



Figure S2. Estimated resonance wavelength of the cavity as a function of the length of the phase-matching sections.

Here we present a simple but useful tool for designing the resonance wavelength of the cavity by tuning the length L_0 of the phase-matching sections under even mode injection conditions (EMIC). Consider the fundamental TM-like mode propagation diagram shown as the red lines in Figure S1. As the wave propagated through the tapered MIM waveguide and was reflected by the fully reflective mirror, the total phase delay $\Delta\varphi$ could be expressed as $\angle t_t + \angle r_r + 2\Delta\varphi_r$, where “2” indicated that the mode passed the phase-matching section Δx_2 twice, and where the symbols are defined in Figure S1. To a first-order approximation of neglecting mode mismatching, the direct reflections from the taper sections r_t could be neglected. The inputs on either side of the taper section could then be guaranteed to be in phase or satisfy even mode injection conditions by ensuring phase-matching, according to $\Delta\varphi = 2k\pi$, $\forall k \in \mathbb{Z}$. We could

then derive the length of the phase-matching section Δx_2 . For the fixed Δx_2 , the length of the input-side phase-matching section Δx_1 could be adjusted to provide an appropriate phase delay and ensure that the structure was resonant at the designed wavelength. Figure 5 in the main text shows that by neglecting the intrinsic phase delay difference from the PLCs and the reflections in the taper structure, a symmetrical condition $\Delta x_1 = \Delta x_2$ may be found to satisfy the resonance condition to a first-order approximation. In this paper, we used $\Delta x_1 = \Delta x_2 = L_0$ to simplify the discussions. We compared the resonance frequencies measured in the 3D simulations (fitting the electric field intensity and modal reflectance curves in Fig 7 (a) and (b) in the main text) with those predicted by our EMIC model (solid line by solving the resonant condition $\Delta\varphi = 2k\pi$) in Figure S2. The plot confirmed that the EMIC provided reasonably accurate estimates of the actual resonance frequencies, with <1% error. The model can serve as a simple yet handy design tool.