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A METHOD FOR BOUNDING UNCERTAINTIES IN COMBINED PLATE RECONSTRUCTIONS

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Abstract. We present a method for calculating uncertainties in plate reconstructions that does not describe the uncertainty in terms of uncertainties in pole positions and rotation angles. If a fit of magnetic anomalies of the same age and fracture zones that were active as transform faults at that time can be found, such a reconstruction can be perturbed and degraded by small rotations about each of three orthogonal axes (partial uncertainty poles). If the uncertainty in the reconstruction is a consequence of independent, small, but acceptable, rotations about these axes, then the uncertainties in reconstructed points will be elliptical in shape. The dimensions and orientation of such ellipses will depend upon the magnitudes of the perturbing rotations and upon the relative geometry of the partial uncertainty poles and the points in question. In a sequence of rotations, each rotation will contribute an elliptical region of uncertainty for each reconstructed point, and these ellipses can be combined as independent statistical quantities to obtain a confidence ellipse for the sequence of rotations. As a test, we calculated uncertainties for three points on the Pacific plate with respect to North America at the time of anomaly 6 (20 Ma). The computed uncertainties are similar in shape to those that we previously obtained for a sequence of marginally acceptable rotations, but the major axes of the ellipses presented here are about 25% shorter.

Introduction

The relative position of two plates at a given time in the past is described by a finite rotation of one plate relative to the other. Such rotations, or reconstructions, can be implemented using either 3 x 3 rotation matrices [e.g., McKenzie and Sclater, 1971, Appendix] or 2 x 2 matrices constructed from Cayley-Klein parameters [Francheteau, 1970, chapter 1; Le Pichon et al., 1973, Appendix]. In either case, three independent quantities uniquely describe the reconstruction: the coordinates of a pole or axis about which the rotation is performed and the angle of the rotation. These quantities are easy to visualize, but determining the uncertainties in them is more difficult because the uncertainties in the three quantities are tightly coupled to one another.

In three-dimensional, latitude-longitude-rotation angle space, the "best" pole position and angle can be represented by a point with the latitude and longitude of the pole and with the angle given by the distance from the origin along the radial axis. Other pole positions about which finite rotations bring magnetic anomalies and fracture zones from one plate into near coincidence with those of the other plate will surround the point describing this "best" pole and angle. (Although the best rotation and its uncertainty can be described in this way, one cannot, however, treat the best pole and angle as a vector, even if it seems to be represented as one.) The angles associated with those pole positions close to the positions of the reconstructed magnetic anomalies and fracture zones will be larger than those for pole positions far from these data [e.g., Stock and Molnar, 1983]. Thus the family of rotations representing the uncertainties in the pole position and rotation angle will form a plunging blimp-shaped volume in latitude-longitude-angle space. The point representing the "best" rotation should lie toward the center of the blimp but may not be exactly the center. All other acceptable pole positions and angles would then lie within the blimp.

The uncertainty of a rotation, as represented by this three-dimensional blimp-shaped region of latitude-longitude-angle space, is difficult to depict geometrically. The normal representation of the uncertainty in a finite rotation as a confidence region surrounding the best pole (a projection of the blimp onto the latitude-longitude plane) underemphasizes the strong dependence of the value of the rotation angle (and its possible variation) on the location within the confidence region. This can lead to an underestimate of the uncertainties in combined finite rotations because of the loss of information regarding acceptable values of the rotation angle for a given pole position. It follows that the detailed analysis of the uncertainties in the finite rotations themselves is mathematically complex and difficult to implement because of the strong coupling between the allowable values of latitude, longitude, and angle. Moreover, because of this dependence the relationships of the uncertainties in the pole positions and rotation angles to uncertainties in reconstructed positions of points on different plates are particularly complicated [see Stock and Molnar, 1983].

In most cases our ultimate goal is not to know the uncertainty in the pole position or in the angle but to know the uncertainties of the reconstructed positions of the plates. It turns out that to estimate the uncertainties in reconstructed positions of plates, we need neither to determine nor to use the uncertainties in the pole positions and the rotation angles. Below we discuss a method for estimating the uncertainties in reconstructed positions of specific points on individual plates, building on observations of certain geometrical aspects of finite rotations discussed by Stock and Molnar [1983].

