ON THE QUANTUM THEORY OF THE RAMSAUER EFFECT

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The Ramsauer effect does not correspond to the decrease in optical scattering by a particle when the wave-length of the light is increased. For one may compute the cross-section for elastic collision of an electron with an atom, without making the approximations used by Born. If one does this, and neglects the resonance of the colliding electron with the atomic electrons one finds that, for atoms with no dipole moment in the normal state, the cross-section approaches a finite limit when the electronic velocity is decreased. The only general result of this calculation is that the distribution of scattered electrons tends, in this limit, to become uniform over all angles.

If, however, one considers the electronic resonance and spin, one obtains results which seem adequate to account for the effects observed by Ramsauer. Thus one may compute two first order cross-sections for the elastic collision of an electron and a hydrogenic atom, corresponding to initial orbital wave functions respectively symmetric and anti-symmetric in the coördinates of the impacting electron and the atomic electron. These turn out to be of the form

$$|f(v, \delta) + g(v, \delta)|^2$$

and

$$|f(v, \delta) - g(v, \delta)|^2.$$

Here v is the electronic velocity, δ the angle of deflection; f and g are positive; f corresponds to transitions in which the atomic electron remains undisturbed, and the free electron changes its direction of motion by the angle δ; g corresponds to transitions in which the free electron takes the place of the atomic electron and this latter electron leaves the atom with a velocity v and a direction of motion δ. For fixed δ and large v, f is much greater than g, while for $v = 0$, $g = 6f$; and f becomes equal to g for velocities of the order of a volt. Thus the symmetric wave function gives a cross-section which is always greater than that computed without resonance: The anti-symmetric one gives a cross-section which, for each angle of deflection, vanishes for some small velocity; and the corresponding total cross-section

$$\int_0^\pi \sin \delta d\delta \, |f(v, \delta) - g(v, \delta)|^2$$

has a sharp minimum at about a volt.

Now, in general, both symmetrical and anti-symmetrical wave functions will occur; and the orbital wave function will involve terms which are symmetrical in the colliding electron and one of the atomic electrons. But if the atom has only paired electrons, the symmetric orbital solution will be excluded by Pauli's principle. Hence in this case (e.g., helium, argon, methane) the cross-section will be given by the anti-symmetric wave function alone, and may pass through a minimum for sufficiently low velocities.

The position and shape of the minimum will depend essentially upon the series of wave functions of the atom, and may be computed only by complete solution of the collision problem; the first order cross-section is inadequate for quantitative prediction. But the occurrence of the minimum depends upon the chemical properties of the atom; this is in good agreement with the available experiments,[1] and particularly with those of Rusch, who reports a sharp minimum in the cross-sections of argon and krypton. Details of the theory will be published elsewhere.

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[1] *Hdb. d. Phys.*, **23**, 646, 1926.

THE FIFTH DIMENSION IN RELATIVISTIC QUANTUM THEORY

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O. Klein[1] has shown that the Schrödinger equation

$$\frac{1}{\sqrt{-g}}\sum_{j,k=1}^{4}\frac{\partial}{\partial x^j}\cdot\sqrt{-g}\,g^{jk}\frac{\partial\psi}{\partial x^k}+\frac{4\pi i e}{hc}\sum_{j=1}^{4}\varphi^j\frac{\partial\psi}{\partial x^j} - \frac{4\pi^2e^2}{h^2c^2}\left[\sum_{j,k=1}^{4}g^{jk}\varphi_j\varphi_k+\frac{m^2c^4}{e^2}\right]=0 \quad (1)$$

may be cast into the form

$$\frac{1}{\sqrt{-\gamma}}\sum_{j,k=0}^{4}\frac{\partial}{\partial x^j}\sqrt{-\gamma}\,\gamma^{jk}\frac{\partial\Psi}{\partial x^k}=0, \quad (2)$$

where

$$\Psi(x^0, x^1, x^2, x^3, x^4) = e^{\frac{2\pi i x^0}{h}}\psi(x^1, x^2, x^3, x^4),$$

and the matrix γ_{ij} has the components

$$\gamma_{00}=\frac{1}{m^2c^2};$$