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# TWO-DIMENSIONAL SEISMIC MODELS WITH CONTINUOUSLY VARIABLE VELOCITY DEPTH AND DENSITY FUNCTIONS*

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ABSTRACT

A method for fabricating two-dimensional ultrasonic seismic models with variable velocity and density is described. The method is justified theoretically. It is tested by comparing the experimental and theoretical dispersion of Rayleigh waves in a model of a two-layered earth crust.

## INTRODUCTION

Many problems of interest to the seismologist are impossible to solve with available mathematical techniques. Where exact solutions are not available, seismic models can facilitate the development of approximate solutions that are almost as useful as the exact solution, and the seismic model provides the additional advantage of a clear illustration of the results with the model seismograms. Thus, seismic model studies can play a very important role in the study of wave propagation problems.

The assembly of the electronic part of a seismic model system has become a simple task as a result of advances in design and the ready availability of the components. However, practical difficulties involved in the fabrication of the seismic model itself seriously limit the scope of problems that can be studied.

One large class of problems that is important both in exploration seismology and in earthquake seismology is concerned with media that have a continuously variable velocity depth function. This paper describes a method of modeling a medium with variable velocity depth and density functions and illustrates the technique by a study of a problem in Rayleigh wave dispersion. Two dimensional model techniques, as described by Oliver, Press and Ewing (1954), are the basis of this method which makes use of the fact that waves propagating in thin sheets obey the two-dimensional wave equation. A variable velocity is obtained by the

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use of a laminated sheet in which the relative proportions of the laminae are varied so as to change the average elastic constants (hence the average velocities) of the sheet. The density can be modeled by varying the total thickness of the sheet.

This technique of modeling was suggested originally by Jack Oliver and the method of fabrication used was suggested by H. O. Walker.

## EQUIPMENT

The model set up was essentially the same as described in earlier papers (Oliver, Press and Ewing, 1954). One-eighth inch by one-quarter inch solid cylinders of barium titanate were used as sources and barium titanate bimorph transducers were used as receivers. The bimorph transducers were clamped between rubber sheets to provide sufficient damping to prevent ringing. This simple device proved to be very effective in eliminating the ringing problem.

## MODEL CONSTRUCTION

The models were fabricated from aluminum sheets and an epoxy resin plastic that was poured onto the aluminum and allowed to solidify. The aluminum used was 24ST in .020 inch sheets, and the plastic used was Shell Epon No. 828 with diethylenetrianamene hardener.

The aluminum sheets were cut in circles 24 inch in diameter and mounted to a 26-inch face plate with parafin wax.

A horizontal milling machine available in our shops was converted to a stub lathe so that a contour could be cut in the surface of the aluminum sheet. After the desired contour was cut in the aluminum, the face plate was removed from the lathe and a layer of plastic was poured over the aluminum and allowed to harden.

A final contour was cut in the surface of the plastic so as to provide control of two parameters, the total thickness of the model, and the ratio of plastic to aluminum. Figure 1 shows a portion of the model with source and receiver.

This process, while simple in concept, is a difficult machine shop problem. The tolerances desired exceed the capabilities of the standard machine tools and extreme care is required to obtain a satisfactory model. A great deal of credit is due our experimental machinest, Mr. Carl Holmstrom, for the successful construction of these models.

## THEORY OF TWO-DIMENSIONAL LAMINATED PLATE MODELS

If the the three-dimensional equations of motion and boundary conditions are solved to yield the period equation for a laminated plate with two sheets, it can be shown that for wave lengths, long compared to plate thickness, the phase velocity reduces to a constant value, reflecting the average elastic parameters of the plate.

Another approach is to proceed from the assumption of plane strain and derive

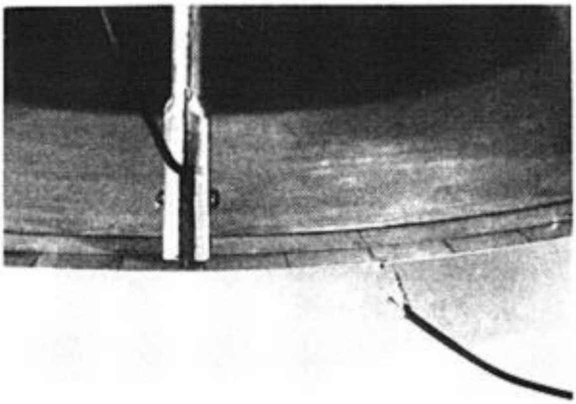

FIG. 1. Layered plate model with a damped bimorph receiver and cylindrical source.