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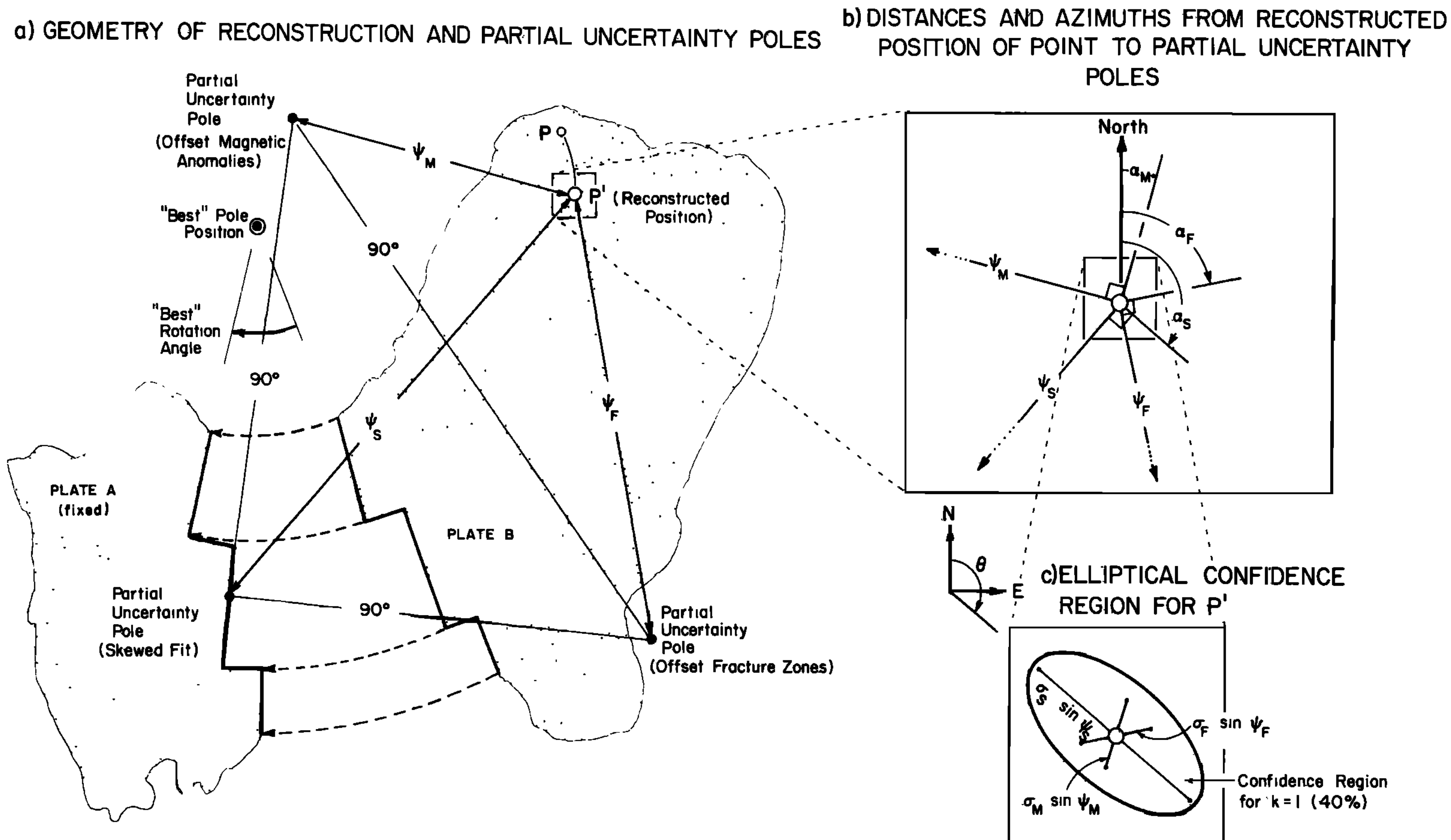

Fig. 1. Geometry of reconstruction, partial uncertainty poles, and confidence region for a particular point. (a) Plate B is rotated about a "best pole" to plate A, arbitrarily held fixed. The relative motion cannot be described by the rotation about this best pole; a change in direction of spreading has occurred. The positions of three partial uncertainty poles are shown 90° from one another. Point P on plate B is rotated to P', its reconstructed position with respect to plate A. The angular distances Ψ_S, Ψ_F, Ψ_M are also shown. (b) The contributions of partial uncertainty rotations about the three partial uncertainty poles are in directions, with azimuths α_S, α_F, and α_M, perpendicular to the great circles through P' and the three partial uncertainty poles. (c) Each partial uncertainty rotation independently contributes a small uncertainty ($\sigma \sin \Psi$) to the position P', and the combination yields an ellipse with an orientation of the semimajor axis given by θ. The ellipse given by k=1 corresponds to the 40% confidence region.

