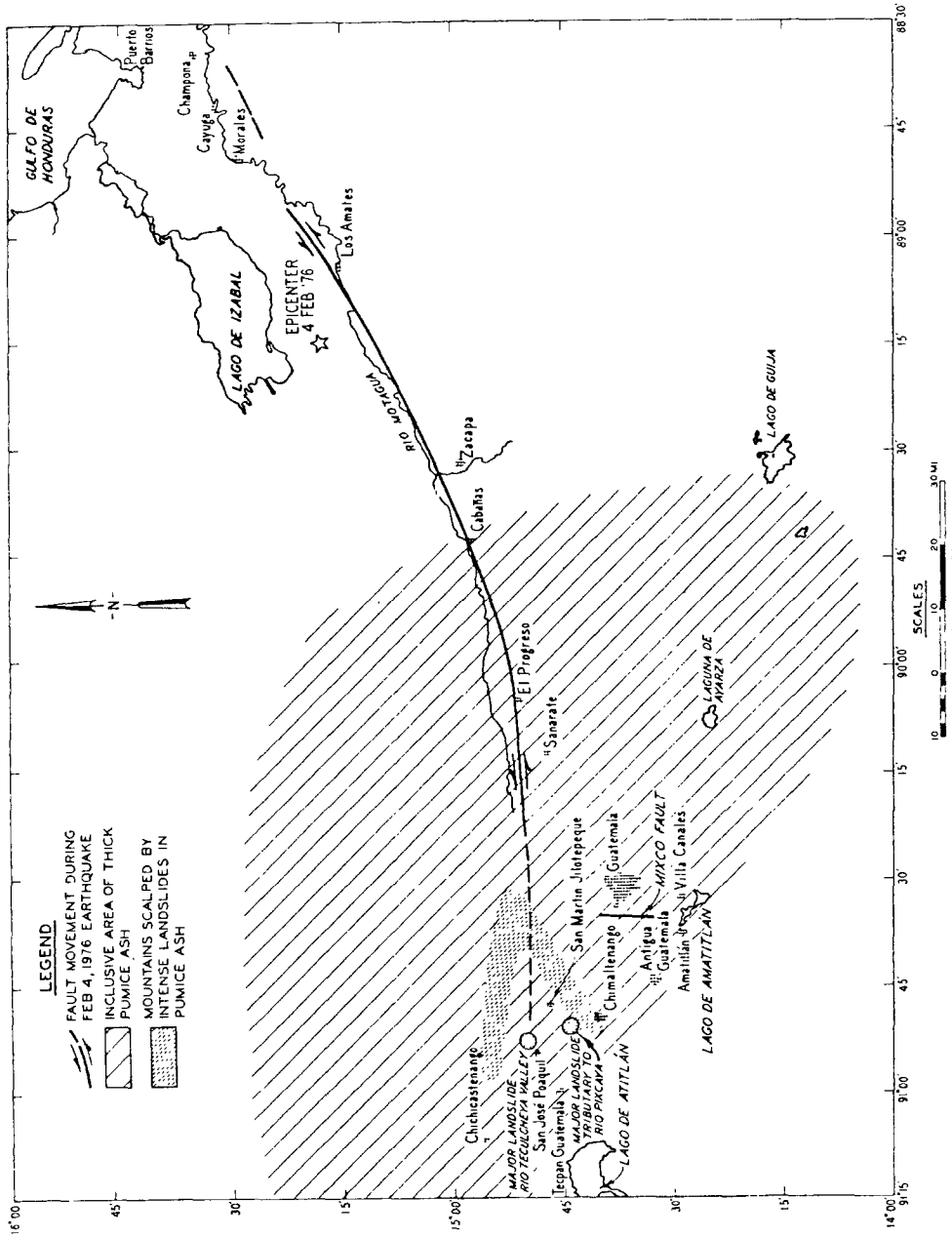

FIG. 6. Map of the area affected by the Guatemala earthquake of February 4, 1976. The observed and inferred rupture of the Motagua fault and some of the locations of principal damage are indicated. Note the easterly location of the epicenter. (Modified from Espinosa, 1976).

bob weighing 8.15 kg. A long pin passes freely thru a vertical hole in the middle of the bob and records on smoked glass below, with very little friction." The gain of the pendulum is given as 1.1±. The instrument was installed at the Veteran's Home (latitude 38°24′N; longitude 122°22′W) at a reported epicentral distance of 54 km. The foundations are described as alluvium over trachite.

