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The Shadow of the Earth's Core[1]

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Abstract. There is very good agreement between the observed decrease in amplitudes of longitudinal waves diffracted at the boundary of the earth's core and corresponding theoretical results. Especially, no rapid decrease of amplitudes at the beginning of the shadow zone is to be expected. Theory and observations show that the amplitudes of diffracted short-period P waves decrease faster with distance in the shadow zone than those of longer waves. At epicentral distances of over 110° short-period diffracted P waves emerge gradually and their beginning can rarely be ascertained. Corresponding long-period waves arrive within the limits of error at the time calculated on the assumption of a straight-line travel-time curve.

Amplitudes of waves diffracted from a caustic decrease rapidly with distance; this usually limits the range of their observation to roughly 10°. On the contrary, amplitudes of P waves diffracted at the core may be visible on records of a great earthquake at distances of 103° to 180°. Observations for diffracted S waves cover a much smaller range of distances, since S waves appear in a portion of seismograms which is disturbed by earlier motion. Otherwise, observations indicate similar behavior for diffracted P and S waves.

Introduction. In his first investigation of the earth's core, the author [*Gutenberg*, 1914, p. 192; 1925*b*, p. 23] explained the observed longitudinal waves at epicentral distances beyond 102° as waves diffracted around the earth's core. Although this hypothesis has been widely accepted, it has been doubted by *Lehmann* [1953, p. 303; 1958], who suggests that the small amplitudes of P beyond about 104° may result from a decrease in wave velocity at depths approaching the core.

Records of the 'diffracted P waves' at distances over 140° are rare, since they require large earthquakes with epicenters in regions which have distances of 120° to at least 160° from many well-equipped stations. Earthquake epicenters which fulfill these requirements are in the western East Indies with the recording stations in North America, and in the New Zealand area with the recording stations in Europe. The most extended range of useful data seems to have been furnished by the shock of June 26, 1924, at 01:37:34 UT. This originated at 56°S 157½°E (near Macquarie Island) and had a magnitude of about $m = 7\frac{3}{4}$, corresponding to about $M = 8\frac{1}{4}$. It was studied by *Macelwane* [1930, with reproduction of seismograms] and by *Krumbach* [1934] who reproduced seismograms separately [*Krumbach*, 1931]. The diffracted P is clearly recorded to a distance of 162° (Paris) and doubtfully [*Macelwane*, 1930, p. 214] at 167° (Oxford).

Fundamental data. According to *Jeffreys* [1959, p. 98] the mean radius of the core is 3473±3 km. This value is based partly on travel times of longitudinal and of transverse waves reflected at the core boundary. The flattening of the core is about 1/460 [*Gutenberg*, 1959*a*, p. 162], so that the difference between the equatorial and the polar radii of the core is probably between 7 and 8 km. According to *Jeffreys* [1939*a*, p. 511], longitudinal waves arriving at an epicentral distance of $\theta = 90°$ have their deepest point at a distance from the earth's center of about 3660 km, where their velocity is 13.61 km/sec. Rays arriving at $\theta = 96°$ have their deepest point at a distance of about 3545 km from the center; the velocity there is 13.64 km/sec. This velocity remains constant, according to Jeffreys, down to the core boundary. The longitudinal waves grazing the core arrive at a distance of about 102°.

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Gutenberg and Richter [1939, p. 134] found a velocity of 13.8 km/sec at a distance of 3570 km from the center and from there a small decrease to 13.7 km/sec at the core boundary. They calculated that in shocks starting at the surface the longitudinal waves grazing the core arrive at an epicentral distance of about 103°.

Transverse waves arriving at distances of 91.6°, 96.2°, and 103° have, according to *Jeffreys* [1939*a*, p. 513], their deepest points at distances of 3667, 3549, and 3486 km, respectively, from the earth's center, with corresponding velocities of 7.26, 7.31, and 7.30 km/sec. *Gutenberg and Richter* [1939, p. 134] found a decrease in velocity from 7.3 km/sec to 7.25 km/sec in the 125-km-thick portion of the mantle immediately above the core. All these rates of decrease in velocity are definitely smaller than the critical rate of decrease given by $dV/dr = V/r$ [see, e.g., *Gutenberg*, 1959*a*, p. 25] for formation of a complete shadow. Near the core boundary, this critical rate is about 0.4 km/sec per 100 km for longitudinal waves and slightly over 0.2 km/sec per 100 km for transverse waves. On the other hand, at an epicentral distance of about 90° the amplitudes of P and S begin to decrease, but at a noticeably smaller rate than in the shadow zone produced by the low-velocity layer of the asthenosphere [see, e.g., *Gutenberg*, 1959*a*, pp. 76–89]. Combined with the nearly straight travel-time curves, this indicates nearly constant velocities in the portion of the mantle near the core; there is no indication of an appreciable decrease in the velocities of P or S.