Uncertainties in Reconstructed Positions

Single Finite Rotation

First, let us assume that we can find an acceptable pole and an angle to describe the relative position of one plate with respect to another at some past time. We would use the best pole and angle that we could, derived in one way or another from the available magnetic and bathymetric data [e.g., Hellinger, 1979, 1981; Pilger, 1978; G. Cole, manuscript in preparation, 1985]. If no acceptable pole and angle could be found, then either the interpretation of some of the data would be erroneous or more than two plates would be represented by the data. The following analysis is thus based on the assumption that an acceptable (good) reconstruction exists and can be found. Using this reconstruction points on the rotated plate can then be reconstructed to their "best fit" positions with respect to the (arbitrarily chosen) fixed plate.

We can then perturb the reconstructed positions by performing small rotations about three orthogonal axes (Figure 1a). Rotations about an axis through the middle of the reconstructed positions of the various data used will skew the fit by causing an overlap at one end of the reconstructed plate boundary and a gap at the other end. A second axis lies 90° from the first in the direction of the trend of fracture zones, which were active as transform faults when the reconstructed plate boundary was active. Small rotations about this axis will misalign fracture zones, but reconstructed positions of magnetic anomaly lineations will still be coincident, at least for spreading centers orthogonal to fracture zones. The third axis, perpendicular to the others, lies on the great circle through the first axis and perpendicular to the trends of the fracture zones. Small rotations about this axis will misalign magnetic anomalies but not fracture zones. Stock and Molnar [1983] called these three axes partial uncertainty poles and discussed some of the pathological consequences of small but allowable rotations about these three axes on the uncertainties in the reconstructed positions of points not on the plate boundary under consideration.

Note that the positions of the partial uncertainty poles for mismatched magnetic anomalies and mismatched fracture zones are defined here differently from the positions given by Stock and Molnar [1983]. They are equivalent for cases in which the position of the finite pole is close to that of the instantaneous pole for the reconstructed time. If this is not the case, however (e.g., for boundaries that have experienced change in a spreading direction), it is preferable to define the partial uncertainty poles as outlined above.

Note that the positions of the partial uncertainty poles are governed by the distribution of reconstructed data that define the plate boundary. The positions of these poles are virtually independent of the quality or density of the data. Given a maximum uncertainty of 10° in trends of the fracture zones, the uncertainties in the positions of the partial uncertainty poles are about 10°, which in most cases is an insignificant amount for the analysis described below.

The uncertainties in the small rotation angles that describe the partial uncertainty rotations depend on the quality (and quantity) of the data. They depend both on the uncertainties in the positions of magnetic anomalies and fracture zones defining the plate boundary and on the misfits of the best fitting reconstructed positions of these data (and the number of such data). Let us now make an assumption that is difficult to justify (and that we discuss further below); let us assume that we can treat the angles used to describe the partial uncertainty rotations as statistical quantities and specifically as standard errors. Call these angles σ_S, σ_F, and σ_M, where S, F, and M stand for skewed fits, mismatched fracture zones, and mismatched magnetic anomalies. Also let us describe these angles with units of kilometers (1° = 111.4 km) so that the angles correspond to the uncertainty in the positions of points 90° from the particular partial uncertainty poles.

We are interested in the uncertainty in the reconstructed position P' of a point, originally at P. Let us set the origin of a Cartesian coordinate system at P', with the x and y directions as east and north (Figure 1). Insofar as the uncertainty in the position P' results from random and independent sources of error, then the probability density function describing the expected reconstructed position, P', will be a two-dimensional Gaussian distribution:

$$f(x,y) = \frac{1}{2\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right)\right] \quad (1)$$

[e.g., Fisz, 1963, p. 158]. In this expression σ_x and σ_y represent standard deviations in the x and y coordinates, and ρ, for which $-1<\rho<1$, is the correlation coefficient between x and y.