The record (Figure 7) has a maximum peak-to-peak amplitude of 49 mm in the N-S direction and, inferring the closure of the easternmost peak, the same value for E-W response. For small, linear response the gain of a simple pendulum is 1, and the given value of 1.1 is interpreted as including the effect of the mechanical extension

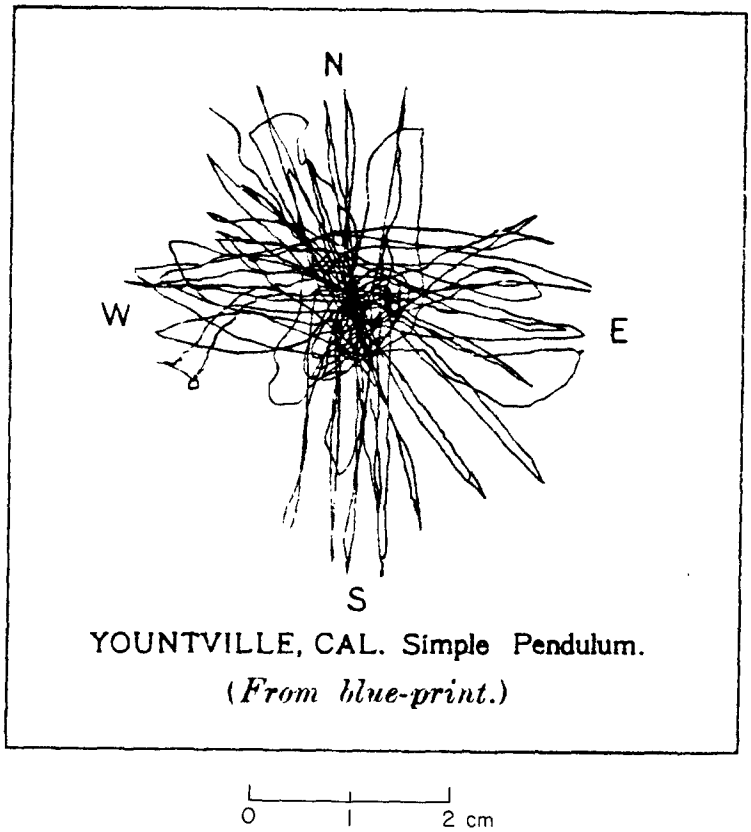

FIG. 7. Record obtained from a simple 1-m pendulum at Yountville, California during the 1906 San Francisco earthquake. From Reid (1910).

for scribing the record. For a length of 1 m, the period of the instrument is 2.0 sec. The damping is more difficult to estimate, but considering the given information, it seems reasonable to assume that the fraction of critical damping lies within the range from 0.02 to 0.10. Using these properties and the standard values of the Wood-Anderson seismograph in equation (9) gives a Wood-Anderson response (one-half peak-to-peak) ranging from 2.5 m for $\zeta_{sc} = 0.02$ to 5.6 m for $\zeta_{sc} = 0.10$. The distance to the closest approach to the fault is very nearly equal to the epicentral distance of 54 km and if this distance is used, the local magnitude estimate is from 6.0 to 6.35, again depending on the damping of the pendulum. If the epicenter is taken near the Golden Gate, in accordance with studies by Boore (1977) and Bolt (1968), the distance to Yountville increases to about 65 km, and the estimated values of M_L ranges from 6.1 to 6.5.

