THE PROCESS OF FORMATION OF SEISMIC SURFACE WAVES[1]

By B. Gutenberg

The results of studies concerning the characteristics of the surface waves of earthquakes, carried on by many investigators, lead to the conclusion that: (*a*) in the case of near earthquakes, quite short and mostly irregular waves predominate, though at the same time, it is true, there appear as well very long waves; (*b*) the waves tend, with increasing distance, to become continuously longer and more regular; (*c*) at a distance equal to about a quarter of the earth's circumference, waves with periods of from twelve to twenty-four seconds contain the maximum energy. At still greater distances from the focus, the short waves are missing altogether; on the other hand, the waves of from thirty to fifty seconds period again become, in general, weaker, and after one complete circuit of the earth or still later (W_3 and W_4) one finds, as a rule, only very long waves (L) and waves with periods of from sixteen to twenty seconds. The two wave trains are propagated independently, the first with a velocity of from 4.4 to 4.5 kilometers per second, the second with approximately 3.3 kilometers per second and less.

The newer theoretical investigations concerning surface waves enable us to explain these various phenomena. In the first place, Sezawa[2] has shown that, for an elastic-viscous medium—and the earth is such a medium—each movement, however irregular, must gradually develop into waves more and more regular, which tend to become longer and flatter. From the equations of Sezawa the following new equation for the period T, in the case of plane waves of velocity V, at distance Δ may be deduced:

$$T^2 = T^2_0 + \frac{\alpha\Delta}{V^3}$$

wherein α is a constant which depends on the form of the wave. Although this law serves only for plane waves, and, from purely kinetic

[1] Translated by Ernest A. Hodgson.

[2] K. Sezawa, *Bulletin of the Earthquake Research Institute,* Tokyo, **3**, 43, 1927, and **4**, 107, 1928.

considerations (as pointed out by Uller)[3] the wave-length probably increases, nevertheless this law now serves to give us a very satisfactory representation of the growth of the period of surface waves. If we choose, for example:

$$\alpha : V^3 = 1 : 40$$

and express the distance Δ in kilometers, thus obtaining the form:

$$T^2 = T^2{}_0 + \frac{\Delta(\text{km})}{40}$$

we arrive at the following related values:

T_0	$\Delta =$	100	500	1,000	4,000	10,000	40,000 km.
0.....	$T =$	1.6	3.5	5	10	16	33 sec.
6.....	$T =$	6.2	7	8	12	17	33 sec.
60.....	$T =$	60	60	60	61	62	68 sec.

The growth of the periods is thus explained but not the great amplitudes of waves with certain periods. For this we must turn to the result of an investigation by H. Jeffreys.[4] He found that waves are particularly strong whose periods lie close to such periods as T_m, for which the group velocity is a minimum. Now it is quite certain that such group-velocity minima do appear. Several cases have been investigated theoretically. If, for example, a homogeneous layer of thickness d lies above another homogeneous layer, then, according to Jeffreys, the desired period T_m depends, under certain probable assumptions as to the elastic constants in both layers, in the following manner, upon the thickness d of the upper layer:

$$T_m = ca.\ 2/3\ d, \text{ and thus for } \begin{cases} d = 30 \text{ km. } T_m = ca.\ 20 \text{ sec.} \\ d = 45 \text{ km. } T_m = ca.\ 30 \text{ sec.} \end{cases}$$

In the case of three layers of thickness d_1, d_2, and ∞, Stoneley[5] found:

$$T_m = ca.\ 3/4\ d, \text{ and thus for } d_1 = d_2 = 24 \text{ km.},$$
$$d_1 + d_2 = 48 \text{ km.}, \qquad T_m = ca.\ 18 \text{ sec.}$$

We see that these values actually lie within the period range in which the great energies are observed.

[3] K. Uller, Contribution to the Program of the "Deutschen Naturforscher und Arzte" in Hamburg, 1928.

[4] H. Jeffreys, *Monthly Notices of the Royal Astronomical Society, Geophysical Supplement,* **1**, 286, 1925.

[5] R. Stoneley, *Monthly Notices of the Royal Astronomical Society, Geophysical Supplement,* **1**, 527, 1928.

