

nuclei singles out the isotopic spin exchange part, V_b , of the neutron-proton interaction inside nuclei. Thus a comparison of the experimental cross section for this reaction with a detailed theoretical calculation of the cross section ought to provide the potential V_b of (2). It is important to note that no one has yet succeeded in calculating the correct absolute magnitude of the cross section for a direct interaction process. Levinson and Banerjee⁶ have given the most complete treatment in their study of proton inelastic scattering from C^{12} . They found it necessary to use a direct interaction with a strength of more than twice the free nucleon-nucleon potential. These authors suggested that the increased effective interaction may arise from a polarization of the nucleons in the target nucleus. This effect would of course be absent in the interaction (1), if it is correct to think of the target nucleus as an inert core plus one neutron in a well-defined state. Moreover the (p, n) reaction involves unambiguously a nucleon-nucleon interaction, whereas inelastic scattering by direct interaction may proceed by particle excitation in the target, or by excitation of a collective state, and these two modes are not necessarily easily distinguished.⁷

Thus the (p, n) reactions connecting the ground states of mirror nuclei are particularly suited to a rather direct measurement of the effective pro-

ton-neutron interaction in nuclei, or more specifically the charge exchange part of the interaction.

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¹Many references to the earlier work can be found in J. S. Blair and E. M. Henley, *Phys. Rev.* **112**, 2029 (1958). See also N. K. Glendenning, *Phys. Rev.* (to be published).

²S. A. Moszkowski, Office of Ordnance Research Technical Report No. 2, University of California at Los Angeles, April, 1959 (unpublished).

³Recent measurements of the angular distribution of neutrons from this reaction are reported in S. D. Bloom and R. D. Albert, *Bull. Am. Phys. Soc.* **4**, 321 (1959); see also Albert, Bloom, and Glendenning (to be published).

⁴This point as well as some others briefly referred to throughout this Letter are developed in detail in a University of California Radiation Laboratory Report now in preparation.

⁵Evidence strongly supporting the extreme $j-j$ coupling shell model for this case is given in M. K. Banerjee and C. A. Levinson, *Ann. Phys.* **2**, 499 (1957).

⁶C. A. Levinson and M. K. Banerjee, *Ann. Phys.* **2**, 471 (1957); **2**, 499 (1957); **3**, 67 (1958).

⁷H. Ui, *Progr. Theoret. Phys. (Kyoto)* **18**, 163 (1957); N. K. Glendenning, Ph.D. thesis, Indiana University, 1958 (unpublished).

ELECTROMAGNETIC CORRECTIONS TO THE B^{12} - N^{12} β -SPECTRUM RATIO*

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It has recently been suggested¹ that a precise comparison of the β spectra of B^{12} and N^{12} transitions to the ground state of C^{12} would provide a test of the nature of the vector interaction in β decay. Let the spectrum of each transition, divided by the corresponding Fermi spectrum, be called $S(E)$, where E is the total energy of the β ray. Then define

$$R(E) \equiv S(E, B^{12})/S(E, N^{12}). \quad (1)$$

In reference 1 it was shown that the conserved vector current theory of β decay² predicts the result³

$$R(E) \approx \text{const}(1 + AE), \quad (2)$$

where A is determined by the width of the 15.11-

Mev level in C^{12} for γ transitions to the ground state and comes out

$$A = 1.33\% \pm 0.15\% \text{ per Mev}, \quad (3)$$

using the measurements of Hayward and Fuller,⁴ subsequently confirmed by Garwin.⁵

According to the more usual theory of β decay, in which the pion is not assigned any intrinsic β -decay "charge," we may expect a formula similar to (2), but with a much smaller value of A ; the reduction factor in A should be roughly the factor by which $\mu_p - \mu_n$ is reduced if the pion current contributions to this quantity are omitted. (Here μ_p and μ_n are the proton and neutron magnetic moments.) A reasonable guess is that $\mu_p - \mu_n$ would become about one Bohr magneton

rather than almost five if the pion contributions were omitted and thus in the old theory A should be something like one-fifth of the value given in (3). The B^{12} - N^{12} experiment, if carefully performed over a range of several Mev, should thus be able to distinguish the two theories.