The Ewing Duplex Pendulum at Carson City, Nevada produced the record seen in Figure 8. Reid (1910) gives the following instrumental data: lat. 39°10′N; long. 119°46′W; epicentral distance 291 km; and gain, $V = 4$. A close examination of the record in Figure 8 suggests that the instrument went off-scale slightly in both the N-S and E-W directions, although this is not noted by Reid (1910). In order to apply the present technique to this record, it is necessary to know the period and damping of the seismograph and this presented a problem. We were fortunate, eventually, to

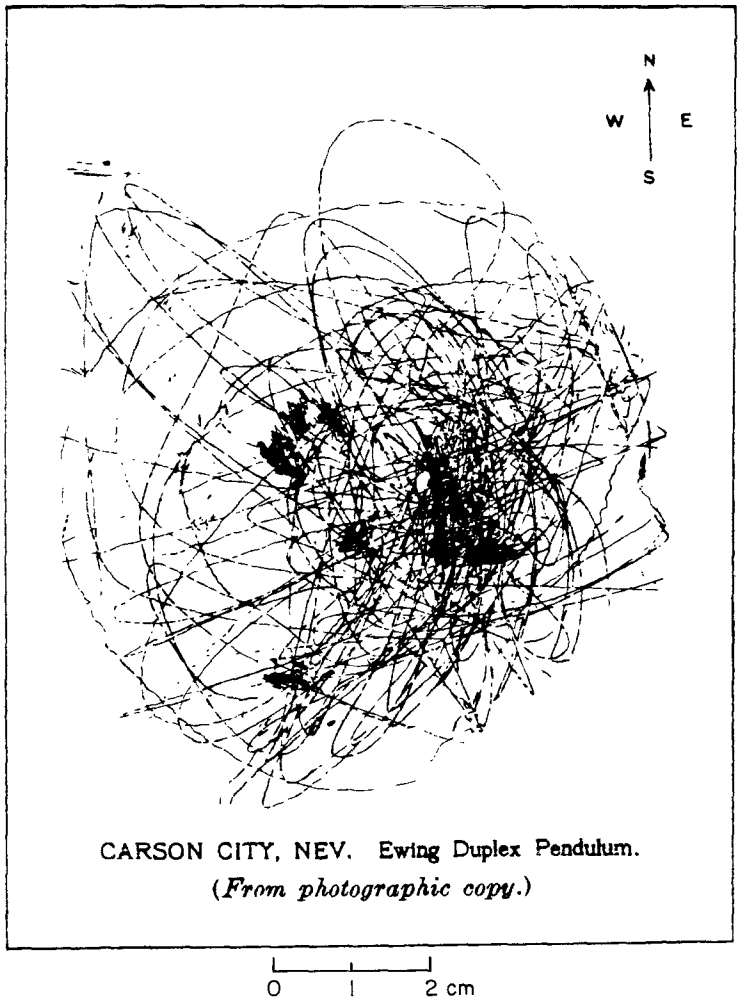

FIG. 8. Record obtained from the Ewing duplex pendulum seismograph at Carson City, Nevada during the San Francisco earthquake of 1906. From Reid (1910).

locate the instrument that recorded the record in Carson City, with the help of Doug Van Wormer and Bruce Douglas of the University of Nevada at Reno, and to find a similar instrument in the London Science Museum, which was analyzed for us by N. N. Ambraseys of Imperial College, with the assistance of Anita McConnell of the museum.

The Nevada instrument was manufactured in 1887 at Paul Seiler's electrical works in San Francisco. It was originally installed as part of a seven-instrument network established by E. S. Holden, Director of the University of California's Lick

Observatory. It operated at Carson City until 1910, when it was moved to the University of Nevada (Reno) campus. It was retired from service in 1916.

