PHYSICAL PROPERTIES OF THE ATMOSPHERE UP TO 100 KM[1]

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ABSTRACT

Results of new calculations of temperature, pressure, density, and other physical quantities in the stratosphere up to an elevation of 100 km are given and discussed.

The development of jet-propelled missiles and rockets and their use in obtaining more information concerning the structure of the atmosphere requires the utilization of all data on the stratosphere available at present. For this reason such results are presented here, although it is to be expected that they will become obsolete in the near future. (For discussions of earlier data and literature, see Gutenberg, 7, 8, or Haurwitz, 10.)

There are two fundamental quantities on which all calculations are based: the composition of the atmosphere and its temperature. For both, accurate measurements are available only up to an elevation of about 30 km, but inferences may be drawn regarding higher elevations by using theory in combination with observations of other phenomena.

Air samples taken at elevations between the ground and 29 km (see Paneth, 17) indicate that the oxygen content of the air varies from 20.9 per cent in volume between the ground and an elevation of about 20 km to 20.4 per cent at an elevation of $28\frac{1}{2}$ km. However, this (possibly partly local) difference of about $\frac{1}{2}$ per cent was not compensated by a corresponding larger amount of the light gases; especially, helium remained at about 0.0005 per cent up to 25 km (0.00052 per cent at the ground and 0.00054 at 25 km). Relatively small amounts of hydrogen and helium are to be expected at even greater elevation since, according to Jeans (11), both gases escape at the top of the earth's atmosphere; Peterson (19) found that under such circumstances these gases are of little importance throughout the whole atmosphere.

Theoretical investigations as to the time which is required to restore equilibrium after gases of the atmosphere have been mixed, by Epstein (1) and Maris (15, 16), indicate that it takes many months to restore even half of the difference between the actual distribution and the equilibrium distribution of the gases at elevations below 100 km, after the atmosphere has been mixed by air currents. Consequently, the composition cannot be expected to differ appreciably over this range. This result is confirmed by the interpretation of the spectrum of the aurora by Kaplan (14). Thus, in the following, it is assumed that the atmosphere has the same composition between the ground and an elevation of nearly 100 km.

If a homogeneous atmosphere is assumed, all major mechanical properties can be expressed as functions of the temperature alone. The fundamental equations for average air consisting of molecules of the gases are given in equations 1–5.

Sound velocity in m/sec:

$$V = 20.1\sqrt{T}, \tag{1}$$

T = absolute temperature centigrade.

Molecular velocity in m/sec:

$$\sqrt{\frac{\sum c^2}{n}} = 1.46\ V, \qquad \text{average } c = 1.35\ V. \tag{2}$$

Pressure p at elevation h in km:

$$\log p = \log p_0 - 14.837 \int_0^h \frac{dh}{T}. \tag{3}$$

Density in g/m^3:

$$d = 348\ p/T, \qquad p \text{ in millibars.} \tag{4}$$

Mean free path of molecules in cm:

$$f = 0.007648/d. \tag{5}$$

The best information regarding the temperature between 30 and 60 km is derived from observations which give the velocity of sound waves at these altitudes. At higher levels the absorption of sound increases so fast with elevation that the usefulness of sound waves for temperature determinations decreases rapidly above the level of 60 to 70 km. In Figure 1[3] the fundamental data of Schrödinger (20)

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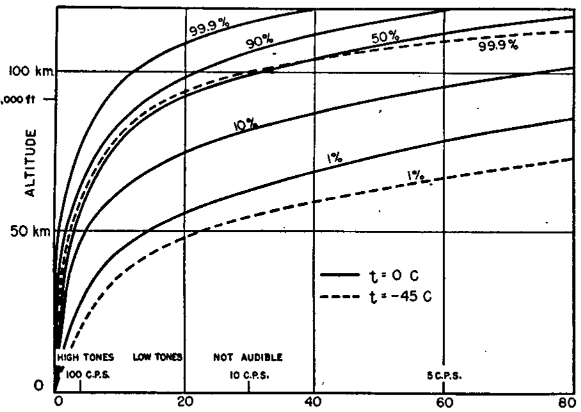

FIG. 1. Absorption of sound energy per km in an isothermal atmosphere after Schrödinger.