It has been pointed out, however, by Morita⁶ and Schwarzschild⁷ that the spectrum ratio $R(E)$ is altered somewhat by electromagnetic effects that were ignored in reference 1. It is our purpose here to supply a new theoretical estimate of the correction factor that should be applied to Eq. (2) to take account of these effects. With respect to the processes considered by Morita, our numerical results are in agreement with his, although our method is somewhat different.

Let us state the answer first. Instead of Eq. (2), we should use

$$R(E)/f(E) = \text{const}[1 + (A + \delta A)E], \quad (4)$$

where

$$f(E) = 1 + \frac{\alpha}{2\pi} \left\{ 4(z-1) \ln[(k_0 - E)/(k_0' - E)] + \left(z - \frac{4}{3} \right) \frac{(k_0 - k_0')}{E} + z \frac{k_0^2 - k_0'^2}{6E^2} \right\}, \quad (5)$$

with α signifying the fine structure constant, k_0 the end-point total energy for B^{12} , k_0' the end-point total energy in N^{12} , and with

$$z = \frac{1}{2} \frac{E}{(E^2 - m^2)^{1/2}} \ln \left[\frac{E + (E^2 - m^2)^{1/2}}{E - (E^2 - m^2)^{1/2}} \right] \approx \ln(2E/m) \text{ for } E \gg m. \quad (6)$$

The calculated value of δA is -0.25% per Mev, to which we would like, for the sake of safety, to attach a theoretical "error" of 0.15% per Mev. This error, and, indeed, the electromagnetic correction itself, is considerably smaller than the effect being sought, and therefore the existence of the correction does not appreciably weaken the B^{12} - N^{12} experiment as a test of the conserved current theory.

In deriving this correction, we have considered the following effects, many of which are negligible:

(a) The effect of the B^{12} - N^{12} mass difference on the spectra. Insofar as the spectra are allowed, this is taken care of by factoring out the Fermi spectra as in Eq. (1). However, there are small forbidden corrections to the spectra caused by retardation effects in the axial vector coupling. These cancel between B^{12} and N^{12} except for those terms that involve the end-point energy,

which is different for the two nuclei. The resulting correction is a contribution to δA of the form

$$\delta A^{(a)} = (k_0 - k_0') \{ \int \vec{\sigma} \}^{-1} \left\{ \frac{8}{9} \int \vec{\sigma} \gamma^2 - \frac{4}{9} \int \vec{\sigma} \vec{r} \vec{r} \right\}. \quad (7)$$

(b) The effect of the B^{12} - N^{12} mass difference on the inner bremsstrahlung. The probability of inner bremsstrahlung for a given electron energy depends on the end-point energy. The ratio of the correction factors in B^{12} and N^{12} is finite without an infrared cutoff and is given⁸ by $f(E)$ in Eqs. (4) and (5).

(c) The difference between B^{12} and N^{12} wave functions caused by electromagnetic violations of charge independence. This alters ft values but changes the spectra only through very small changes in the already small retardation terms.

(d) The effect of virtual transverse photons exchanged between β ray and nucleus. This is of order nucleon v/c compared to the Coulomb interaction between β ray and nucleus, which we go on to consider.

(e) The effect of the Coulomb interaction between β ray and nucleus on the retardation corrections to the axial vector matrix element $\int \vec{\sigma}$. This is often considered as the sum of three terms—the "finite de Broglie wavelength effect," the effect of finite nuclear size, and the Coulomb corrections to forbidden matrix elements. We treat them all together.

Since Z is small, we can treat the Coulomb field in first order perturbation theory. Between the β decay and the Coulomb interaction the nucleus is in a virtual state which may be excited. It is difficult to evaluate the sum over all the excited states and we therefore look at two opposite approximations:

I. The excitation energies of the important nuclear levels are neglected by comparison with $\hbar c/R$, where R is the nuclear radius. In other words, the nuclear motion is assumed slow compared to the velocity of light with which the β ray is traveling. Under these conditions the sum over nuclear levels can be done by closure. In physical terms, the electron sees the instantaneous positions of the various protons that are sources of the Coulomb field. This approximation should be practically valid. As a check, however, we also consider the opposite case.