Lehmann's [1953, p. 293] main objection to the hypothesis that the P and S waves observed beyond epicentral distances of about 103° are diffracted at the core boundary is that P 'does not seem to fall off suddenly at this or a neighboring distance, nor does its appearance undergo a sudden change.' Moreover, she reported that during her investigation of records of a Chilean earthquake 'serious doubt arose respecting the existence of a shadow having a sharp edge.' She investigated data for many shocks and concluded [*Lehmann*, 1953, p. 302]: 'The fact that there are rarely short-period waves in P at great distances may be in favor of the assumption that it is diffracted, but it cannot be taken to prove it. Actually, there is no need for such an assumption if there is no clearly defined shadow, and the result of the present investigation is that there is not.' On the other hand, *Lehmann* [1958, 1959] pointed out that at distances up to 108° short-period P waves have been recorded just as well as long-period waves. In addition, she pointed out that the conditions for the transmission of P and S waves in the portion of the mantle immediately above the core differ, since beyond the critical distance of about 105° the amplitudes of S are frequently considerable.

From the preceding results it appears that the explanation of the P and S waves observed beyond about 103° as diffracted waves depends on answers to the questions of how amplitudes of such waves decrease theoretically in the shadow zone and whether we have to expect a 'sharp edge' of the shadow zone.

Theory and model experiments concerning waves diffracted at the core boundary. Waves diffracted at the boundary of a cylinder were investigated by *Friedlander* [1954]. He found that a diffracted plane unit pulse should flatten into a wave with its maximum moving more slowly than the beginning of the disturbance, as the distance from the beginning of the shadow zone increases. *Jeffreys* [1959, p. 90] pointed out that this theoretical delay of the maximum 'appears to increase more rapidly than linearly with the distance traveled in the shadow.'

Scholte [1956] developed solutions for somewhat simplified equations for elastic waves in the earth. However, he considered the curvature of the layers. He applied his solutions to the shadow zone [*Scholte*, 1956, p. 32] and found that the amplitudes of longitudinal waves with a period of 8 sec should decrease to about one-half at a distance of 8° beyond the boundary of the shadow zone. He concluded that 'this rather slow decrease explains the fact that P waves have been observed far inside the shadow zone.'

Rykunov [1957] investigated the problem by using ultrasonic modeling. The similarity between his model and the earth is good except for the periods which, in the model, are about ten times the equivalent of the periods observed in the diffracted P. His result shows no 'sharp edge' of the shadow. The decrease in amplitudes is faster for short than for long waves and depends on the rigidity in the core near its boundary.

Knopoff and others [1959] included the problem of diffraction of elastic waves by the core of the earth in an investigation of scattering of seismic waves. They found that similar to the diffraction at the boundary of a cylinder, mentioned above, a wave diffracted by a sphere gets flatter, and that its maximum is delayed as the wave arrives at increasing distances in the shadow zone [*Knopoff and others*, 1959, Fig. 9, p. 124]. Slightly over 30° within the shadow zone the amplitudes of waves having a period of about 2½ sec have decreased to about one-tenth, and they have a minimum of about 0.03 about 50° from the beginning of the shadow zone. In their calculations, Knopoff and his collaborators suppose that the shadow zone begins at an epicentral distance of 120°. Near the anticenter of the disturbance which corresponds to 60° within the shadow zone, according to their assumption, the amplitudes have increased again to about one-sixth of their original value as an effect of the concentration of energy near the anticenter. Thus it follows that in great earthquakes the diffracted waves could well be observed as far as the anticenter.

The results discussed in this section agree very well. Theoretically as well as from model experiments it follows that the amplitudes of the diffracted waves should decrease relatively slowly from the beginning of the shadow zone where no well-marked discontinuity should be observed. Only a gradual decrease in amplitudes should be found. Waves having small periods should show a greater decrease of amplitude with distance than those with large periods. Approaching the anticenter, which has the properties of a focus, the amplitudes of the diffracted waves should increase correspondingly. The smallest waves should be observed roughly 10° from the anticenter where waves having periods of 2½ sec should show amplitudes of the order of 1/100 of those at the beginning of the shadow zone.

