GRADUATE AERONAUTICAL LABORATORIES
of the
CALIFORNIA INSTITUTE of TECHNOLOGY
Pasadena, California 91125

INVISCID INSTABILITY CHARACTERISTICS OF FREE SHEAR LAYERS WITH NON-UNIFORM DENSITY

by

M. M. Koochesfahani* and C. E. Frieler#

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* Post-Doctoral Research Fellow, Aeronautics. Member AIAA.
# Graduate Student, Aeronautics.

ABSTRACT

The linear spatial instability of two-dimensional two-stream plane mixing layers has been studied extensively in the past. In the case of uniform density, Michalke (1965) investigated the single-stream shear layer and Monkewitz & Huerre (1982) considered the effect of the velocity ratio. Maslowe & Kelly (1971) studied the stratified (non-uniform density) shear layers and showed that density variations can be destabilizing. In all these studies, the mean velocity profile has been assumed to be monotonically increasing from the value on the low-speed stream to that on the high-speed stream and usually the hyperbolic tangent form is used. It should be noted, however, that under experimental conditions the initial mean velocity profile almost always has a wake component due to the boundary layers on the two sides of the splitter plate. The effect of the wake component has only recently come into consideration with the investigations of Miau 1984 and Zhang et al. 1984 for the uniform density case.

The purpose of the present work is to study the instability characteristics of both uniform and non-uniform density plane shear layers taking into account the wake component of the initial velocity profile. The inviscid, linear, parallel-flow stability analysis of spatially growing disturbances is utilized to numerically calculate the range of unstable frequencies and wave-numbers.

We consider the general case of a two-stream plane shear layer with U_1, ρ_1 as the free-stream velocity and density on the high-speed stream and U_2, ρ_2 as the corresponding quantities on the low-speed side of the layer. All quantities used here are normalized with the average velocity $(U_1+U_2)/2$, average density $(\rho_1+\rho_2)/2$ and the local layer thickness δ as the length scale. The mean velocity profile is composed of the usual hyperbolic tangent profile plus a wake component represented by a Gaussian distribution and has the form

$$U(Y) = 1 + \lambda_u \tanh(Y) - W e^{-Y^2}$$

where $\lambda_u = (U_1-U_2)/(U_1+U_2)$ and W is the normalized wake deficit. The mean density profile has a hyperbolic tangent profile and is given by

$$\rho(Y) = 1 + \lambda_\rho \tanh[(Y-Y_0)/\sigma]$$

where $\lambda_\rho = (\rho_1-\rho_2)/(\rho_1+\rho_2)$ and Y_0 and σ adjust the lateral position and thickness of the density profile relative to the velocity profile.

The disturbance stream function is written in the form

$$\psi = \phi(Y)\, e^{i(\alpha x - \beta t)}$$

where $\alpha = \alpha_r + i\alpha_i$ is the complex wave-number and β is frequency which is real for the spatial case. For the case of incompressible flow and negligible buoyancy effects (i.e. gravity is ignored), it can be shown that the disturbance eigenfunction ϕ satisfies the equation

$$\phi'' + (\rho'/\rho)\,\phi' - \left[\alpha^2 + \frac{U'' + \rho'\,U'/\rho}{U-\beta/\alpha}\right]\phi = 0$$

where ( )' corresponds to d/dY. The equation above reduces to the Rayleigh equation when the density is uniform. A "shooting" technique is used to solve this eigenvalue equation and calculate the spatial growth rate, $-\alpha_i$, of unstable disturbances and the corresponding wave-number, α_r, versus frequency β.

Representative results for the uniform density case are shown in figure 1. The main result is that when the wake component is present there are two unstable branches as opposed to one in the case of the hyperbolic tangent. These two branches do not have much in common with the "sinuous" and "varicose" modes of instability calculated by Mattingly & Criminale (1972) in the case of pure wakes since the symmetry conditions that exist in the pure wake do not hold here. The stronger branch leads to the usual Kelvin-Helmholtz roll-up patterns observed in shear layers and it is referred to here as the shear layer branch. The lower branch is suspected to correspond to roll-up patterns that resemble those in pure wake flows and, therefore, we call it the wake branch. This branch, in the uniform density case, is rarely observed since its amplification rate is quite lower than the shear layer branch.