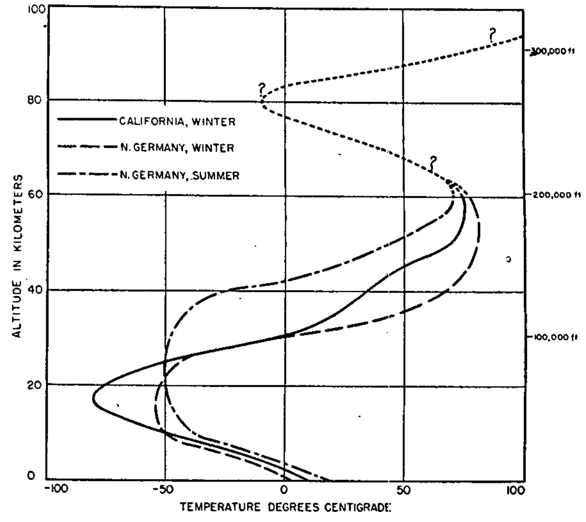

FIG. 2. Examples of vertical temperature distributions.

have been used to indicate the loss of energy in sound waves of various wave lengths and at various altitudes.

Thus far, no difference in sound propagation has been found when the energy was changed within wide limits. However, it is to be considered that for elevations above 60 km the change in pressure due to sound waves may approach the pressure itself, and that under such conditions equation (1) must be replaced by equations for shock waves at greater heights. (See, for example, Gutenberg, 8, p. 143.)

The results on the propagation of sound waves (Fig. 5, up to an elevation of 60 km) indicate that their velocity increases to a maximum at elevations of roughly 60 km, where it exceeds the velocity near the ground. (For details see, for example, Gutenberg, 9.) Data are available only for central and western Europe in summer and winter, California in winter, and some data for arctic areas during summer and winter (Wölcken, 26).

From equation (1) it follows that the temperature must reach a maximum at the same level of about 60 km as the sound velocity, and that it must be higher