Observations. Seismograms showing diffracted P waves have been reproduced for the shock of June 26, 1924, by *Macelwane* [1930] and *Krumbach* [1931]. In addition to these, original records have been used of the earthquake of April 16, 1957, 04 04 04, in the East Indies, adopted epicenter at 4.5°S, 107.5°E, focal depth 600 km. Records of this shock show unusually clear P and S waves (Figure 1*a* to 1*d*, 1*k*), as well as PKP and SKS waves [*Gutenberg*, 1959*b*]. The author is grateful to many stations for the loan of records of this shock or for copies. All data used in the following discussion are based on the author's measurements.

In general, it has been found by investigators of diffracted P waves that at epicentral distances of more than 103° they frequently arrive later than the time calculated on the basis

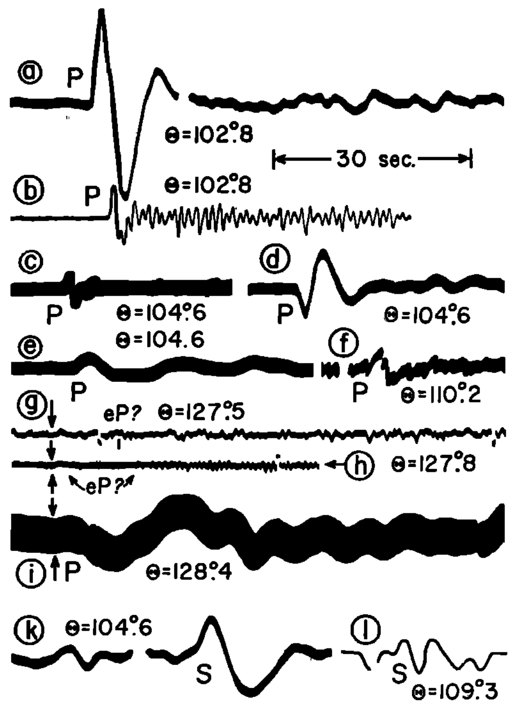

Fig. 1. Portions of seismograms of shock on April 16, 1957, 04 04 04 UT, $h = 600$ km, all on the same time scale; (*a*) long-period and (*b*) short-period vertical components of P at Tamanrasset; (*c*) short-period, (*d*) long-period vertical and (*e*) EW components of P at Kew; (*f*) vertical component of P at Malaga; (*g*) and (*h*) short-period vertical components of P, (*g*) at Eureka, Nevada and (*h*) at Woody; (*i*) ultra-long-period vertical component of P at Pasadena; (*k*) and (*l*) NS components of S, (*k*) at Kew and (*l*) at Toledo. The arrows in (*g*) to (*i*) indicate the time at which P has been calculated from *Gutenberg and Richter* [1936, p. 349] for $h =$ 600 km.

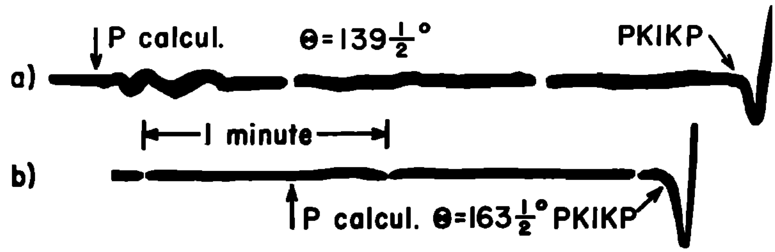

Fig. 2. Beginning of seismograms written by vertical Galitzin seismographs; (*a*) at Eskdalemuir, June 26, 1917, 05 49 40, based on original record; compare *Gutenberg* [1925*a*]; (*b*) at Paris, June 26, 1924, 01 37 26, based on *Macelwane* [1930, Fig. 30]. Estimated accuracy of adopted epicenters about 1½°, of origin times about 6 sec.