To investigate the effect of non-uniform density, we consider the case of low-speed stream having the higher density (see figure 2). The density profile has been arranged to have its inflection point at the minimum of the velocity profile and its thickness much thinner than that of the velocity profile. These conditions are expected to hold in the initial region of the flow near the splitter plate. The qualitative features of the results are not sensitive to these conditions as long as the density profile is reasonably thin relative to the velocity profile.

The results for the case of non-uniform density are shown in figure 3. In figure 3a, the uniform density case, the two instability branches similar to figure 1b can be seen. The most important result is that when the high density is on the low-speed side, the two branches have similar growth rates. In fact, the normally weak branch (wake branch) in the uniform density case, now seems to have a slightly larger growth rate than the shear layer branch. This means that, depending on the spectrum of disturbances in the flow and the extent of the persistence of the wake component in the downstream region, a shear layer of non-uniform density may not roll-up like the usual Kelvin-Helmholtz structures but more like a wake. The finding in figure 3 also suggests that, under the right flow conditions, both shear layer and wake modes of instability may exist simultaneously and interact with each other.

To show that the features described above are, in fact, possible, the shear layer between a high-speed stream of Helium and low-speed stream of Argon was forced acoustically. The flow visualization by Schlieren photography, figure 4, shows that both wake and shear layer modes can be generated in a two-stream shear layer.

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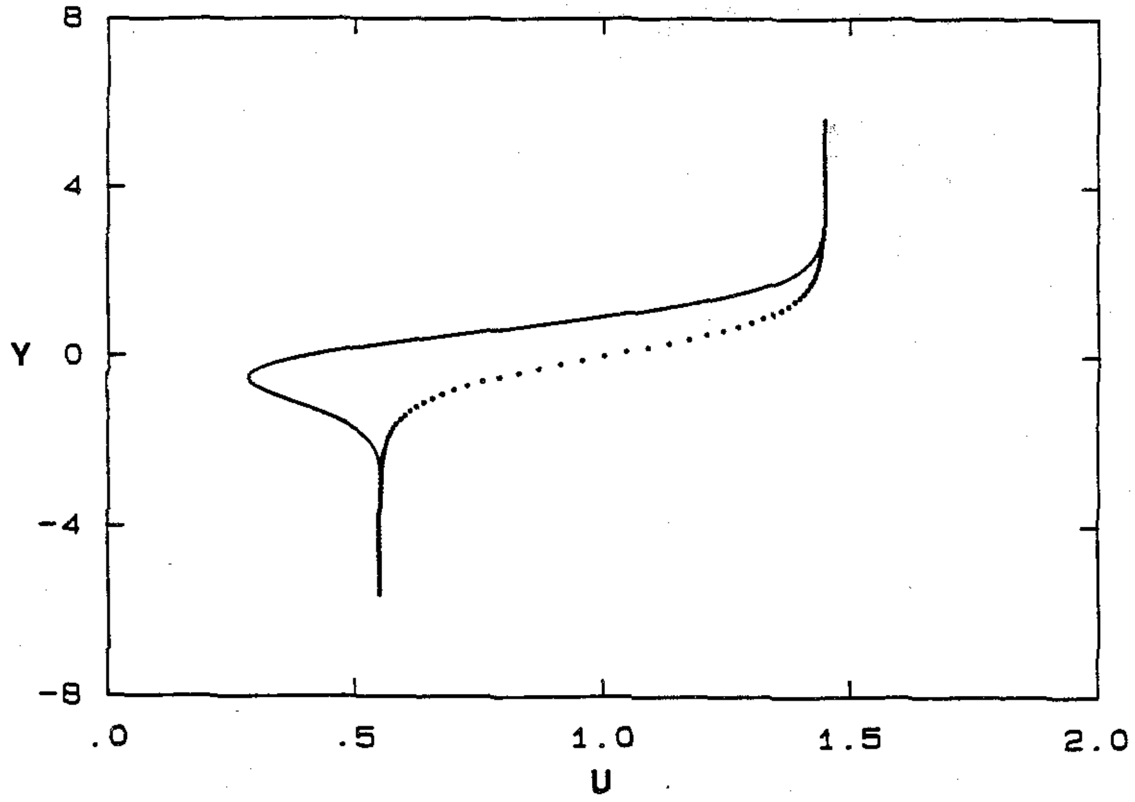