When recovered from storage, the instrument was found to be damaged and missing some of its original parts. It was repaired at the California Institute of Technology by Ivar Sedleniek of the Seismological Laboratory and Raul Relles of the Earthquake Engineering Laboratory. Photographs of the instrument in London, and Ewing's papers describing the instrument (Ewing, 1882; 1883) were used to guide the rehabilitation. A photograph of the restored seismograph is shown in Figure 9.

The instrument in the London Museum was manufactured in 1888 by the Cambridge Scientific Instrument Company. It too had been damaged in storage and had to be repaired before measurements could be made. The repaired instrument is illustrated in Figure 10. Although the case differs from that of the Carson City seismograph, the transducers of the two instruments appear identical. (The scribing mechanism of the Carson City instrument was missing, and the new mechanism was modeled after that of the London instrument.)

After restoration, the Carson City seismograph was leveled carefully, adjusted to operating condition, and then subjected to a number of free-vibration tests to determine the period and damping of the seismograph. The measuring system consisted of a small (approximately 2 x 4 mm) piece of reflective tape attached to the upper bob, an Optron (Model 1701, Displacement follower, Optron Corporation, Santa Barbara, California) which projects a beam of light onto the reflective tape and tracks its reflection, and a recording oscillograph. The basic experiment consisted of giving the instrument a small initial impulse and observing the subsequent response. Three sets of tests were performed: one with a clear, unsmoked glass recording plate, a second with the instrument in complete operating condition, including a smoked glass recording plate, and a third in which the scribe was lifted from the plate and the upper gimbal locked. This last test was performed to investigate the roll of friction in changing the effective damping and period.

A result from one of the tests is shown in Figure 11; this particular curve is from the first set of tests in which the instrument was in operating condition, but the glass plate was unsmoked. The initial impulse received by the joined pendula is not measured, but after the first peak, the motion is equivalent to free vibrations from rest with that initial displacement, and can be analyzed on this basis. An analysis of the response in Figure 11, and similar tests in this set, leads to an effective undamped period of about 4.1 to 4.3 sec, depending on amplitude, and an equivalent viscous damping factor, also depending on amplitude, ranging from 0.22 to 0.30 with the smaller values associated with the larger amplitudes of response. Giving more weight to values at larger amplitudes, an undamped period of 4.2 sec and a damping of 0.22 are considered representative of earthquake conditions. The test was then repeated with a smoked glass plate installed, and with enough stylus pressure to produce a good record. In this case, the result showed the equivalent undamped period to be in the range of 3.7 to 3.9 sec, and the equivalent viscous damping factor representative of larger amplitudes was found to have increased to about 0.25. Finally, with the stylus lifted and the upper gimbals locked, the undamped period was found to be about 3.6 to 3.8 sec, and the effective damping was in the range of 0.15 to 0.20, with 0.16 being representative of larger amplitudes. As expected, the response of the seismograph showed that the instrument does not respond truly as a viscously-damped oscillator, but the differences from viscously-damped behavior are not large, and are thought to be acceptable for the present purposes.