Note that ellipses defined by

$$\frac{1}{1-\rho^2}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right] = k^2 \quad (2)$$

describe loci of equal probability density. The areas enclosed by such ellipses are confidence regions for particular probabilities. For instance, k=2.448 corresponds to 95% confidence. Ellipses described by (2) can also be parameterized by the lengths of major and minor axes, σ_1 and σ_2, and by the azimuth, θ, of the major axis:

$$\sigma_1^2, \sigma_2^2 = \frac{1}{2}\left[\sigma_x^2 + \sigma_y^2 \pm \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\rho^2\sigma_x^2\sigma_y^2}\right]$$

$$\tan 2\theta = \frac{2\rho\sigma_x\sigma_y}{\sigma_y^2 - \sigma_x^2} \quad (3)$$

If two (or more) independent sources of error, each associated with its own values of σ_{xi}, σ_{yi}, and ρ_i, contribute to the uncertainty in P', then the combined probability density function is also Gaussian in two dimensions, with

$$\sigma_x^2 = \sum_i \sigma_{xi}^2$$

$$\sigma_y^2 = \sum_i \sigma_{yi}^2 \tag{4}$$

$$\rho = \frac{\sum_i \rho_i \sigma_{xi} \sigma_{yi}}{\sqrt{\sum_i \sigma_{xi}^2 \sum_i \sigma_{yi}^2}} = \sum_i \rho_i \sigma_{xi} \sigma_{yi} / \sigma_x \sigma_y$$

(see, for instance, Fisz [1963, pp. 105-113, 147-151, 158-159]).

Before proceeding, notice that if $\rho^2 = 1$, equation (1) loses meaning; the probability density function collapses to a one-dimensional distribution in which x and y are linearly dependent on one another. As can be seen from equation (2), for $\rho = \pm 1$, the ellipse collapses into a straight line $y = \rho(\sigma_y/\sigma_x)\,x$. Notice also, however, that if there are two different sources of error such that either or both sources contribute errors in x and y that are linearly, but differently, related, then the combined uncertainty will be Gaussian in two dimensions. Moreover, for such a case the correlation coefficient cannot be equal to 1 or -1.

Let us consider the uncertainty in P' due to one partial uncertainty rotation. A small rotation about the partial uncertainty pole will displace the point P' a small amount along a small circle about this pole. Assume that the displacement is sufficiently small that we can approximate the displacement as a straight line on a flat surface centered at P'. For skewed fits, σ_S is of the order of 500 km or less, and for mismatched fracture zones or magnetic anomalies, σ_F and σ_M, in general, are less than 50 km. Thus the curvature of the earth is negligible. For a partial uncertainty pole at distance Ψ from P' (Figure 1a) the magnitude of the uncertainty in the position of P' due to this partial uncertainty rotation can be approximated by $\sigma_S \sin \Psi_S$, $\sigma_F \sin \Psi_F$, or $\sigma_M \sin \Psi_M$, depending upon which partial uncertainty rotation is considered (Figure 1c). Let α_S, α_F, α_M be azimuths (measured clockwise from north) describing the directions that P' can be displaced about the particular partial uncertainty poles (Figure 1b); these directions are perpendicular to the great circles through P' and the appropriate partial uncertainty poles. Insofar as statistical significance can be attached to σ_s^2, σ_f^2, and σ_m^2, the displacement due to each of the three poles can be considered to be a one-dimensional Gaussian probability density function, represented in x-y coordinates as follows:

$$\sigma_{xj}^2 = \sigma_i^2 \sin^2\Psi_i \sin^2\alpha_i \qquad i = S,\ F, \text{ or } M$$

$$\sigma_{yi}^2 = \sigma_i^2 \sin^2\Psi_i \cos^2\alpha_i$$

and $\rho_i = \pm 1$ depending upon whether the product $\sin\alpha_i \cos\alpha_i$ is positive or negative. We assume that the uncertainties in P' from these three partial uncertainty rotations are independent of one another. Since these three probability density functions are not colinear, they can be combined into a single two-dimensional probability density function, with $p^2 \neq 1$, representing the final position of point P'. This probability density function will have the following values:

$$\sigma_x^2 = \sum_{i=S,F,M} \sigma_i^2 \sin^2\Psi_i \sin^2\alpha_i$$

$$\sigma_y^2 = \sum_{i=S,F,M} \sigma_i^2 \sin^2\Psi_i \cos^2\alpha_i$$

$$\rho = \frac{\sum_{i=S,F,M} \sigma_i^2 \sin^2\Psi_i \sin\alpha_i \cos\alpha_i}{\sigma_x \sigma_y}$$

Substitution of (5) into (2) can then be used to define confidence ellipses for P'.