the wave equation for a laminated plate. Consider a plane wave propagating in a layered plate in the $X$ direction so that the particle motion is perpendicular to the surface of the plate. Figure 2 shows the cross section of the plate with the differential element which we will consider. With the following definitions,

$u$ = particle motion in $X$ direction
$T_i$ = thickness of $i$th lamination
$\bar{\rho}$ = density of laminated plate (mass per unit area)
$\beta$ = shear velocity in laminated plate
$C_p$ = plate longitudinal velocity in the laminated plate
$\alpha_i$ = compressional velocity of $i$th lamination
$\beta_i$ = shear velocity of $i$th lamination
$E_i$ = Young's modulus of $i$th lamination
$\sigma_i$ = Poisson's ratio of $i$th lamination
$\rho_i$ = density (mass per unit volume) of $i$th lamination
$C_{pi}$ = plate longitudinal velocity in a homogeneous plate of the composition of the $i$th lamination

the equation of motion in the $X$ direction is,

$$\frac{\partial}{\partial X}\left[\frac{\partial U}{\partial X}\left(\frac{E_1}{1-\sigma_1^2}T_1 + \frac{E_2}{1-\sigma_2^2}T_2\right)\right] = (\rho_1 T_1 + \rho_2 T_2)\frac{\partial^2 U}{\partial t^2}\cdot \qquad (1)$$

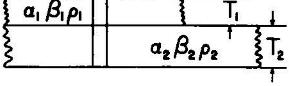


FIG. 2: Cross section layered plate.

This reduces to a wave equation,

$$\frac{\partial^2 U}{\partial X^2} = \frac{\rho_1 T_1 + \rho_2 T_2}{\dfrac{E_1}{1-\sigma_1^2} T_1 + \dfrac{E_2}{1-\sigma_2^2} T_2} \frac{\partial^2 U}{\partial t^2} . \tag{2}$$

Thus, the plate longitudinal velocity is given by,

$$C_p^2 = \frac{\dfrac{E_1}{1-\sigma_1^2} T_1 + \dfrac{E_2}{1-\sigma_2^2} T_2}{\rho_1 T_1 + \rho_2 T_2} = \frac{\rho_1 C_{p1}^2 T_1 + \rho_2 C_{p2}^2 T_2}{\rho_1 T_1 + \rho_2 T_2} . \tag{3}$$

Similarly for shear waves one finds,

$$\beta^2 = \frac{\rho_1 \beta_1^2 T_1 + \rho_2 \beta_2^2 T_2}{\rho_1 T_1 + \rho_2 T_2} . \tag{4}$$

From comparison with the homogeneous plate it is evident that $\bar{\rho} = \rho_1 T_1 + \rho_2 T_2$ replaces the density factor and $\rho_1 \rho_{p1}^2 T_1 + \rho_2 \rho_{p2}^2 T_2$ or $\rho_1 \beta_1^2 T_1 + \rho_2 \sigma_2^2 T_2$ replaces the elastic factor.

We have verified equation (3) by solving the exact three-dimensional equations of motion for an infinite laminated plate subject to the exact boundary condition at the free surfaces and the interface. Equation (3) is a limiting form of the solution for phase velocity which emerges from the long wave length condition $\lambda >> T_i$. This result has also been experimentally verified as will be seen in the next section.

It is more difficult to establish the use of equations (3) and (4) and to specify exact boundary conditions when the thicknesses of the lamina are varied to model the velocity and density parameters. One might justify the use of these equations by arguing intuitively from the results of Oliver et al. (1954) for homogeneous plates. We prefer to justify these equations experimentally by study of Rayleigh wave dispersion in a model of a double layered crust overlying a mantle.

Note that the mass per unit area will be the density that must be considered when we vary the thickness of the lamina to model the velocity and density parameters. By suitable choice of $T_1$ and $T_2$ one can model density and one of the elastic constants provided it falls between the corresponding values of the model materials chosen.

## EXPERIMENTAL VERFICATION OF THE THEORY

An experimental verification of the results derived was obtained with a series of layered plates constructed, as described above, of a layer of plastic over aluminum. The total thickness of the plate was $\frac{1}{8}$ inch and the ratio of plastic to aluminum was varied in 11 steps from all aluminum to all plastic. Velocities of compressional and shear waves in these plates were measured and the results are shown in Figure 3. Good agreement was found for the shear waves, but a marked variation from the theoretical values was shown by the compressional waves.