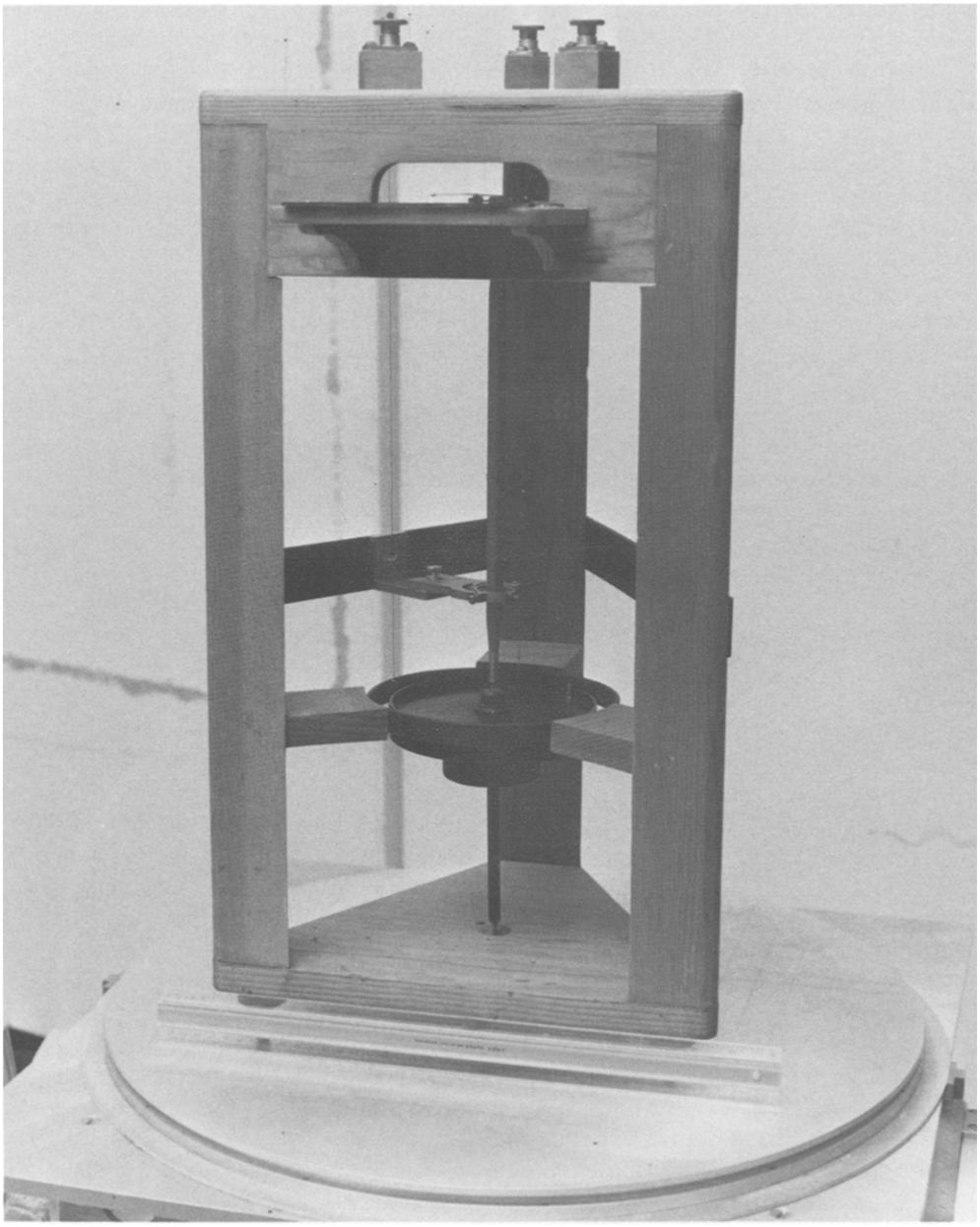

FIG. 9. The Carson City instrument that wrote the record in Figure 8, after restoration. The ruler is 16 in long.

Professor Ambraseys was asked to determine the gain, period, and damping of the instrument in the London museum. He found the gain to be 3.8 to 4.0. Depending on the adjustment of the two pendulums, the period could be adjusted from 3.2 to 5.5 sec, with the most likely value of the period for operating conditions being about 4.0 sec. The amount of equivalent viscous damping depended on the adjustment of the gimbals and whether or not there was contact between the scribe and the smoked glass plate. With free gimbals and no contact, the damping was about 0.10. With pressure on the scribe sufficient to produce a reasonably good record, the damping increased to about 0.22 to 0.33. Considering the results of the two tests, the