Successive Finite Rotations

Suppose that we seek the uncertainties in points whose positions are calculated by a series of rotations each of which is uncertain. For instance, suppose we want to know the uncertainties in the reconstructed positions of points on the Africa plate with respect to the Eurasia plate. Since the Africa-Eurasia boundary is convergent, we cannot calculate the uncertainties directly; instead, we must use the relative positions of these two plates with respect to North America in order to calculate their relative positions. We must first rotate Africa to North America and then both Africa and North America to Eurasia. Two rotations, each of which is uncertain, must be performed.

For each rotation there will be three partial uncertainty poles and three corresponding angles. First, we rotate the points P_i on the first plate (Africa) to their "best fit" positions P_i' with respect to the second plate (North America), and we calculate the partial uncertainty poles and the corresponding values of σ_{Si}', σ_{Fi}', and σ_{Mi}', in the frame of reference of the second plate. We then perform the second rotation (North America to Eurasia) to move the points to their "best fit" positions P_i'' with respect to the third plate (Eurasia). We must also rotate the partial uncertainty poles by the same amount so that they describe the uncertainty of the positions of points on the first plate (Africa) due to the first rotation (Africa to North America) but in the frame of reference of the third plate (Eurasia). This procedure is outlined mathematically in the appendix. To estimate the uncertainty in the positions of the points (on Africa moved to North America) due to the second rotation (North America to Eurasia), we must estimate the partial uncertainty rotations for this second rotation. We can then repeat this process for subsequent rotations; for each new rotation we must rotate the existing set of partial uncertainty poles, and we must add a new set.

Each of the successive rotations is independent of the previous rotations, and the uncertainties in each are independent of the others. Thus once all of the partial uncertainty poles are rotated to the reference frame of the last, fixed plate (Eurasia in the case outlined above), we can

TABLE 1. Parameters Describing Finite Rotations and Partial Uncertainties

	Pacific→North America	Antartica→North America	India→North America	Africa→North America
		"Best" Poles and Angles		
Latitude	47.09°N	-60.88°S	-36.12°S	81.03°N
Longitude	-65.18°W	-41.28°W	-134.74°W	35.03°E
Angle	11.76°	6.76°	13.67°	-5.32°
		Skewed Fit*		
Latitude	-57.68°S	-61.71°S	-23.66°S	28.50°N
Longitude	-121.73°W	105.51°E	59.22°E	-42.65°W
σ_S, km	20.6	13.2	143.6	32.3
		Mismatched Fracture Zones*		
Latitude	-8.20°S	27.26°N	20.92°N	-8.90°S
Longitude	-18.57°W	122.26°E	139.59°E	42.48°E
σ_F, km	5.0	5.0	5.0	5.0
		Mismatched Magnetic Anomalies*		
Latitude	31.01°N	6.98°N	57.52°N	59.90°N
Longitude	-103.60°W	28.66°E	12.69°E	116.82°E
σ_M, km	5.0	5.0	5.0	5.0

*Partial uncertainty poles and angles (in the frame of North America)

simply combine contributions of each as we did to obtain (5):

$$\sigma_x^2 = \sum_{j=1}^{N} \sum_{i=S,F,M} \sigma_{ij}^2 \sin^2\Psi_{ij} \sin^2\alpha_{ij}$$

$$\sigma_y^2 = \sum_{j=1}^{N} \sum_{i=S,F,M} \sigma_{ij}^2 \sin^2\Psi_{ij} \cos^2\alpha_{ij} \qquad (6)$$

$$\rho = \frac{\sum_{j=1}^{N} \sum_{i=S,F,M} \sigma_{ij}^2 \sin^2\Psi_{ij} \sin\alpha_{ij} \cos\alpha_{ij}}{\sigma_x\sigma_y}$$

Here N finite rotations are performed, and all distances Ψ_{ij} and azimuths α_{ij} must be measured in the reference frame of the fixed plate with respect to which the plate in question has been reconstructed. Once again the confidence regions for the calculated positions of particular points will be given by (2), into which we can substitute (6).