The cause of this discrepancy was thought to be a violation of the assumption that the wave length of the waves was sufficiently long as compared to the plate thickness. A second series of plates was constructed whose total thickness was only $\frac{1}{16}$ inch and good agreement was found with the theoretical values.

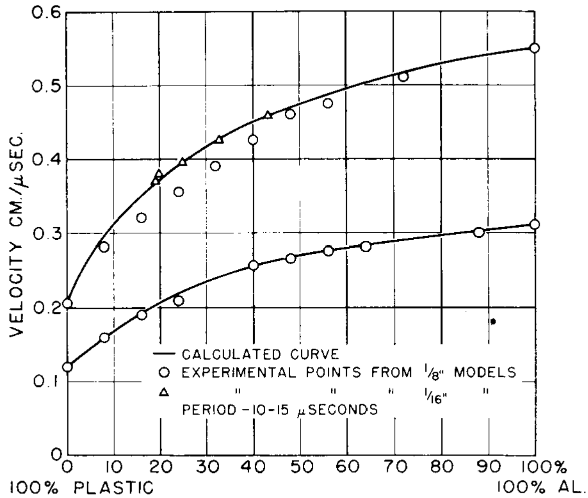


FIG. 3. Compressional and shear wave velocity in plastic and aluminum laminated plates.

As further check on the effect of lamina thickness, the exact phase velocity curve for a two-layer plate with a plastic to aluminum ratio of .040/.085 was computed. The exact period equation was formulated according to Haskell's matrix method (1953) and solved numerically on the Seismological Laboratory's electronic computer, the Bendix G15D. The results are shown in Figure 4. These curves verify that for the periods used, 10–15 micro/sec, there is marked dependence of phase velocity on period in a $\frac{1}{8}$ inch plate. It is interesting to note that the dispersion is more severe for the laminated plate than for the homogeneous plate. We believe this is a result of coupling between the symmetric and antisymmetric waves in the laminated plate. The dispersion in the $\frac{1}{16}$ inch plate is negligible for these periods.

It was concluded that with the materials and frequencies used in our experiment, total plate thickness should be less than $\frac{1}{16}$ inch. In the particular experiment described below, all models were less than $\frac{1}{32}$ inch in thickness.

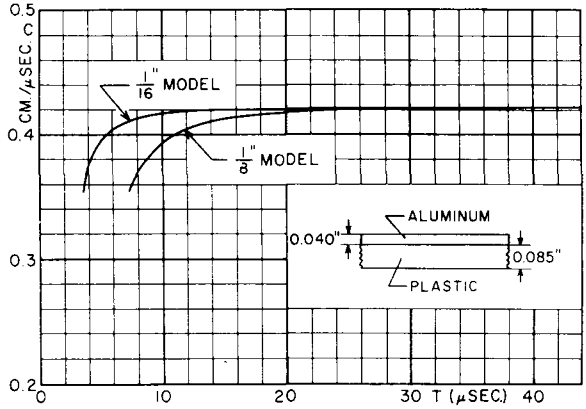


FIG. 4. Phase velocity for a two layer plate .040 inch aluminum and .085 inch plastic.

## PHASE VELOCITY IN A TWO-LAYER CRUST

As a preliminary to use of this method in the study of wave propagation in the earth's crust, an attempt was made to reproduce the Rayleigh wave phase velocity in a two-layer crust. Since dispersed surface waves may be regarded as multiply reflected and refracted body waves, a powerful test of a model is a comparison of experimental and theoretical dispersion curves. For this comparison we have selected a two-layered crust overlying a mantle, as computed by Haskell. The first model constructed modelled only the shear wave velocity, neglecting the density and compressional wave velocity.

As the Rayleigh wave velocity is predominately controlled by the rigidity of the medium, one would expect that variations in density and compressional velocity would have only secondary effects on the Rayleigh wave velocity.

The model was a disk 24 inch in diameter with thickness as shown in the top cross section of Figure 5. Both phase and group velocity were measured on this model and the results are shown in Figure 6. The phase velocity falls below the expected values, which was anticipated since we failed to model density.

In the second attempt, we had improved our technique so that we were able to cut a contour in the aluminum sheet and in the plastic, and were able to model both the density and the shear velocity. The lower cross section of Figure 5 shows the thicknesses used in this model.