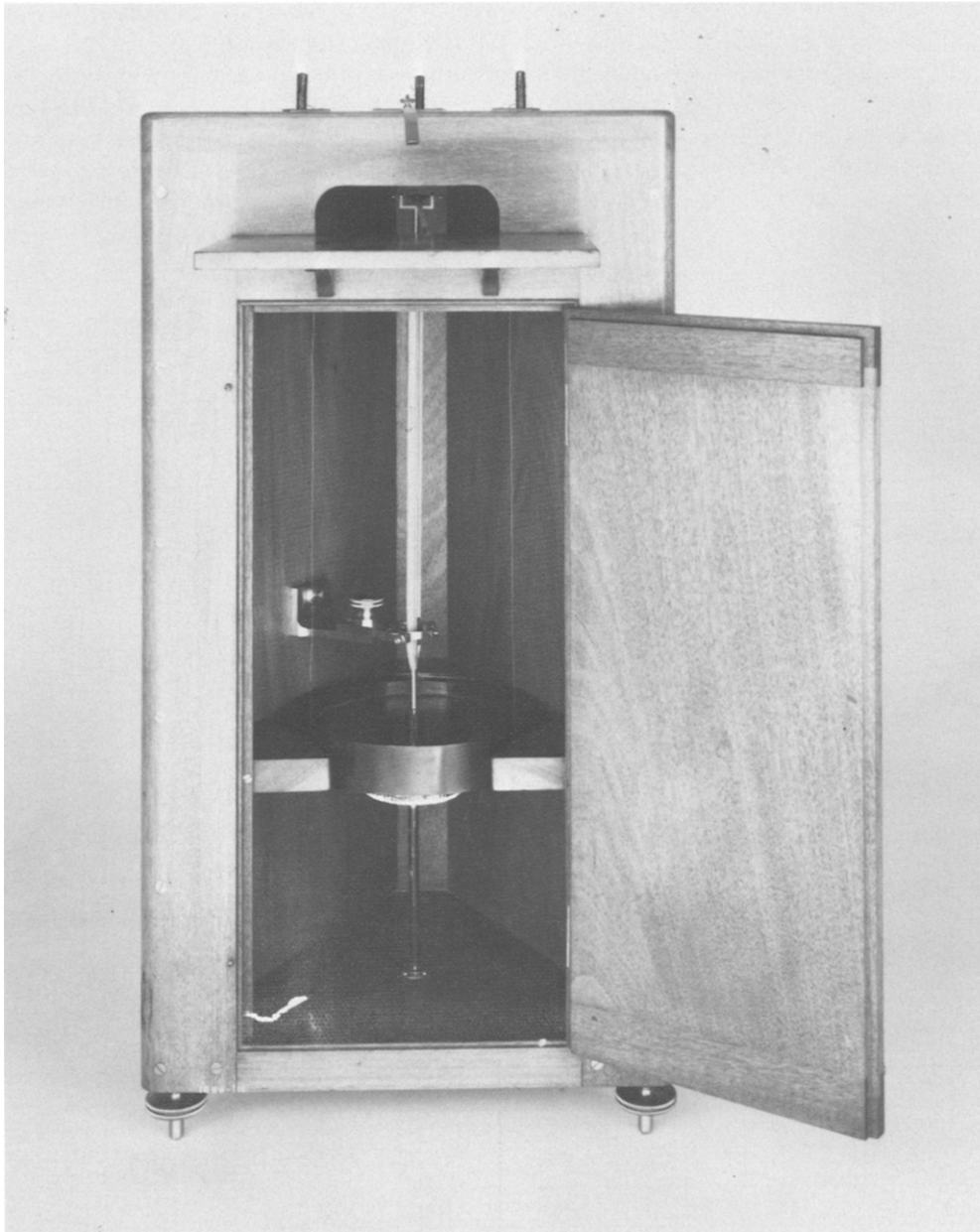

FIG. 10. The Ewing duplex pendulum seismograph that forms part of the collection of the London Science Museum (photo courtesy of Anita McConnell).

