PHYSICAL REVIEW A VOLUME 43, NUMBER 11 1 JUNE 1991

Light incident on a Bose-condensed gas

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(Received 17 January 1991)

The coherent interaction of light with a Bose-condensed gas via atomic electric-dipole transitions produces strong mixing of photons and excited atoms. Consequent optical properties near resonance include substantial reflection from the edge of the gas.

Continuing advances in trapping and cooling neutral atoms[1] may someday lead to the realization of a gaseous Bose condensate. For example, a temperature of 10^{-6} K has been achieved with cesium atoms at a density of 10^{10} cm^{-3}.[2] An increase of 10^5 in density at that temperature, decrease of 10^{-3} in temperature at that density, or some combination of the two would reach conditions needed for Bose condensation of an ideal gas. (Of course, interatomic interactions dictate whether a given density is low enough for an ideal-gas description.)

The optical properties of a Bose-condensed gas at frequencies near, say, an electric-dipole transition will be dominated by coherence of scattering off different condensate atoms. Quantum fields yield a simple description of this coherence, which is a resonant optical analog of the Anderson-Higgs phenomenon (e.g., in superconductivity, where the condensate is charged). In the present case, it is demonstrated below that, within the condensate, resonant photons and excited atoms will mix strongly. The range of frequencies over which this mixing occurs is determined by the parameter

$$\delta \equiv 3\pi\gamma\rho_0(\lambda_0/2\pi)^3\,, \tag{1}$$

with γ the natural linewidth, ρ_0 the condensate density, and the wavelength $\lambda_0 = 2\pi/\omega_0$ with ω_0 the nominal resonant frequency. δ arises explicitly in the dispersion relations for propagating modes in the condensate [Eqs. (5)–(7)]; it is roughly the natural linewidth times the number of condensate atoms per photon wavelength cubed; and it is a measure of the photon-excited atom mixing induced by coherent scattering off the condensate. If the density is large, i.e., $3\pi\rho_0(\lambda_0/2\pi)^3 = \delta/\gamma \gg 1$, physics at temperature $T=0$ is particularly simple, i.e., there is 100% reflection of incident light for frequencies within the δ interval. However, at high densities and finite T, there are analogous coherence effects from photon scattering off noncondensate atoms that lie within a photon wavelength of each other.[3] These effects exist even above the Bose-Einstein transition temperature and, hence, do not give an unambiguous signal of the transition. At low densities ($\delta/\gamma \ll 1$), scattering is not very significant, coherent or otherwise.

After developing the basic simple formulas that follow from ignoring noncondensate atoms as either initial or final particles I will discuss, at least qualitatively, the interplay of dissipative processes that produce such particles with coherent phenomena.

The Lagrange density describing electric-dipole transitions (henceforth with $\hbar = c = 1$) is

$$\begin{aligned}\mathcal{L} = &-F_{\mu\nu}^2/4 + \varphi^\dagger(i\partial_t + \nabla^2/2m')\varphi \\ &+ \varphi_i^\dagger(i\partial_t + \nabla^2/2m)\varphi_i \\ &+ (3\pi\gamma/\omega_0^3)^{1/2}F_{ti}(\varphi_i^\dagger\varphi + \varphi^\dagger\varphi_i)\,,\end{aligned} \tag{2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ is the vector potential or photon field, φ and φ_i $(i = x,y,z)$ are respectively, the nonrelativistic annihilation fields for the atomic ground state and ℓ=one excited state, the masses satisfy $m - m' \equiv \omega_0$, and μ and ν run over four space-time indices (t,x,y,z). The particular form of the interaction is the simplest allowed by electromagnetic gauge invariance for photon, neutral spin-zero, and neutral spin-one fields; the strength of the interaction is chosen to match the natural width. Realistic magnitudes for the parameters are $\gamma/\omega_0 \sim 10^{-6}$ and $m/\omega_0 \sim 10^{10}$, while conditions thus far achieved,[2] correspond to $k_BT/\omega_0 \sim 10^{-10}$ and *total* density $\rho \sim \omega_0^{-3}$. A uniform condensate of ground-state atoms is described by a constant c-number component of φ, φ_0, such that $|\varphi_0|^2 = \rho_0$. Fluctuations of φ about φ_0 describe ground-state atoms not in the condensate.