Figure 1a. Mean velocity profile, $\lambda_u = 0.45$ ($U_2/U_1 = 0.38$), ••••• tanh profile, ——— tanh plus wake component.

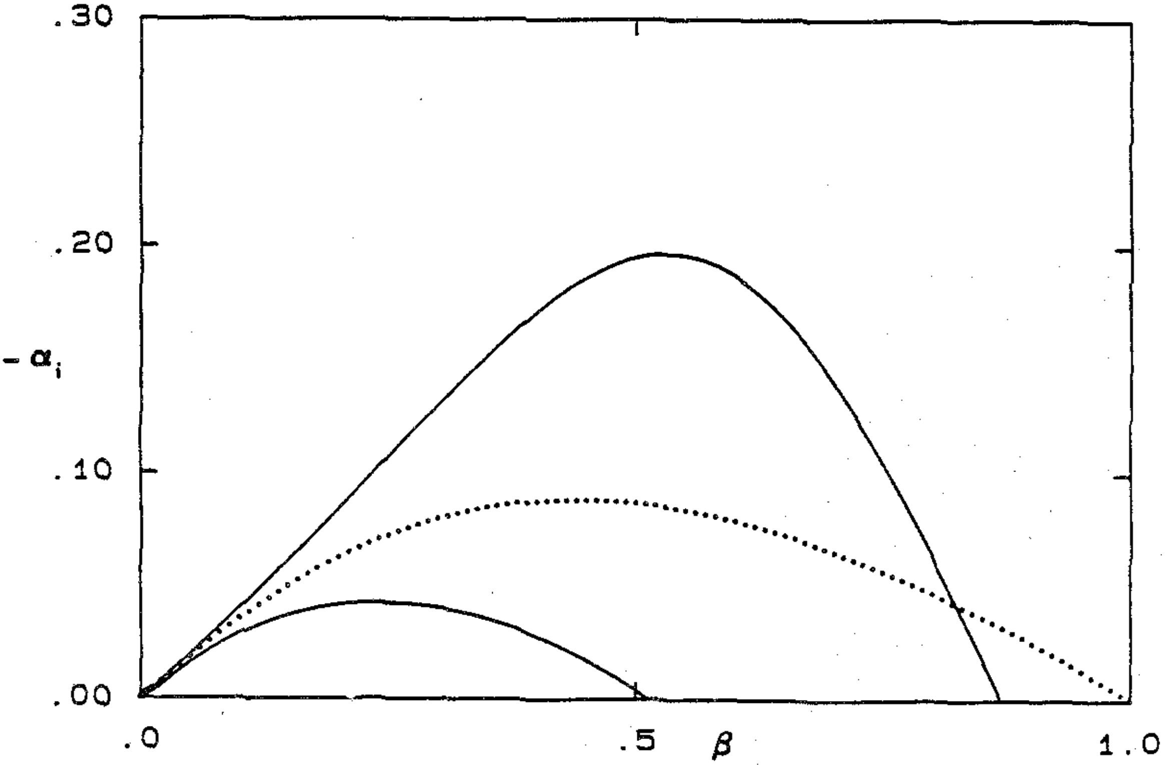


Figure 1b. Growth rate vs. frequency, same conditions as above.