For the second model, the phase velocity was measured in three ways. One method was to excite the disk model with a continuous sine wave source so as to set up a standing wave on the edge of the disk composed primarily of the Rayleigh wave trains traveling in opposite directions around the disk. This is analogous to exciting a free mode of the earth. The phase velocity could be determined by counting the nodes around the disk. This method was most effective with the longer wave lengths (Figure 7). In the second method the Rayleigh wave

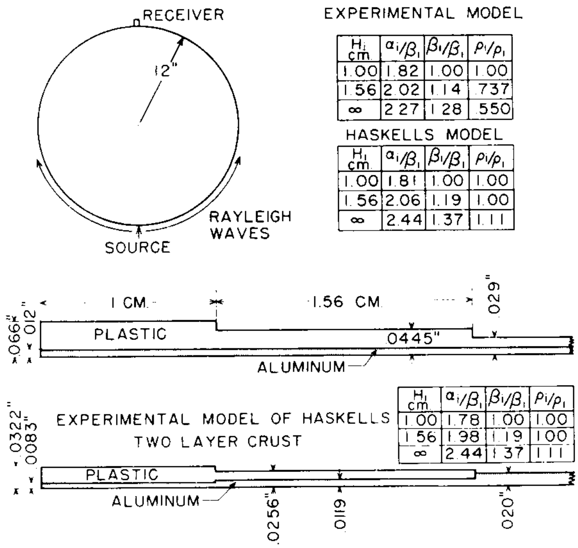


FIG. 5. Cross-section of two models with tabulated values of constants.

train traveling in one direction from the source was damped so that the wave train traveling in the other direction propagated without interference, and the phase velocity could be determined by following a particular peak or trough around the disk (Figure 8).

The results of these two methods are shown in Figure 9. Good agreement exists

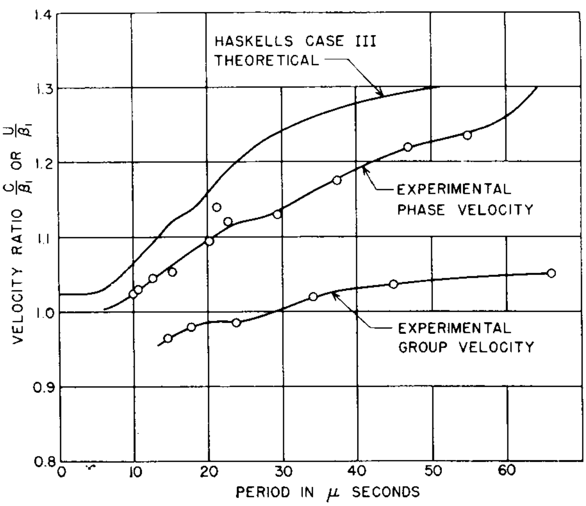


FIG. 6. Rayleigh wave phase and group velocity for a two layer crust (see upper Figure 5 for constants).

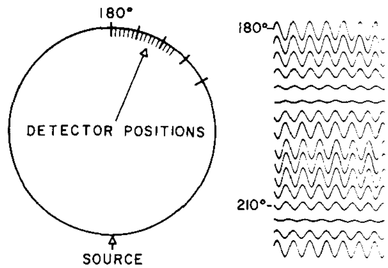


FIG. 7. Seismogram showing standing Rayleigh wave.

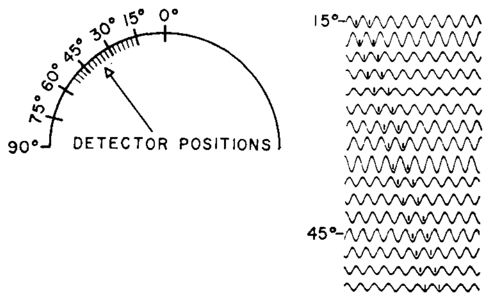


FIG. 8. Traveling Rayleigh wave from continuous source.

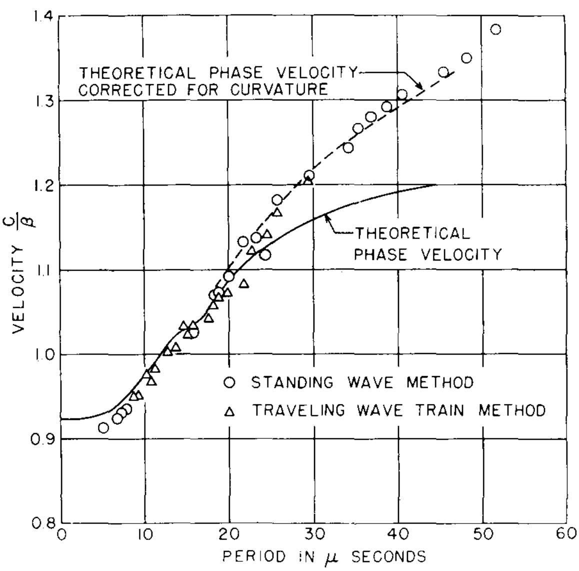


FIG. 9. Theoretical and experimental phase velocity (constants given in lower part of Figure 5).

between the theoretical and experimental results. Since the theoretical curve was for a half space it was necessary to apply a correction for the curvature of the disk model. An approximate correction was made for each theoretical phase velocity point by multiplying the phase velocity by a factor which would be correct for a homogeneous model. The correction for cylindrical curvature was originally given by Sezawa and later corrected by Oliver (Ewing, Jardetzky and Press, p. 265).