Statistical Significance of the Uncertainties in the Reconstructions

If errors in the positions of the magnetic anomalies and fracture zones defining segments of plate boundaries obeyed a Gaussian distribution, then we probably could use a least squares approach to estimate σ_S^2, σ_F^2, and σ_M^2. We could then proceed with the analysis described above and use (2) or (3) and (6) to define 95% (or other) confidence regions for the positions of reconstructed points.

Errors in the positions of magnetic anomalies or fracture zones arise from two causes: errors in navigation and subjective decisions about what part of a magnetic anomaly or topographic profile defines a particular age of seafloor or the location of a fracture zone. Errors in navigation probably are random, and since the advent of satellite navigation, such errors are generally small (a few kilometers). Uncertainties in defining either a particular magnetic anomaly or the plate boundary within rough fracture zone topography are probably not random. It is likely that we make systematic errors in picking the plate boundary in fracture zones, and the same may be so for some magnetic anomalies. Moreover, sometimes we can reject reconstructions because of mismatches that cannot be quantified easily. For instance, often fracture zones can be inferred solely from offsets in magnetic anomaly lineations, each of which is defined by only a few crossings. We will reject a reconstruction that brings magnetic anomaly crossings from one segment on one plate to a position where magnetic anomalies on the other plate require a match with a different segment. Perhaps ultimately there will be enough data to use statistical methods to evaluate the probablility (or improbability) of such a mismatch, but at present, most of us think that we must rely on some subjective judgement in estimating uncertainties of reconstructions.

Thus we do not feel that we can be confident of making statistically meaningful estimates of uncertainties in partial uncertainty rotations (σ_S, σ_F, σ_M). At the same time, if we consciously overestimate these quantities, we can combine them to obtain maximum likely uncertainties in reconstructed positions of plates in which successive reconstructions are involved.

Ignorant of how to proceed, Stock and Molnar [1983] simply combined families of reconstructions to estimate the full range of possible reconstructed positions of the Pacific plate at the time of anomaly 6. They derived uncertainties by implicitly assuming that sequences of marginally acceptable reconstructions are as equally probable as the sequence of best fitting ones. Thus their uncertainty regions included several combinations of marginally acceptable reconstructions. The method outlined above gives a smaller uncertainty region than theirs by

TABLE 2. Positions of Three Points on the Pacific Plate With Respect to North America at the Time of Anomaly 6

	A	B	C
Present positions	51.90°N, -147.30°W	38.60°N, -127.65°W	26.95°N, -118.40°W
Position reconstructed to North America	44.05°N, -140.42°W	31.69°N, -121.88°W	20.81°N, -112.05°W

rendering such combinations less probable than those comprising combinations of better fitting reconstructions.

An Example: Pacific-North America at Anomaly 6 Time

To illustrate the use of expressions (2), (3) and (6) above, we calculated uncertainties in reconstructed positions of three points on the Pacific plate with respect to North America, the same three points considered by Stock and Molnar [1983]. Using their choice of best rotation parameters for Pacific-Antartica, for Antartica-India (Australia), for India-Africa, and for Africa-North America, we calculated the parameters for rotating these plates to North America (Table 1). We then took the partial uncertainty poles and angles that they gave for each of these rotations and rotated the pole positions about the "best" poles to lie in the frame of reference attached to North America (Table 1). As noted above their partial uncertainty poles are defined slightly differently from here, but we use their poles in order to make an exact comparison. For this case partial uncertainty poles derived as outlined above would not be very different. The partial uncertainty angles used by Stock and Molnar [1983] correspond to a misfit of 10 km in the reconstructed data. Let us assume that these correspond to 95% or 1.96σ confidence limits in a one dimensional normal distribution. We then calculated the rotated positions of the three points (Table 2), which are the positions of three points on the Pacific plate reconstructed to their positions relative to the North American plate at the time of anomaly 6 (~20 Ma). Using these positions, we calculated the contributions to the uncertainties in the positions of points A, B, and C (values of $\sigma \sin\Psi$ and α in Table 3) from each partial uncertainty rotation.