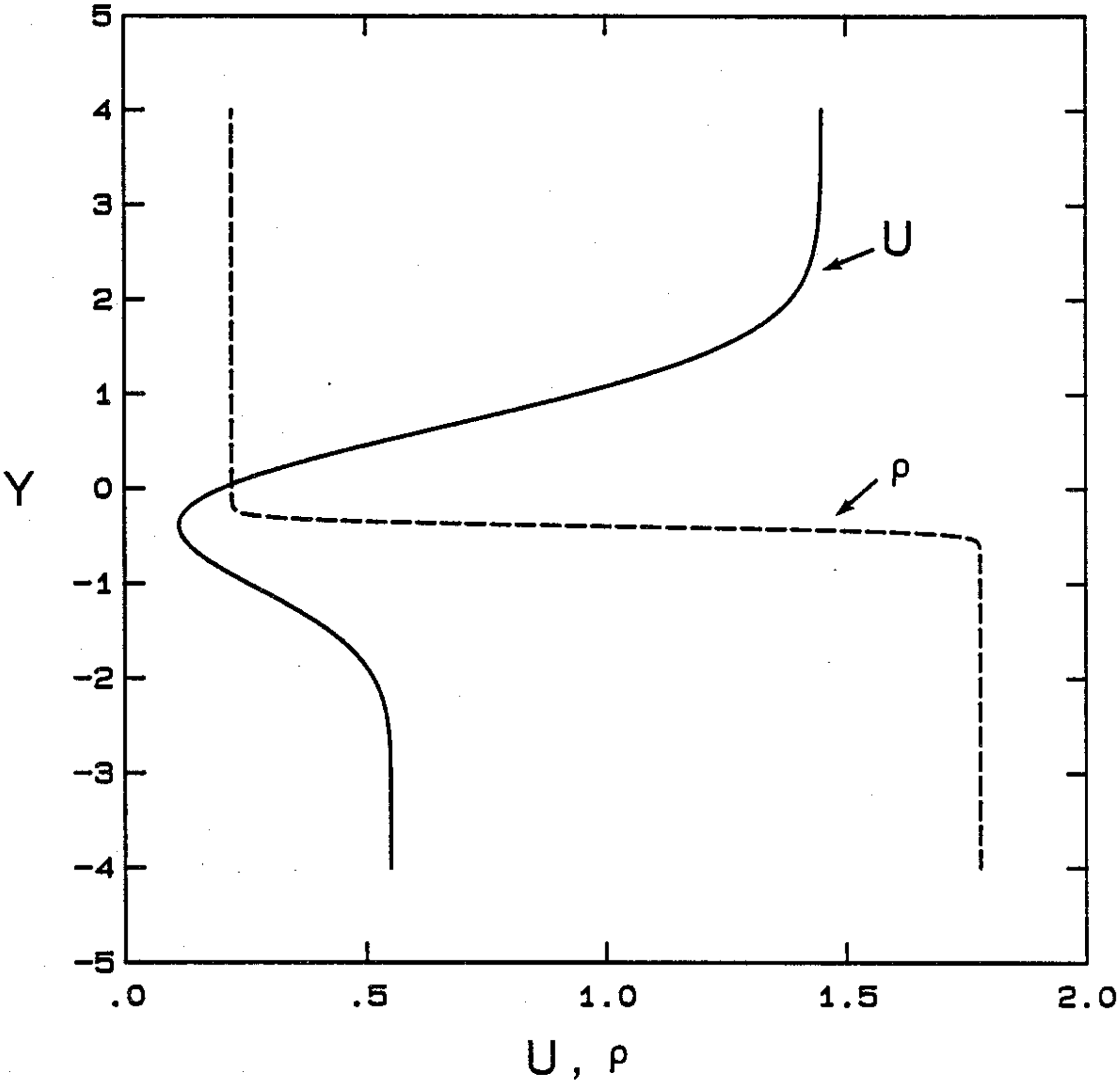


Figure 2. Mean velocity and density profiles, $\lambda_u = 0.45$ ($U_2/U_1 = 0.38$) and $\lambda_\rho = -0.78$ ($\rho_2/\rho_1 = 8$).