of travel-time curves extrapolated by straight lines beyond the end of the direct waves at an epicentral distance of about 103°. *Jeffreys* [1959, p. 89] and *Lehmann* [1953] pointed out that the beginning of diffracted waves is frequently doubtful and may be too small to be observable. The seismograms available for the present investigation show that diffracted P waves having periods of more than about 10 sec (Figs. 1*i*, 2) usually arrive within the limits of error at the time calculated on the basis of the adopted elements for the shock and of the extrapolated travel-time curve. However, the beginnings of waves having periods of less than about 3 sec (Fig. 1*g*, 1*h*) are more and more indefinite as the distance in the shadow zone increases. Beyond an epicentral distance of about 120° the existence of such short-period

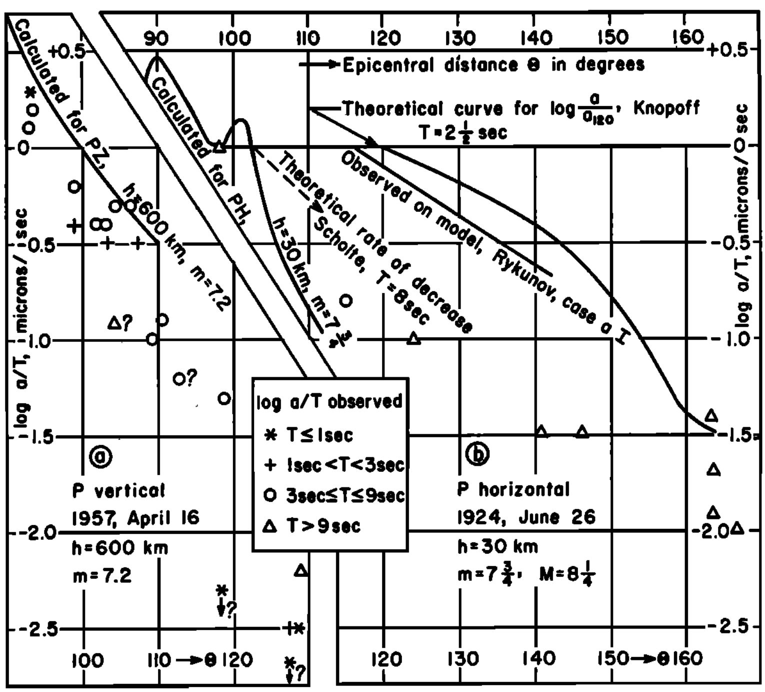

Fig. 3. Logarithms of ratio of observed amplitudes *a* in microns of longitudinal waves P to periods *T* in seconds as function of epicentral distance *θ* in degrees for two earthquakes; theoretical curves after *Scholte* [1956] and after *Knopoff and others* [1959], and curve from model experiment after *Rykunov* [1957]. Calculated curves are based on *Gutenberg and Richter* [1956] for the respective magnitudes and depths of foci.

diffracted P waves can be established beyond reasonable doubt rarely earlier than about 15 sec after the calculated time, and they are usually observable only in records of large earthquakes. The delay in arrival may well be related to the delayed maximum, predicted by the theory for a rectangular pulse.

The beginning of diffracted S waves is disturbed by the motion preceding it. In the shocks investigated here no S waves could be identified beyond a distance of 125°; the observed travel times of the diffracted S waves (Figures 1*k* and 1*l*) agree within about ±15 sec with the values calculated from the extrapolated travel-time curve for S.

For the shocks of June 26, 1924, and April 16, 1957, trace amplitudes of diffracted P and S waves have been measured and ground amplitudes have been calculated as far as possible. The instrumental constants for 1924 were taken from *Krumbach* [1931] for the mechanical instruments and from *McComb and West* [1931] for the Galitzin instruments. For 1957, the constants were taken from data furnished by the stations or from station bulletins for 1957. The resulting ground amplitudes a and periods T are listed in Tables 1 and 2; the logarithms of a/T for P are plotted in Figure 3 and those for

TABLE 1. Approximate Ground Amplitudes a and Periods T for *Horizontal Components* PH of P and SH of S for the Earthquake on June 26, 1924, 01 37 UT, Based on Records Reproduced by *Macelwave* [1930] or *Krumbach* [1931]

		PH		SH	
Station	θ, deg	a, μ	T, sec	a, μ	T, sec
Rio de Janeiro	98.7	15±	14	60	15
Hyderabad	98.8	?	?	25	12
Pasadena	114.8	0.5?	7	5	10
Victoria, B. C.	123.4	1.2	10	4?	10?
Sverdlovsk	140.6	0.5	18	...	...
Ottawa	145.4	0.3	10?	...	...
Uccle	163.4	0.2	15	...	...
Paris	163.6	0.4	14	...	...
De Bilt	163.6	0.3	15	...	...
Oxford	167.0	0.1?	10	...	...