Before proceeding, let us recall some of the peculiarities of this anomaly 6 reconstruction described by Stock and Molnar [1983]. First, the very short segment of ridge along which magnetic anomalies and fracture zones lie in the northwest Indian Ocean allows a large angle (2.585°) for a skewed misfit of 10 km in the India-Africa reconstruction, corresponding to a maximum possible error of σ_s=143.5 km. Indeed the largest uncertainty ($\sigma \sin\Psi$) contributed to any of the three point positions is due to this possible skewed misfit: 63 km in the direction N45°E for point A (Table 3). Note, however, that the reconstructed positions of points B (31.70°N, 121.88°W) and C (20.81°N, 112.05°W) are very close to the antipodal position of this partial uncertainty pole, when reconstructed to the reference frame of North America (23.66°N, 120.88°W). Consequently, the uncertainties in the positions of points B and C contributed by a large, skewed misfit of the India and Africa plates (20 km and 21 km, respectively) are not as

TABLE 3. Partial Uncertainties of Three Points on the Pacific Plate With Respect to North America at the Time of Anomaly 6.

	A		B		C	
	$\sigma\sin\Psi$,km	α,deg	$\sigma\sin\Psi$,km	α,deg	$\sigma\sin\Psi$,km	α,deg
Pacific-Antartica						
Skewed fit	21	80	21	90	21	95
Mismatched fracture zones	4	163	5	0	5	6
Mismatched magnetic anomalies	3	12	1	178	1	125
Antarctica-India (Australia)						
Skewed fit	9	131	9	121	10	113
Mismatched fracture zones	5	25	5	37	5	40
Mismatched magnetic anomalies	4	104	4	131	4	148
India-Africa						
Skewed fit	64	45	21	83	22	21
Mismatched fracture zones	5	9	5	22	5	26
Mismatched magnetic anomalies	5	105	5	113	5	116
Africa-North America						
Skewed fit	32	154	30	160	29	158
Mismatched fracture zones	3	85	2	126	2	158
Mismatched magnetic anomalies	4	55	5	64	5	68

TABLE 4. Parameters Describing Elliptical Uncertainty Regions for Three Points on the Pacific Plate With Respect to North America at the Time of Anomaly 6

	A	B	C
σ_x^2, km^2	2814	1106	761
σ_y^2, km^2	3000	926	1260
ρ	0.59	-0.27	-0.21
semi-major axis, km	68(166)*	36(88)	36(89)
semi-minor axis, km	35(85)	27(66)	26(64)
Azimuth of semi-major axis	N43°E	N126°E	N160°E

*Numbers in parentheses for 95% confidence limits.

large as the uncertainties contributed by a skewed fit of Africa to North America (30 km for B and 29 km for C in Table 3). This comparison illustrates how the relative positions of the plate boundaries that are reconstructed and the positions of points that are reconstructed affect the uncertainties in the reconstructed positions of these points.

Using the values of σ's and α's in Table 3, we calculated values of σ_x^2, σ_y^2, and ρ for the three points and the sizes and orientations of the corresponding ellipses (Table 4; Figure 2). The shapes and orientations of these ellipses vary markedly from being elongated and oriented northeast-southwest (A), to more circular and oriented west-northwest-east southeast (B) and north northwest-south southeast (C). These variations illustrate again the complexity of the interactions among the various geometrical controls on the uncertainties.

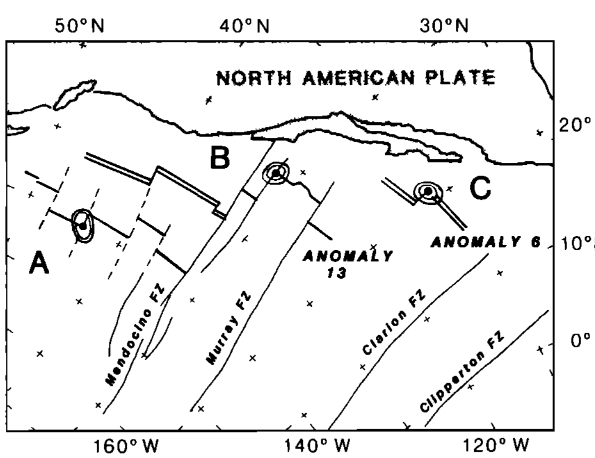

Fig. 2. Positions and uncertainties in the positions of three points in the Pacific plate reconstructed to North America at the time of anomaly 6 (~20 Ma). Large ellipses surrounding the points enclose all possible positions calculated by Stock and Molnar [1983], and smaller ellipses give 95% confidence ellipses calculated here.