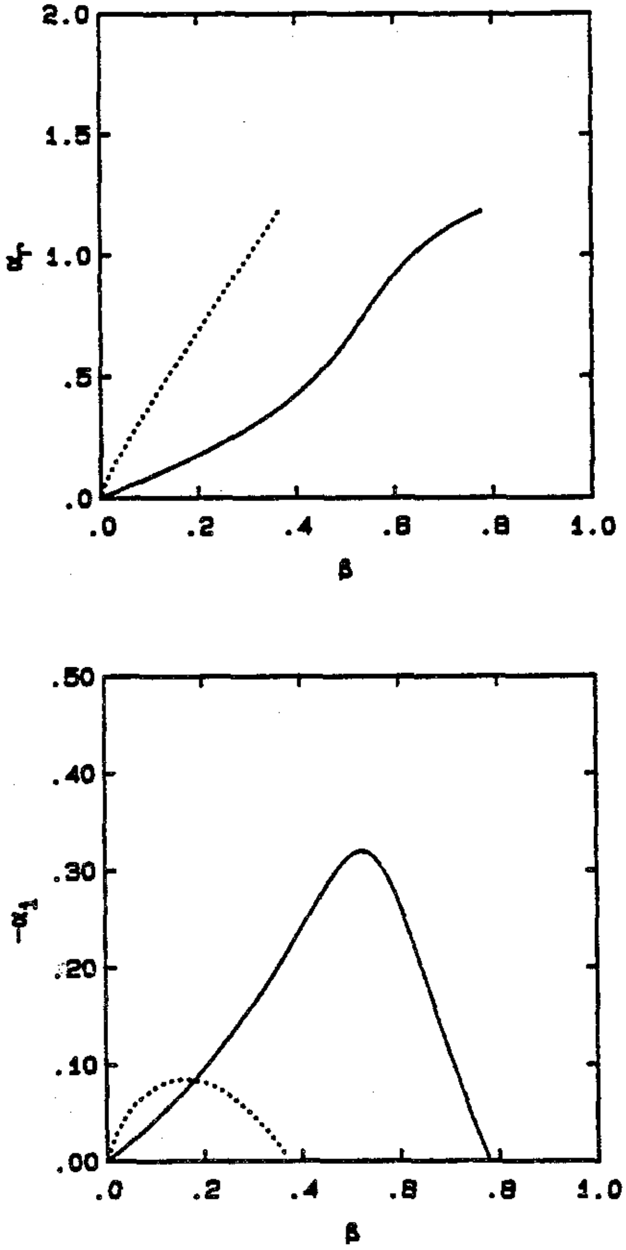


Figure 3a. Instability characteristics of the mean velocity profile of figure 2 with uniform density.