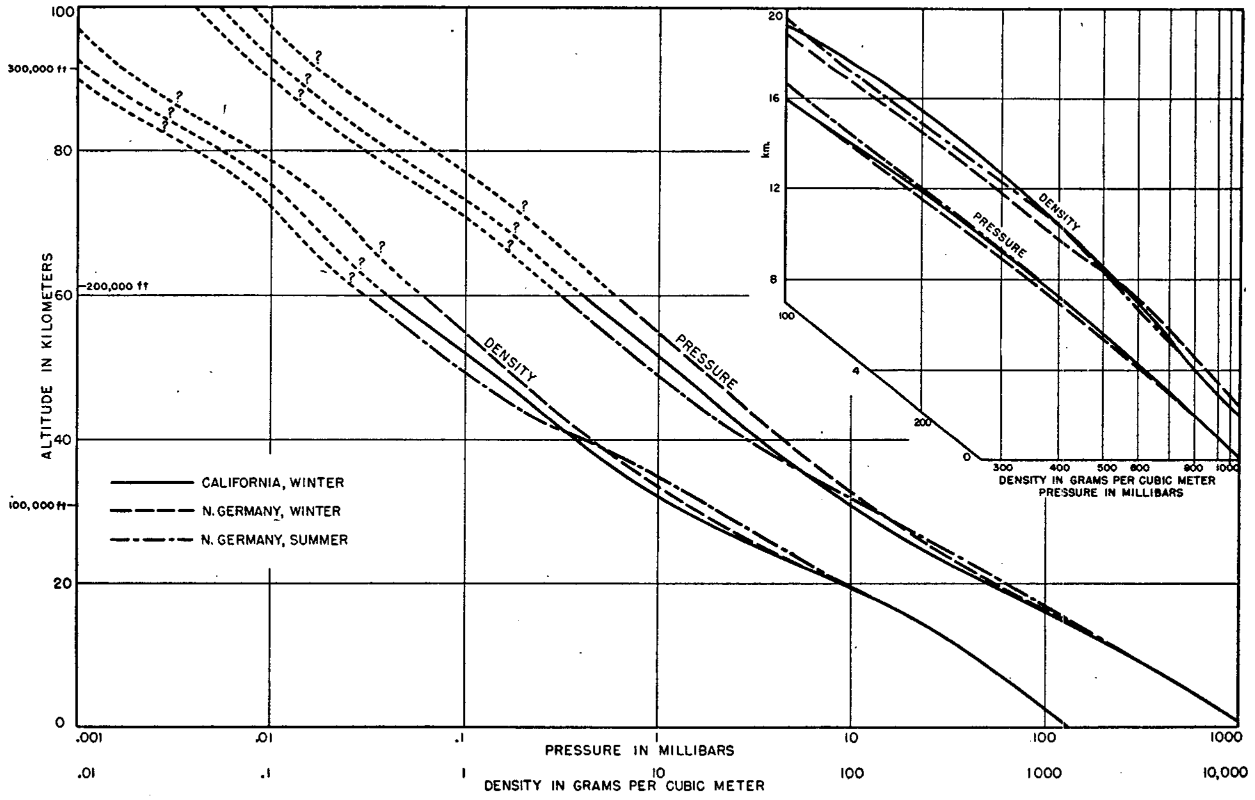

FIG. 3. Vertical distributions of pressure and density.

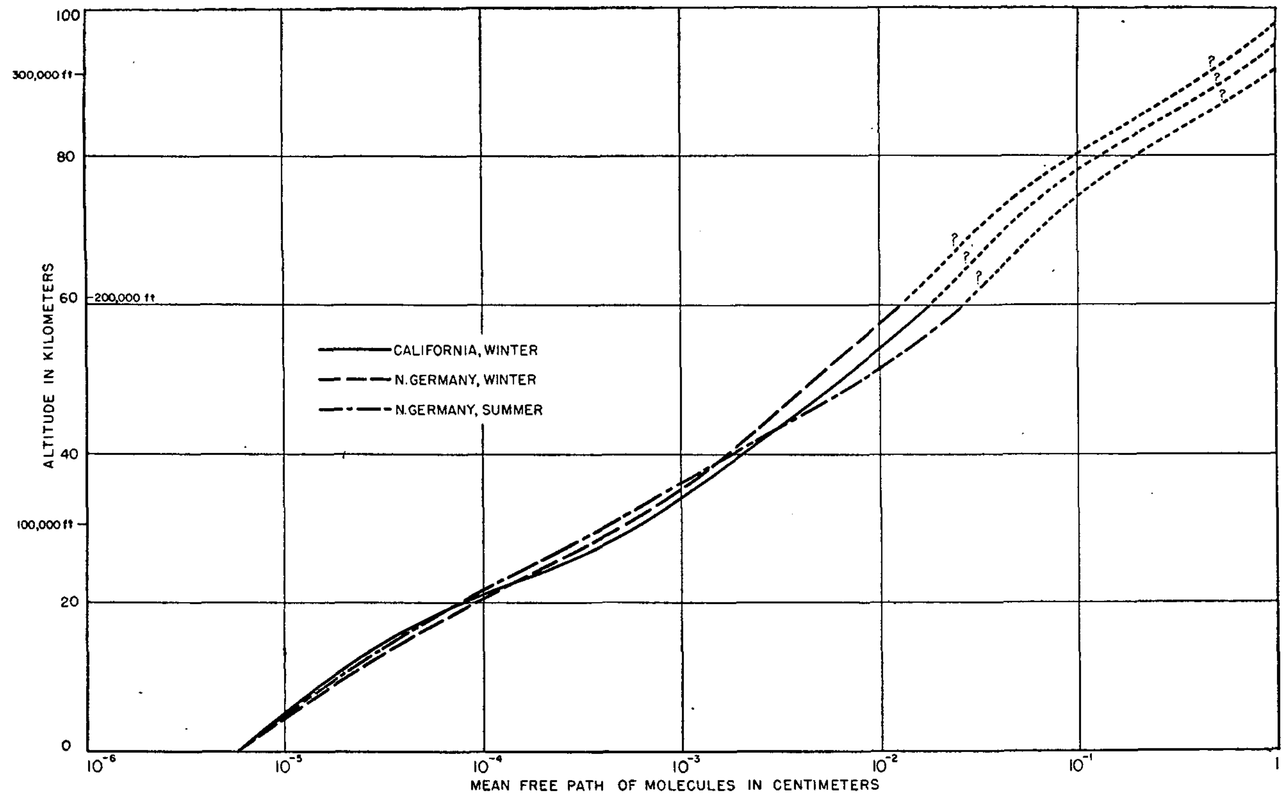

FIG. 4. Vertical distributions of mean free path of molecules.