Moreover, in the absence of layers, a group-velocity minimum exists, provided the wave velocity increases with the depth. Thus the group-velocity minimum does not yield conversely any certain information as to the structure of the territory concerned.

A further group-velocity minimum is to be expected in the case of very great periods, as a consequence of the fact that at a depth of from sixty to seventy kilometers the velocity of the waves becomes temporarily constant, or even diminishes. To that fact the L-waves owe their long persistence.

Consider then the propagation of surface waves in the light of these results. In any given earthquake for example there may originate short, irregular surface waves only. The waves then become longer and more regular, in a gradual manner, according to the foregoing table. As soon, however, as the period T_m is reached, for which the group velocity becomes a minimum, there appears a concentration of energy; there are then developed maxima with periods of from twelve to twenty-four seconds, depending on the magnitude of T_m. Moreover, these waves tend to grow longer. But the energy concentration effect of T_m operates against this, so that waves with greater periods remain comparatively weak. On the other hand, according to our table, no surface waves with periods essentially less than ten seconds are to be met at a distance of more than 4,000 kilometers from the focus. This is, furthermore, in accord with our observations. Exceptions may appear in the case of some stations, but in that case we deal with inherent periods which were excited by the arriving waves.

If the waves pass, now, into an area in which T_m is greater than before, then the period of the surface waves will become greater. If, on the other hand, in the course of propagation, T_m should be found less, the period of the waves cannot diminish, as the elasto-viscosity prevents this. If then, say, the waves pass from an area where T_m is equal to sixteen seconds, into one where T_m is twelve seconds, the period of sixteen seconds would remain unchanged; there must then be a recrudescence only in the case of waves with periods greater than sixteen seconds, for the existing period is already quite far removed from the T_m value. Indeed we observe, for example, that in Eurasia, in the case of surface waves, and particularly in the coda, periods of about twelve seconds often occur as long as the waves have not actually left that continent, but that the repeat waves of the same earthquake show periods of from sixteen to twenty seconds. We find here a strong confirmation of our theory.

We assumed then that only short waves were present. But if, at the

onset of the surface waves, there are some with periods considerably above twenty seconds, then these must continue to grow until the second group-velocity minimum is reached, where, again, there appears a concentration of energy. We get thus L-waves which must be particularly noticeable, if, in the vicinity of the focus, waves with large periods were present. As a matter of fact, the records, for example, obtained in the vicinity of the Chile earthquake of November 11, 1922, are distinguished by the presence of very strong waves with periods of twenty-four seconds and more; and, further, we may note that it was in connection with this very earthquake that the L_3 and L_4 waves were first definitely established.

These laws concerning surface waves serve, in like manner for the coda, in territory without inherent native oscillations. There are, without doubt, essentially surface distortional waves whose oscillations appear to be not perfectly polarized, and, according to Uller,[6] are propagated the more slowly as their oscillations are more nearly circular.

The investigation is complicated by the frequently mentioned inherent native oscillations of layers; these inherent oscillations, however, play only a subordinate rôle. They are almost altogether lacking at stations having solid foundations.

Other disturbing factors are the absorption of waves, which is seemingly less for long waves than for short ones, and the change of waveform due to wave-kinetic causes. In spite of this, however, the theory here briefly outlined[7] now represents all the essential phenomena which were established in connection with the appearance and the period of the surface waves. Since, moreover, the theoretical investigation as to the velocity of the waves has led to results which are in accord with the observations, there are here established the fundamentals of the theory of surface waves. Nevertheless there still remain many details of the question to be solved.

For this purpose the collaboration of investigators in all parts of the world is necessary. Moreover, the results gained hitherto may only be established on the basis of records from all parts of the world.

Manuscript dated at
DARMSTADT, GERMANY,
November 6, 1928

Translation completed at
OTTAWA, CANADA,
March 9, 1929

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[6] K. Uller, "Elastische Oberflächenplanwellen," *Annalen der Physik* (IV), **56**, 463, 1918.

[7] B. Gutenberg. A somewhat similar discussion is to be published by the author, in German, in Bd. 4, *"Handbuch der Geophysik"* (Gebrüder Borntraeger, Berlin).