Ignoring these fluctuations and setting $\varphi = \varphi_0$ in $\mathcal{L}$ yields a linear system in which photons and excited atoms mix with a coupling proportional to $\delta^{1/2}$. Physically, a photon of frequency ω_0 [actually $m - (m^2 - 2m\omega_0)^{1/2} \simeq \omega_0$] can excite a condensate atom. Since we cannot keep track of individual condensate atoms, the photon and the produced excited atom are effectively degenerate. The decay of the excited atom back into the condensate and a photon of the original frequency and momentum is stimulated by the presence of the condensate (as well as by the rest of the photon beam, if present). The field equations for $\varphi = \varphi_0$ are

$$\partial_\mu F_{\mu i} - \delta^{1/2}\partial_t(\varphi_i + \varphi_i^\dagger) = 0\,, \tag{3a}$$

$$\partial_i F_{ti} + \delta^{1/2}\partial_i(\varphi_i + \varphi_i^\dagger) = 0\,, \tag{3b}$$

$$K\varphi_i + \delta^{1/2}F_{ti} = 0\,, \tag{3c}$$

$$K^\dagger\varphi_i^\dagger + \delta^{1/2}F_{ti} = 0\,, \tag{3d}$$

where $K = i\partial_t + \partial_i^2/2m$.

We can eliminate φ_i by inverting K and view these as modifications to Maxwell's equations in the medium. The longitudinal part of φ_i, $\varphi_i^L = \partial_i\partial_j\partial^{-2}\varphi_j$, does not mix with the photon (conservation of angular momentum), and the resulting wave equations for the electromagnetic fields can be reduced to

$$\nabla^2\mathbf{E}=[1+\delta(K^{-1}+K^{\dagger -1})]\partial_t^2\mathbf{E}\,. \qquad (4)$$

Wave solutions with frequency ω and wave number k satisfy

$$k^2=\omega^2[1+\delta(\omega-\omega_0-k^2/2m)^{-1}] \qquad (5)$$

($K^{\dagger -1}$ does not contribute for $\omega>0$) or

$$k^2=(\omega^2+2m(\omega-\omega_0)\pm\{[\omega^2-2m(\omega-\omega_0)]^2+8m\delta\omega^2\}^{1/2})/2\,. \qquad (6)$$

For $\delta=0$, we see in Eq. (6) the photon and excited atom (threshold at $\omega=\omega_0$) that have spectra that cross. The $\delta\neq 0$ mixing makes the levels repel, splitting that degeneracy. The mixing also shifts the threshold for the second excitation up to $\omega=\omega_0+\delta$. Except for the tiny region excluded by $|\omega-\omega_0|\gg\omega_0^2/2m$,

$$k^2\simeq\begin{cases}\omega^2-\omega^2\delta/(\omega-\omega_0), & \omega<\omega_0,\omega\geq\delta\,, \qquad (7a)\\ 2m(\omega-\omega_0)+\omega^2\delta/(\omega-\omega_0), & \omega>\omega_0\,. \qquad (7b)\end{cases}$$

These are just the lowest-order perturbative formulas, which then match appropriately at $\omega=\omega_0$.

Alternatively, one can diagonalize the $\varphi=\varphi_0$ propagation for each ω to identify the independent excitations and recover the same dispersion relations. If one writes the independently propagating excitations with (ω,k) as $\alpha A_i+\beta(2m)^{-1/2}\varphi_i$, then

$$\begin{aligned}\alpha/\beta&=(2m\delta)^{1/2}\omega/(k^2-\omega^2)\\&=[k^2-2m(\omega-\omega_0)]/[(2m\delta)^{1/2}\omega]\\&\simeq[(2m/\delta)^{1/2}(\omega-\omega_0)/\omega]^{\pm 1}\end{aligned} \qquad (8)$$

where the sign of the exponent is + [−] for the excitation corresponding to Eqs. (7a) and [7(b)].

The above discussion deals with nondissipative decays in which excited atoms return to the condensate. Dissipative decays, producing noncondensate atoms, are of interest as they affect the reflection of light and the propagation of modes and deposition of energy within the condensate. Because they require atomic creation $\varphi^\dagger-\varphi_0$ and, therefore, excited-state annihilation φ_i, their net effect on the aforementioned issues can be represented by an imaginary contribution to the $\varphi_i-\varphi_i^\dagger$ element of the 2×2 matrix kinetic kernel. Thus, every appearance of ω_0 [i.e., in Eqs. (5)-(8)] is replaced by $\omega_0+i\Gamma/2$, where $\Gamma=\Gamma(\omega,k)$ is a real function of ω and k. In vacuum, $\Gamma=\gamma$. In general, Γ represents the coupling of φ_i and, hence, $\delta^{1/2}F_{ti}(\varphi^\dagger-\varphi_0)$ to physical, propagating states. Γ must be determined self-consistently because the nature of the propagating states in turn depends on Γ. For $\gamma\ll\delta$, the previous discussion which ignored dissipative decays is hardly altered. Γ could be determined recursively, using the modes described by Eqs. (6) and (7) as a first approximation. In that case, while there exist two-body final states for each initial (ω,k), in the interval $\omega_0\lesssim\omega\lesssim\omega_0+\delta$, they have negligible coupling to φ_i. Hence, $\Gamma\ll\gamma$ within that interval, and $\Gamma\approx\gamma$ otherwise.