there than near the ground (Fig. 2). There are some indications (see, for example, Whipple, 25) that above 60 km the temperature decreases with increasing elevation to a minimum at an elevation of about 80 km, but it is believed by many specialists on the ionosphere that much higher temperatures prevail above 90 km. In Figure 2 all values for elevations above 60 km are very uncertain. There are definite indications that the temperature throughout the stratosphere shows seasonal differences and differences in latitude. The distance from the source of the sound to the zone of "abnormal audibility" of sound waves at the ground undergoes strong seasonal variations. The more rapidly the sound wave velocity increases (and correspondingly the temperature) at the elevations of 30–40 km and the lower the level of the beginning of this increase, the less the distance at which the strong sound waves return to the ground. Compilations of data have been published by Whipple (23, 24), A. Wegener (22), and others. Wegener found that in

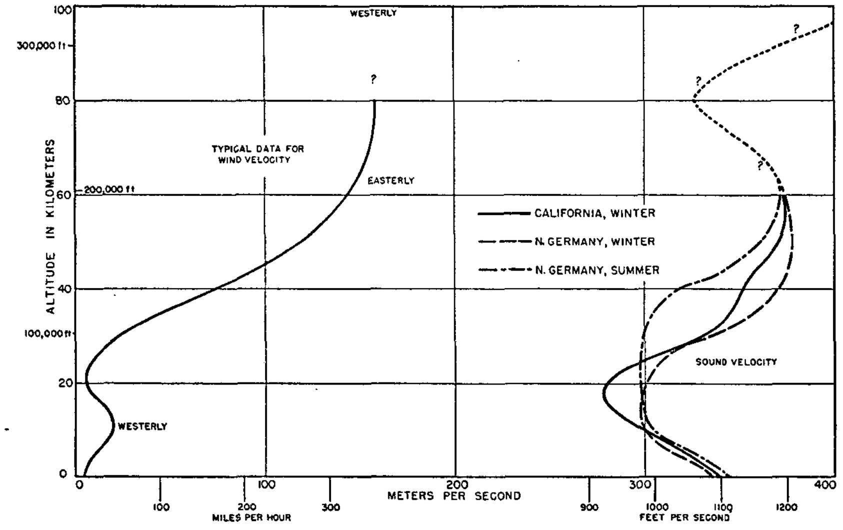

FIG. 5. Approximate characteristic values of the wind speed and direction, and sound velocity.

central Europe the distance of the maximum sound intensity from the source varies between 120 km in February and about 240 km in August.

Many attempts have been made to explain the high temperatures at heights between about 40 and 60 km. The seasonal and geographical changes in the temperature at these levels show fluctuations similar to those in the ozone content at the same levels, and there is little doubt that the absorption of the solar radiation by ozone and water vapor play the major roles in producing these high temperatures. (See, for example, Pekeris, 18, Fabry, 2, Gowan, 4, 5, 6.) As soon as the solar radiation reaches the upper level in which sufficient ozone is present to absorb the solar radiation appreciably, higher temperatures result, and relatively high temperatures persist downward as far as sufficient absorbable radiation remains. By the time the rays reach the layer of maximum ozone content at an elevation of about 25–30 km, practically all the radiation of the wave lengths which the ozone is able to absorb has been absorbed. The high temperatures in the ionosphere are due to other processes.

From equation (3) the pressure can be calculated and from equation (4) the density. Both are given in Figure 3 on the assumption of an atmosphere of uniform composition with the temperatures indicated in Figure 2. Figure 4 shows the mean free paths of the molecules, calculated from equation (5). Again, all data for elevations above 60 km are rather uncertain.

Figure 5 indicates the approximate mean characteristic values of the wind speed and direction and the sound velocity. Data for the wind velocity have been estimated by using observations of noctilucent clouds (elevation about 80 km), smoke from meteors in daytime (30–80 km), sparks from meteors at night (90–120 km), in addition to direct observations. (See also Fennell, 3, for movements of clouds of high ionic density, and Johnson, 12, for direction and velocity of wind over England at an elevation of about 30 km.) In general, the results agree with the pressure field which is indicated by generalization of the calculated values of the pressure in Figure 3.