TABLE 2. Approximate Ground Amplitudes a and Periods T of the Earthquake on April 16, 1957, 04 04 UT, Magnitude m = 7.2, Based on Original Records.

		PZ*		PZ*		PH†		SH†	
Station	θ, deg	a, μ	T, sec	a, μ	T, sec	a, μ	T, sec	a, μ	T, sec
Kiruna	93.0	1.4	0.7	9	7	2.7	6	16	10
Uppsala	93.8	...	...	10	6	3.5	4	16	16
Honolulu	95.7	...	...	...	...	...	...	14	6
Stuttgart	98.8	0.8	2	5	8	1.6	8	4	8
Tamanrasset	102.8	0.7	2½	3	8	...	...	4	9
Alger	104.3	1¼	3	...	...	...	...	...	...
Kew	104.6	...	...	3½	7	{1¼ {¾±	7} 12}	6	12
Relizane	106.4	1.6	3	...	...	...	...	...	...
Rathfarnham	107.7	0.8	2½	...	...	...	...	...	...
Sitka	108.1	...	...	...	...	...	...	7±	7±
Toledo	109.3	...	...	½	5	...	...	...	...
Coimbra	112.6	0.02?‡	7	...	...	...	...	...	...
Victoria, B. C.	118.5	≤0.002	¾	0.35	7	...	...	4±	13
Santa Clara	124.3	...	...	...	...	...	...	1±	6
Saskatoon	124.8	...	...	...	...	...	...	2±	7
Eureka, Nev.	127.5	0.001	¾	0.01±	3	...	...	...	...
Woody, Cal.	127.8	0.002	¾	...	...	...	...	...	...
Pasadena	128.4	≤0.5	7	0.13	20	...	...	...	...

* PZ is the vertical component of P.

† PH and SH are the horizontal components of P and S.

‡ Magnification of instrument based on ratio of trace amplitude to calculated ground amplitude of PP for shock at depth of 600 km and m = 7.2.

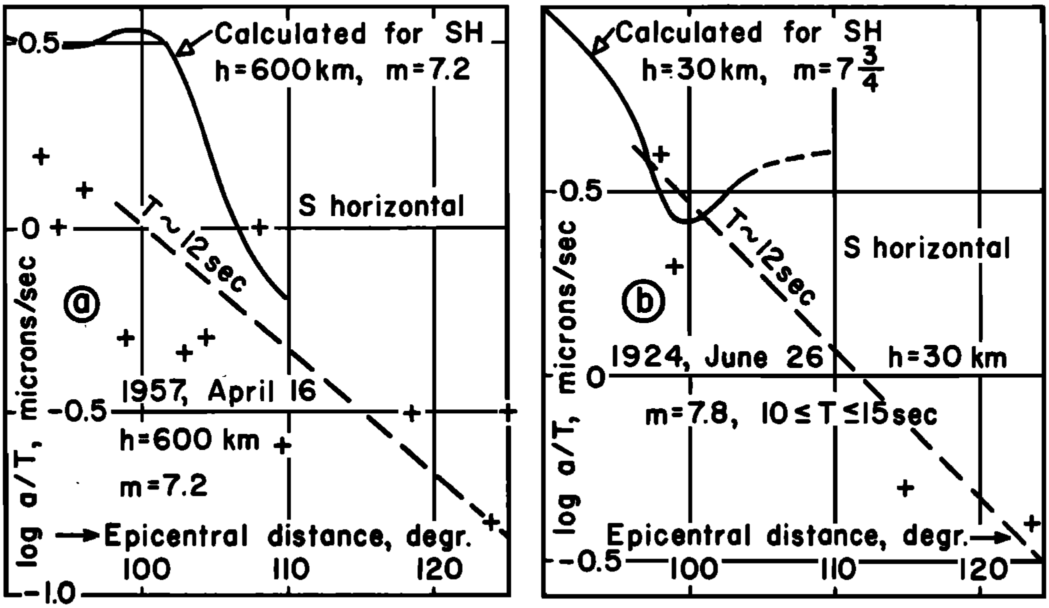

Fig. 4. Logarithms of observed values a/T for transverse waves; details as in Figure 3.