While these two methods give fairly good agreement with the theoretical phase velocity, it is desirable to improve these results further to distinguish the finer features of the phase velocity curve. Two additional methods were tried; the method of Brune and a Fourier analysis method. The precision of the Fourier analysis method indicates that a high accuracy can be achieved by this modeling technique, and the comparison of the four methods of phase velocity determination is a good illustration of an important problem that can be studied with this type of model. Brune's method, based on the stationary phase technique, makes use of the arrival times of peaks and troughs of the dispersed wave train to determine phase velocity (Brune, Nafe and Oliver, in press).

The Fourier analysis made use of the following relationship between the phase of a Fourier component at two distances:

$$C = \frac{(X_b - X_a)\omega}{\phi_b - \phi_a + 2n\pi} = \text{Phase Velocity}$$

$X_b$, $X_a$—are distances to points $a$ and $b$.

$\phi_b$, $\phi_a$—apparent phase angles of frequency component $\omega$ measured with respect to origin time.

$\omega$—angular frequency.

$$\frac{\phi_b - \phi_a + 2n\pi}{\omega} = \text{phase delay time between points } a \text{ and } b.$$

The seismograms in Figure 10 were digitized and subjected to a Fourier analysis on our electronic digital computer. This analysis yielded amplitude and phase of the Fourier spectral components. The latter was used to derive the experimental phase velocity dispersion curve (Figure 11). Excellent precision is obtained by this method as evidenced by the agreement between the experimental data and theoretical curves. The small bend in the theoretical curve at 17 micro/sec is indicated by the experimental phase velocities. The systematic discrepancy that does exist is less than one half of one percent and is well within the error expected from the inaccuracies in the machining process.

## CONCLUSION

This study shows the feasibility of constructing two-dimensional models in which a body velocity and density can be made to vary continually with depth.

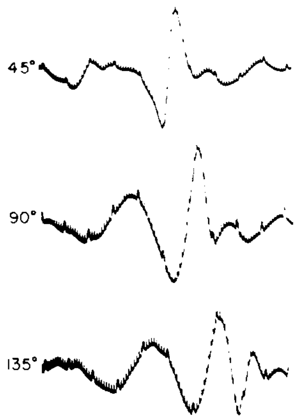


FIG. 10. Seismograms showing Rayleigh wave dispersions at three distances.

A successful test was made by comparing theoretical and experimental dispersion curves of Rayleigh waves in a double layered crust.

In future papers we will report on further applications of this modeling technique: (1) the effect of the low velocity upper mantle layer on body wave amplitudes (the shadow zone problem); (2) methods of deducing the properties of the source by operating on dispersed surface waves.

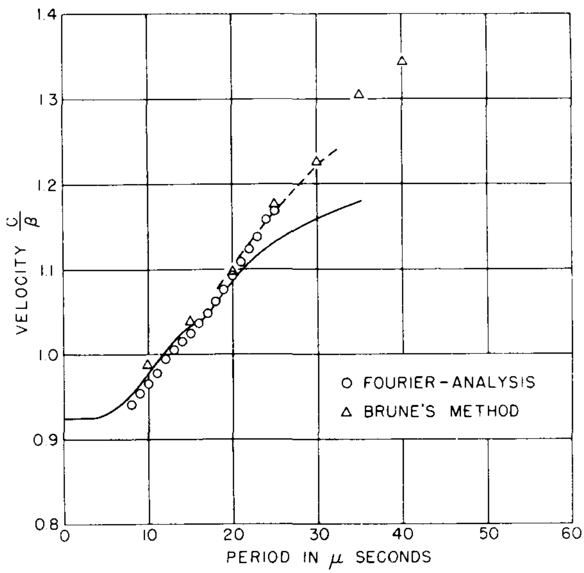


FIG. 11. Experimental dispersion curves according to Fourier analysis method and Brune's method together with theoretical curve. Dashed curve includes correction for curvature.

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