Wood-Anderson response was calculated from equation (9) using a gain of $V_{sc} = 4.0$, $T_{sc} = 3.8$, and $\zeta_{sc} = 0.25$, which are judged to be the most representative values for the instrument. From the record published in Reid (1910), as reproduced in Figure 8, the amplitudes of response were taken as 50 mm in the NS direction, and 45 mm for EW response. These values, which are one-half peak-to-peak, include estimates of the effects of going off-scale. The calculated values of Wood-Anderson response are 1.89 m (NS) and 1.70 m (EW). The epicentral distance given by Reid is 291 km, and the distance to the Golden Gate is nearly the same (286 km). Using these values of Wood-Anderson response and the epicentral distances produces $M_L = 7.2$ (NS)

and $M_L = 7.15$ (EW). Thus, the most representative value of M_L indicated by our analysis of the response of the seismograph at Carson City is 7.2.

There are, of course, considerable uncertainties in this calculated magnitude. M_L is not very sensitive to reasonable variations in distances in this case, but different assessments of the values of the damping and period of the Ewing seismograph could produce different results. It is seen from equation (9) that a value of period greater than 3.8 would reduce M_L, and the use of damping greater than 0.25 would

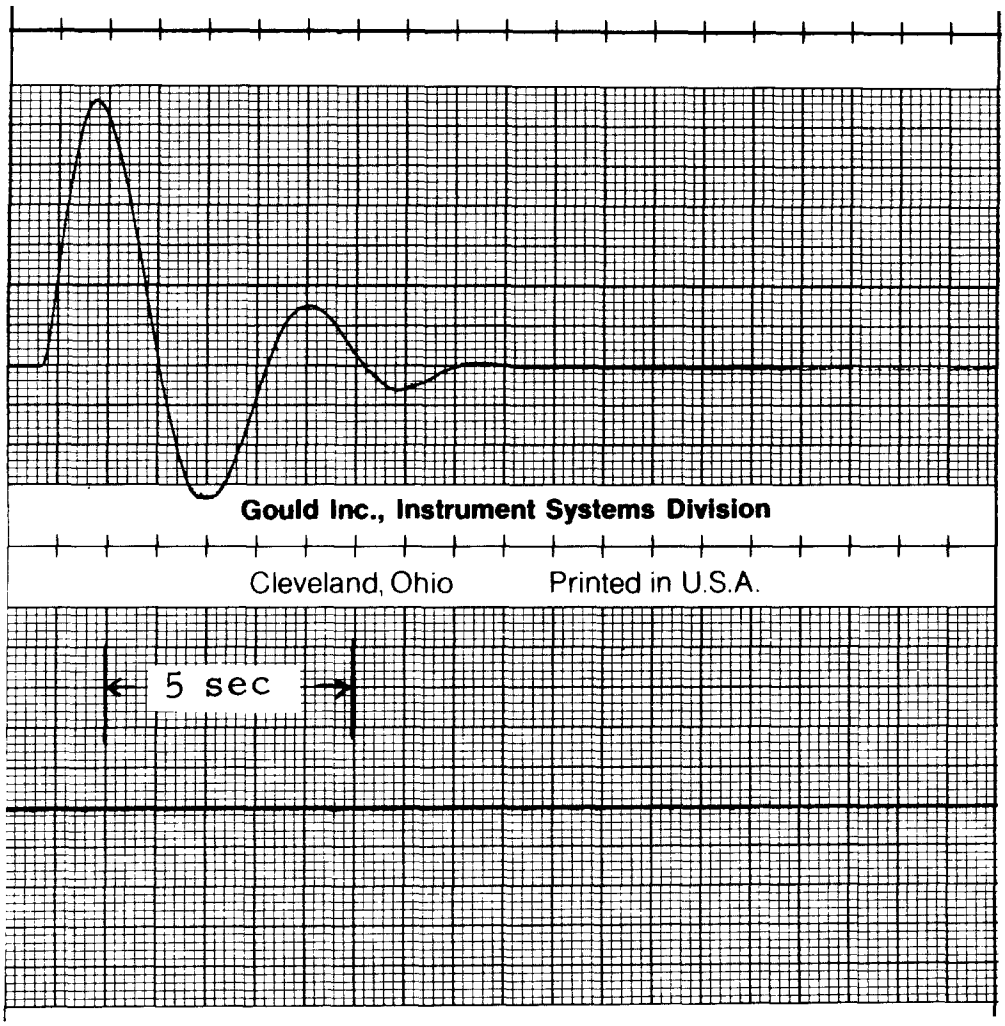

FIG. 11. Free vibration test of the Carson City Ewing seismograph. The amplitude of the trace is proportional to the lateral displacement of the joined pendulums. The motion is initiated by a small impulse.

result in a larger value of M_L. The calculated value of M_L varies from 7.1 to 7.3 as the period ranges between 3.7 and 4.2 and the damping between 0.20 and 0.40.

The value for M_L of 7.2 from the Carson City seismograph is significantly larger than that of about 6.3 indicated by the Yountville pendulum. The Carson City value is a more reliable number in the sense that it was recorded on a standard seismographic instrument upon which some calibration tests have been run. The Yountville pendulum, on the other hand, has the advantage of being situated closer to the fault, within the area of potentially damaging ground motion. To help gain some insight

into the difference between the two indicated values of M_L, the other five records from Ewing instruments were examined. These records are not capable of being analyzed in the same manner as the Carson City record because they were clearly well off-scale. These instruments (Mt. Hamilton, Alameda, San Jose, Oakland, Berkeley) are located from 20 to 36 km from the fault and were all subjected to strong shaking. It is, of course, difficult even to estimate how far off-scale the instruments may have gone, but it is interesting that even if the unconstrained response were to exceed the full scale reading of 50 mm by a factor of 20, the corresponding local magnitude does not exceed 6.8. For an M_L of 7.2, the records from these five seismographs would have to be off-scale by a factor of 50, which seems large, based upon examination and comparison of the records.