The sound velocity (Fig. 5) for the troposphere has been calculated from the temperature; between about 20 km and 60 km the original data are given; above 60 km the values have been calculated from the temperature and consequently are uncertain.

The mean velocity of the molecules is about 1.4 times the sound velocity and has not been plotted separately.

It is very difficult to extrapolate the data beyond an elevation of about 90 or 100 km; the composition of the atmosphere probably begins to change noticeably with elevation and the temperature (over 100 C?) is highly uncertain. In addition, due to the increase in free paths of the molecules and the relative increase in the number of individual atoms the equations used grow more and more inaccurate with higher altitudes.

REFERENCES

1. Epstein, P. S., 1932: Über Gasentmischung in der Atmosphäre. *Gerlands Beitr. Geophys.*, **35**, 153–165.
2. Fabry, Charles (editor), 1929: Rapport de la réunion de l'ozone et de l'absorption atmosphérique. *Gerlands Beitr. Geophys.* **24**, 1–80, 25 authors.
3. Fennell, P., 1944: Winds in the ionosphere indicated by radio reflecting "clouds" of high ionic density. *Bull. Am. Meteor. Soc.*, **25**, 371.
4. Gowan, E. H., 1928: The effect of ozone on the temperature of the upper atmosphere. *Proc. Roy. Soc. London*, **A**, **120**, 655–669.
5. ——, 1930: *Idem*, II. Vol. **128**, 531–550.
6. ——, 1946: Private communication.
7. Gutenberg, B., 1932: Der Aufbau der Atmosphäre, *Handbuch der Geophysik*, **9**, 1–88.
8. ——, 1932: Die Schallausbreitung in der Atmosphäre, *Handbuch der Geophysik*, **9**, 89–144.
9. ——, 1939: The velocity of sound waves and the temperature in the stratosphere in southern California. *Bull. Am. Meteor. Soc.*, **20**, 192–201.
10. Haurwitz, B., 1936–1937: The physical state of the upper atmosphere. *Roy. Astron. Soc. Can.*, reprint, 96 pp.
11. Jeans, J. H., 1940: *An introduction to the kinetic theory of gases*. New York, p. 42.
12. Johnson, N. K., 1946: Wind measurements at 30 km. *Nature*, **157**, 24.
13. Kaplan, J., 1935: Hydrogen in the upper atmosphere. *Nature*, **136**, 549–550.
14. ——, 1936: The excitation of the auroral green line by metastable nitrogen molecules. *Phys. Rev.*, **49**, 67–69.
15. Maris, H. B., 1928: The upper atmosphere. *Terr. Magn. Atm. Elec.*, **33**, 233–255.
16. ——, 1929: *Idem*, Vol. **34**, 45–53.
17. Paneth, F. A., 1939: Composition of the upper atmosphere. *Quart. J. Roy. Meteor. Soc.*, **65**, 304–310.
18. Pekeris, C. L., 1939: Atmospheric ozone as a possible meteorological factor. *Bull. Am. Meteor. Soc.*, **20**, 4–10.
19. Peterson, H., 1928: On the influence on the composition of the air of a possible high temperature in the highest strata of the atmosphere. Inst. Meteor. Danois, *Communications*, Nr. 6, Copenhagen.
20. Schrödinger, E., 1917: Zur Akustik der Atmosphäre. *Physik. Z.*, **18**, 445–453.
22. Wegener, A., 1925: Die äussere Hörbarkeitszone. *Z. Geophys.*, **1**, 297–314.
23. Whipple, F. J. W., 1918: Seasonal variation in the audibility of distant gun-fire. *Quart. J. Roy. Meteor. Soc.*, **44**, 285–289.
24. ——, 1936: The transmission of air wave to great distances. *Procès-Verbaux*, Assoc. Météor., Edimbourg, II, pp. 321–322.
25. Whipple, F. L., 1943: Meteors and the earth's upper atmosphere. *Rev. Modern Phys.*, **15**, 246–264.
26. Wölcken, K., 1934: Schalluntersuchungen im Polargebiet. *Z. Geophys.*, **10**, 222–234.