SH are plotted in Figure 4. It should be noted that the various authors of the theoretical curves reproduced in Figure 3*b* assumed different distances for the beginning of the shadow zone. On the other hand, *Gutenberg* [1957, p. 319] found that in P waves at distances of less than 100° 'the ratio a/T is approximately the same only for periods T of less than 8± seconds, but it decreases appreciably if the periods increase beyond 10± seconds; for periods of 20 seconds it is only roughly ¼ of the corresponding value for waves with periods of 4 seconds.' This has to be considered in comparing the observed values of a/T for $T > 8$ sec for P waves with the theoretical curves. From such a comparison it follows that the observed decrease in amplitudes of P in the shadow zone is approximately the same as the decrease calculated for waves diffracted at the core boundary.

TABLE 3. Approximate Values of the Exponential Factor f (Equation 2) for the Decrease of Diffracted Longitudinal Waves in the Shadow Zone of the Earth's Core

T, sec	f/megameter	Source
110±	0.4	*Rykunov* [1957] from model experiments
70±	0.6	*Rykunov* [1957] from model experiments
20	0.9±	Pasadena, Table 2, discussed in text
12	0.8	Observed, Figure 3*b*, Table 1
8	0.8	Theoretical, *Scholte* [1956]
6	1.2	Observed, Figure 3*a*, Table 2
2½	1.2	Theoretical, *Knopoff and others* [1959]
≤1	1.9	Observed, Figure 3*a*, Table 2

For a numerical comparison we may assume that the amplitudes in the shadow zone decrease from diffraction by a factor e^{-fD} with the distance D in megameters. If θ_1 and θ_2 are the angular distances in degrees of two points in the shadow zones,

$$D = (\theta_2 - \theta_1)/9 \text{ megameters} \qquad (1)$$

Considering that the waves arrive on zones which decrease under otherwise equal circumstances as the anticenter ($\theta = 180°$) is approached, we assume that

$$-f = \frac{2.3}{D} \log \frac{a_2 \sin \theta_2}{a_1 \sin \theta_1} \quad \text{per megameter} \qquad (2)$$

We find from Figure 3*b* that $f = 0.8$ for longitudinal waves having periods of about 12 sec, and from Figure 3*a* that $f = 1.9$ for longitudinal waves having periods of about 1 sec. As has been mentioned, the value of a/T for longitudinal waves having periods of 20 sec at distances less than 100° is roughly one-fourth of those having periods of less than about 12 sec. Similarly, records of PH at Kew give for a/T about 0.2 for $T = 7$ sec and about 0.06 for $T = 12$ sec. Consequently, the observation at Pasadena, $\theta = 128.4°$ (Table 2, Figure 1*i* and 3*a*) would give roughly $f = 0.9$ for P waves having periods of about 20 sec. Effects of the focal depth of 600 km on amplitudes (and distances) are within the limits of errors involved in the present research.

A comparison of the values of f for diffracted longitudinal waves derived from observations with those from theoretical results in Table 3 shows good agreement. Within the limits of the periods T used, all data in Table 3 can be represented fairly well by $f = 1.7 - 0.6 \log T$. This would indicate that the equation for the decrease in amplitudes of P in the shadow zone contains a factor of roughly $T^{D/4}$.

Summarizing, we may conclude that the waves observed in the shadow zone of P beyond $\theta = 103°$ exhibit the properties to be expected theoretically for diffraction at the core boundary. There is no indication that an assumed decrease of the velocity in the lowest portion of the mantle could lead to the observed 'diffracted' waves at distances beyond about 110°. On the other hand, available calculations indicate that P waves arriving at distances of 90° cannot have gone deep enough to graze the core. A constant velocity or one decreasing by about 0.1 km/sec in the portion of the mantle, roughly 100 km thick, just above the core would account for the nearly straight line travel-time curves and the decreasing amplitudes of P waves at epicentral distances between about 90° and slightly over 100°.

The relatively slow decrease in amplitudes of the waves diffracted at the core boundary and arriving at epicentral distances between about 103° and the anticenter contrasts with the much more rapid decrease of the amplitudes near caustics [*Gutenberg*, 1958]. For this decrease, but not for the decrease in the shadow of the earth's core, *Jeffreys* [1939*b*, p. 553; 1959, p. 90] applied Airy's theory of diffraction and found that waves with periods of 1 sec should be traceable back only a few degrees, and those with periods of 10 sec not over about 15°.

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