Considering all the uncertainties involved, it seems reasonable on the basis of our results to assign M_L a range of $6\frac{3}{4}$ to 7. It appears unlikely that an averaged value of M_L would lie outside the range of $6\frac{1}{2}$ to $7\frac{1}{4}$. A most representative single number is hard to determine on the basis of the data that are available, but the center of our estimate of the range is 6.9.

Discussion

The values of M_L for the Guatemala earthquake of 1976, and the San Francisco earthquake of 1906 are both below the values of M_S (7.5 and $8\frac{1}{4}$, respectively) that are assigned to these earthquakes. This feature was also noted for the Kern County earthquakes of 1952 in our previous study (Kanamori and Jennings, 1978), and is taken as further instrumental evidence for the saturation of the local magnitude scale at high values. The Kern County shock has the highest value of M_L, 7.2, of the earthquakes so far studied, which include all major U.S. earthquakes for which strong-motion data are available. Although the data are limited, the inference is fairly clear that this saturation occurs between $M_L = 7$ and $7\frac{1}{2}$. This saturation of M_L, which is the magnitude most representative of strong shaking close to the causative fault, has obvious implication for earthquake resistance design, particularly if the saturation level can be found within narrower limits.

The use of seismoscope records to estimate local magnitude provides an additional broadening of the base from which M_L can be determined. It does not have the inherent accuracy of a direct determination or of a determination from accelerograph records, but the uncertainties in the statistical relation that underlies the basis of determining M_L from seismoscope response appear to be acceptably small. Using seismoscope records to determine M_L does have the significant advantage of producing immediate results without intermediate processes such as developing a record or digitizing an accelerogram. Additionally, the seismoscope has proven to be an exceptionally reliable instrument and it is expected that there will continue to be earthquakes in which seismoscope records form a major part of the ground motion data.

The use of accelerograph records and seismoscope responses both have the intrinsic advantage of determining M_L from near-field motions. This is particularly the case for important, large earthquakes, in which most seismographic instruments are off-scale in the near field.

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