——— shear layer branch

.... wake branch

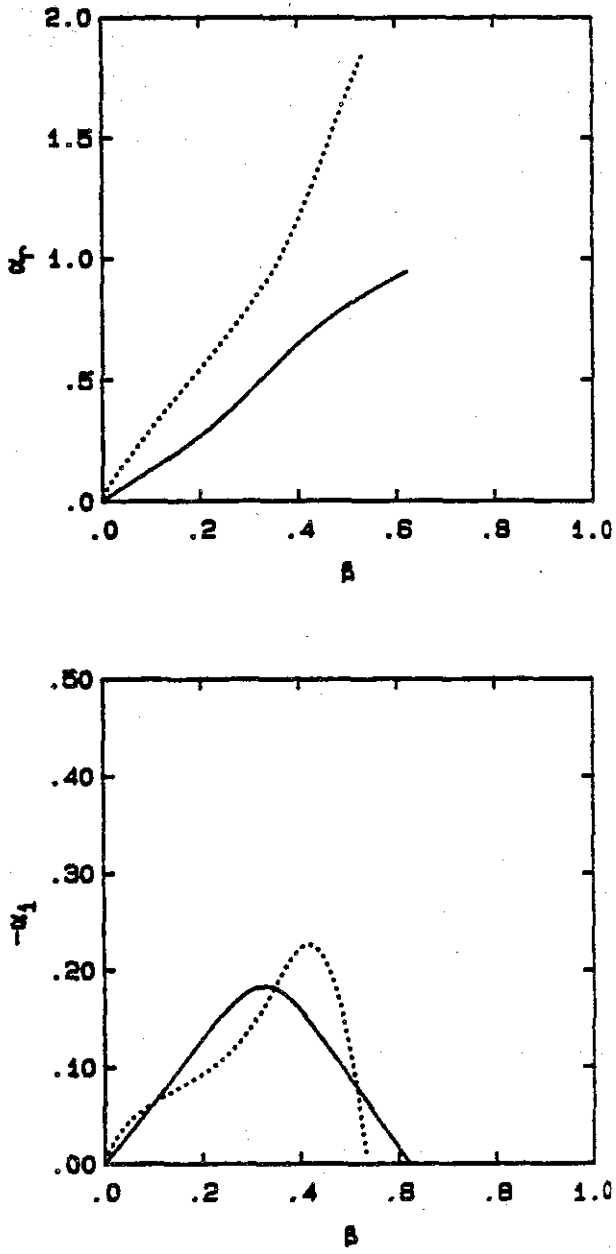
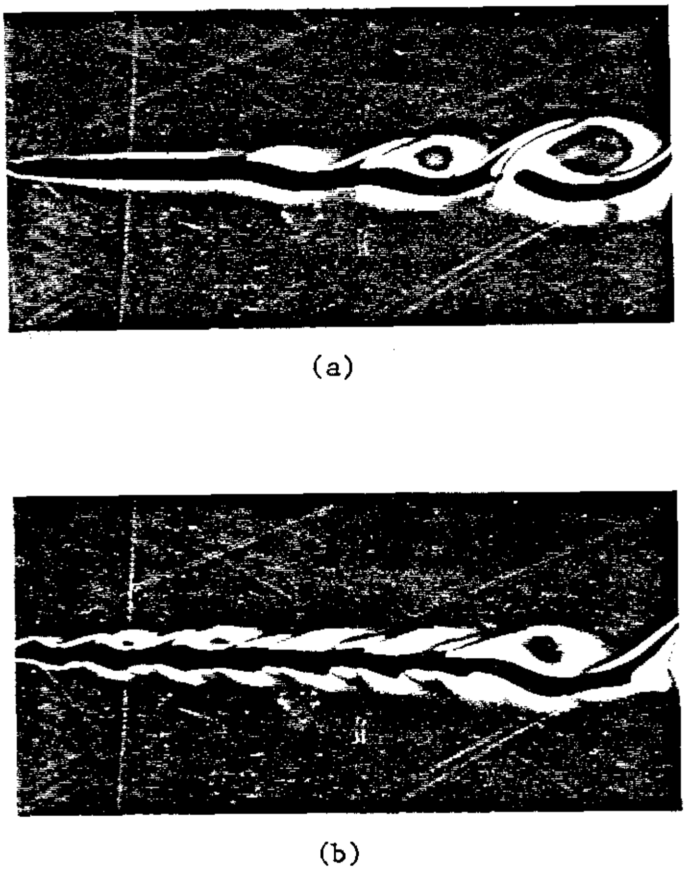


Figure 3b. Instability characteristics of the mean velocity and density profiles of figure

——— shear layer branch

.... wake branch



(a)

(b)

Figure 4. Schlieren photographs of the shear layer mode (a) and the wake mode (b). Flow is from left to right with high-speed stream on top ($U_2/U_1 \simeq 0.38$, $\rho_2/\rho_1 \approx 10$).