In the absence of a proper self-consistent determination of Γ for non-negligible γ/δ, we can see the qualitative consequences of $\Gamma\sim O(\delta)$ by considering the earlier equations (i.e., with $\omega_0\rightarrow\omega_0+i\Gamma/2$) with some substantial Γ. Γ smears out the threshold for Rek at $\omega_0+\delta$, in turn providing accessible final states, further shrinking the interval of suppressed decays. Plausibly, for $\gamma\sim\delta$, $\Gamma\approx\gamma$ for all ω.

There are edge phenomena effecting dissipative decays that do not arise in bulk condensate. The imaginary—or significant imaginary part to—k found in the situations discussed below defines a skin depth for light incident on a condensate. Decays can occur in the skin that are forbidden in bulk, e.g., with a photon tunneling out of the condensate. In such a case, a moving atom will remain behind and heat up the gas.

Interatomic collisions of the excited atoms are another potential source of loss of coherent beam. (The coherent, condensate-enhanced forward, elastic scattering produces only a negligible mass renormalization.) However, the low velocity of the excited atoms and the gaseous density make such processes unimportant. In particular, model the collision rate Γ_c as the excited-atom velocity, ω_0/m, times a cross section times a target density. The cross section is presumably less than the square of the atomic-mean spacing; otherwise it would not be a gas. With that assumption,

$$\Gamma_c/\delta<[\omega_0^2/(m\gamma)](\gamma/\delta)^{2/3}\,. \qquad (9)$$

The term in square brackets is typically 10^{-4}.

To apply these formulas to the problem of light incident on a condensate, one must characterize the interface. The extreme alternatives are sharp or very gradual. To be "sharp" for a given ω, one need only have a substantial change in k in one wavelength. For ω within a few δ's of ω_0, this is likely for a realistic geometry,[4] even if the condensate edge is diffuse relative to the bulk density. The simplest approximation is to treat the interface as absolutely sharp, thus reducing the problem to matching waves across a boundary. In general, attention must be paid not only to the change in k across the boundary, but also the change in propagating basis in the two-dimensional, photon-excited-atom system, as given by Eq. (8). However, a trap might be engineered with density gradients so gradual that photons would evolve adiabatically into the atomlike propagating mode, without substantial reflection.

For a sharp interface with high density ($\delta\gg\gamma$ so that coherence dominates over dissipation), there is total reflection for $\omega_0\leq\omega\leq\omega_0+\delta$, and substantial reflection nearby on the scale of δ. At temperatures well below (e.g., 1/10) the Bose-Einstein transition, the density of atoms not in the condensate is considerably lower than the condensate density. However, at temperatures comparable to (and also above) the transition, coherent scattering off uncondensed atoms[3] is important.

At modest densities ($\delta\sim\gamma$), there would be substantial reflection from a sharp interface in the vicinity of ω_0. The "transmitted" beam would dissipate the unreflected energy (which is also substantial) within a skin depth comparable to the wavelength. This occurs if, indeed, $\Gamma\sim\gamma$ because then Eq. (7a) with $\omega_0\rightarrow\omega_0+i\Gamma/2$ gives k imaginary and real parts of comparable magnitudes. Depending on the incident-beam intensity, the thermal conductivity, and mean free atomic path, the energy thus deposited might allow the incident beam to burn its way through the gas, as local heating destroys the coherence. This loss of coherence between the atoms would make the gas transparent at modest densities along the beam path. With a substantial fraction of the incident beam expected to be absorbed and dissipated in this regime, one could heat the whole sample above its transition temperature with resonant light.

Note added. After this work was completed, Ref. 5 appeared in English translation. It addresses some of these issues as well as studying the interaction of condensate and noncondensate coherence at high density.

Stimulating conversations with V. Telegdi on selective, specular reflection from dense gases are acknowledged. This work was supported in part by the U.S. Department of Energy under Contract No. DEAC-03-81ER40050.

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