It is important to note that although these ellipses represent the 95% confidence limits for the positions of the points at anomaly 6 time, the positions of the points within these ellipses may not vary independently. The three points are all on the same plate and at fixed distances from one another; their relative distances must be preserved in the final reconstruction. Thus, if independent geologic constraints allow the reconstructed position of point A to be restricted to a portion of the 95% confidence region, then the allowable uncertainty regions for points B and C will also decrease to reflect this additional constraint.

Note also that the calculated uncertainties given here are somewhat less than those obtained by combining marginally acceptable, and hence improbable, reconstructions for the various pairs of plates [Stock and Molnar, 1983]. Thus, insofar as we can assume that the allowable misfits due to partial uncertainty rotations are independent of one another, the method outlined above should reduce the uncertainties from those obtained from all combinations of marginally acceptable reconstructions.

Summary

Building on the description given by Stock and Molnar [1983] of errors in reconstructions in terms of partial uncertainty rotations, we calculate uncertainty regions for points on one plate reconstructed to their positions relative to a second plate. Insofar as the uncertainties resulting from small rotations about the three partial uncertainty poles are independent, the resulting confidence region for a particular point will be elliptical, given by equation (2) or (3). The size and orientation of the confidence region will depend upon the quality and quantity of data that define the plate boundaries, which are given by equations (4) and (5). The elliptical uncertainties associated with each rotation in a sequence can then be rotated to the frame of the fixed plate and combined as independent statistical quantities, given by equation (6).

Appendix

As an example of two successive rotations in which we examine the uncertainty in the first rotation, let us consider the sequence Africa to North America to Eurasia. In symbols, let ${}_{NA}\Omega_{Af}$ describe the operation of rotating Africa to North America and ${}_{Eur}\Omega_{NA}$ describe the operation of rotating North America to Eurasia. (Recall that $[{}_{NA}\Omega_{Af}]^{-1} = {}_{Af}\Omega_{NA}$.) If we rotate points P_{Af} on the African plate to P'_{Eur} and let operations proceed from right to left, the "best fit" reconstruction is

$$P'_{Eur_o} = {}_{Eur}\Omega_{NA} \; {}_{NA}\Omega_{Af} \; P_{Af}$$

Let ${}^{P}\Omega_{NA}$ describe one of the appropriate partial uncertainty rotations for the reconstructed position of Africa with respect to North America (in the reference frame fixed to North America).

The reconstructed position including an error describable by a small rotation about one partial uncertainty pole (${}^{P}\Omega_{NA}$) is

$$P'_{Eur} = {}_{Eur}\Omega_{NA} \quad {}^{P}\Omega_{NA} \; {}_{NA}\Omega_{Af} \quad P_{Af}$$

which can be written also as

$$P'_{Eur} = ({}_{Eur}\Omega_{NA} \quad {}^{P}\Omega_{NA} \; {}_{NA}\Omega_{Eur}) \; {}_{Eur}\Omega_{NA} \quad {}_{NA}\Omega_{Af} \; P_{Af}$$

The error in the reconstructed position of a point on Africa rotated to Eurasia, P'_{Eur}, is thus given by

$$\Delta P'_{Eur} = P'_{Eur} - P'_{Eur_o}$$

$$= ({}_{Eur}\Omega_{NA} \quad {}^{P}\Omega_{NA} \quad {}_{NA}\Omega_{Eur} - I) \quad P'_{Eur_o}$$

where I is the identity matrix.

The operator that allows us to calculate the error due to a particular partial uncertainty rotation is $({}_{Eur}\Omega_{NA}[{}^{P}\Omega_{NA} - I]{}_{NA}\Omega_{Eur})$. It rotates Eurasia to North America. Then it calculates the uncertainty of a point on Eurasia with respect to North America (or to Africa rotated to North America) due to the error in the reconstructions of Africa to North America. Finally, it rotates the system to Eurasia to give uncertainties in positions on Africa, which had been rotated to North America, and these uncertainties are now given in the reference frame fixed to Eurasia. This operation is the same as first rotating to Eurasia the position of the partial uncertainty pole that initially was determined in the North America frame and that describes the error in Africa's position with respect to North America and then, second, calculating its effect on reconstructed positions of points on North America (or in points rotated to it in earlier stages).

Obviously, the same procedure should work for any other sequence of rotations.

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