II. The energies of all excited states are taken to be so large (in energy denominators) that only the ground states survive in the sum. Physically this corresponds to the electron's seeing the nucleus as a smeared-out charge distribution.

What is important for the B^{12} - N^{12} ratio is the charge distribution in C^{12} and the average of the B^{12} and N^{12} charge distributions. The latter is just the charge distribution of the excited state of C^{12} at 15.11 Mev. For simplicity we assume that to be the same as the charge distribution of the ground state.

In each case we calculate the terms of order $Z\alpha RE$ in the spectrum ratio, dropping higher orders in RE . The result in case I is a contribution to δA equal to

$$\delta A_I^{(2)} = -4\alpha \{ \int \vec{\sigma} \}^{-1} \sum_i \left\{ \frac{2}{3} \int \vec{\sigma} | \vec{r} - \vec{r}_i | + \frac{1}{3} \int \vec{\sigma} \vec{r} \cdot (\vec{r} - \vec{r}_i) | \vec{r} - \vec{r}_i |^{-1} - \frac{1}{3} \int \vec{\sigma} \cdot \vec{r} (\vec{r} - \vec{r}_i) | \vec{r} - \vec{r}_i |^{-1} \right\}, \quad (8)$$

where the sum is extended over all protons except the decaying nucleon. In case II the result is

$$\delta A_{II}^{(2)} = -4\alpha \{ \int \vec{\sigma} \}^{-1} \int \rho(\vec{r}') d^3r' \left\{ \frac{2}{3} \int \vec{\sigma} | \vec{r} - \vec{r}' | + \frac{1}{3} \int \vec{\sigma} \vec{r} \cdot (\vec{r} - \vec{r}') | \vec{r} - \vec{r}' |^{-1} - \frac{1}{3} \int \vec{\sigma} \cdot \vec{r} (\vec{r} - \vec{r}') | \vec{r} - \vec{r}' |^{-1} \right\}, \quad (9)$$

where ρ is the charge density in C^{12} in units of e .

(f) The Coulomb effect on weak magnetism itself. Since the weak magnetic effect on the spectra comes from V - A interference, it is of opposite sign in B^{12} and N^{12} . The first order Coulomb correction to it thus has the same sign in B^{12} and N^{12} and practically cancels.

(g) The Coulomb effect on the relativistic axial vector matrix element $\int \gamma_5$. Like the uncorrected $\int \gamma_5$, this has a negligible effect on the spectrum.

We have estimated $\delta A^{(1)}$, $\delta A_I^{(2)}$, and $\delta A_{II}^{(2)}$ using the shell model with harmonic oscillator

wave functions adjusted in radius to give the charge distribution of Hofstadter.⁹ The results are that $\delta A^{(1)} = -0.036\%$ per Mev, $\delta A_I^{(2)} = -0.21\%$ per Mev, and $\delta A_{II}^{(2)} = -0.23\%$ per Mev. Since case II is so close to case I and since case I is much closer to the truth, we take $\delta A = \delta A^{(1)} + \delta A_I^{(2)} = -0.25\%$ per Mev.

The largest error in our result probably comes from the use of the shell model. We can estimate how bad the shell model is by using it to calculate the allowed matrix element $\int \vec{\sigma}$ for the β decay of B^{12} . The experimental matrix element is smaller by a factor of 0.4 (in absolute value—presumably the sign is right). The ratios in Eqs. (7)-(9) are probably calculated slightly better than the absolute value and we have therefore assigned an error of about 0.15% per Mev to δA .

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PREDICTION OF DEUTERON POLARIZATION FROM d - α SCATTERING

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There is current interest in possible means of producing and analyzing deuteron polarization. d - α scattering on the 1.07-Mev resonance¹⁻³ and the $T(d, n)He^4$ reaction⁴ have been studied. The present note shows that d - α scattering can be a useful producer (and analyzer) of deuteron polarization at energies between resonances, where the weak energy dependence may be an advantage.

We have used the phase shifts of Galonsky and McEllistrem,⁵ deduced from differential cross sections with the help of dispersion theory, to calculate polarization parameters in the interval between the resonances at 1.07 and 4.6 Mev (deuteron lab energy).

In the region of 2 Mev there is considerable vector polarization, increasing smoothly with