Measuring the Primordial Deuterium Abundance during the Cosmic Dark Ages

Kris Sigurdson* and Steven R. Furlanetto†

California Institute of Technology, Mail Code 130-33, Pasadena, California 91125, USA
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We discuss how measurements of fluctuations in the absorption of cosmic microwave background photons by neutral gas at redshifts $z \approx 7$–200 could reveal the primordial deuterium abundance of the Universe. The strength of the cross-correlation of brightness-temperature fluctuations in the redshifted 21-cm line of hydrogen with those in the redshifted 92-cm line of deuterium is proportional to the value of the deuterium-to-hydrogen ratio [D/H] fixed during big bang nucleosynthesis. Although challenging, this measurement would provide the cleanest possible determination of [D/H], free from contamination by structure formation processes at lower redshifts. We additionally report our result for the thermal spin-change cross section in deuterium-hydrogen scattering.

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Introduction.—After the cosmic microwave background (CMB) radiation decoupled from the baryons at a redshift $z \approx 1100$, most CMB photons propagated unfettered through the neutral primordial medium. This has allowed exquisite measurements of the temperature fluctuations in the primordial plasma at the surface of last scattering, and the statistical properties of these fluctuations have recently been used, in conjunction with other observations, to determine the cosmology of our Universe [1]. After the photons kinetically decoupled from the gas at $z \sim 200$, the latter cooled adiabatically with $T_g \propto (1+z)^2$, faster than the $T_\gamma \propto (1+z)$ cooling of the CMB. This epoch, with most of the baryons in the form of relatively cold neutral atoms and before the first stars formed, is known as the cosmic dark ages.

The reason most CMB photons propagate unimpeded through the neutral primordial gas is elementary quantum mechanics—atoms absorb nonionizing radiation only at the discrete wavelengths determined by the differences of their atomic energy levels. One interesting example is the well-known 21-cm spin-flip transition [2], due to the hyperfine splitting of the ground state of the hydrogen (H) atom. At any given z, CMB photons with wavelength $\lambda_{21} = 21.1$ cm can resonantly excite this transition. By measuring brightness-temperature fluctuations due to density fluctuations in the neutral gas [3], radio telescopes observing at $\lambda = (1+z)\lambda_{21}$ can probe the matter power spectrum at $z \approx 30$–200 [4].

In this Letter, we discuss another application of these measurements. Less well-known than the 21-cm transition of neutral H is the spin-flip transition of neutral deuterium (D) at $\lambda_{92} = 91.6$ cm [5]. We show below that cross-correlating brightness-temperature fluctuations at a wavelength $\lambda_{\rm H} = (1+z)\lambda_{21}$ with those at a wavelength $\lambda_{\rm D} = (1+z)\lambda_{92}$ allows a measurement of the primordial D abundance. In principle, this technique could constrain the primordial value of $[{\rm D/H}] \equiv n_{\rm D}/n_{\rm H}$ to better than 1%. While there is no physical obstacle to such a measurement, it would certainly be technically challenging and require a heroic experimental effort; simply confirming the presence of D via cross-correlation during the cosmic dark ages is a significantly easier goal.

Deuterium has long been recognized as our best "baryometer" because its primeval relic abundance is so sensitive to the baryon-to-photon ratio $\eta = n_b/s$. Moreover, big bang nucleosynthesis (BBN) [6] is the only known natural production mechanism, although mechanisms inside galaxies can destroy it [7]. The measurement we describe below could thus determine the true BBN abundance of D and, in principle, might improve BBN constraints to the baryon density of the Universe $\Omega_b h^2$.

Hyperfine structure of H and D atoms.—The $\boldsymbol{\mu}\cdot\mathbf{B}$ interaction between the magnetic moments of the electron and the nucleus splits the ground state of single-electron atoms into eigenstates of the total spin operator $\mathbf{F} = \mathbf{S} + \mathbf{I}$ with eigenvalues $F_+ = I + 1/2$ and $F_- = I - 1/2$ and $\Delta E = (16/3)F_+\mu_B(g_N\mu_N/a_0^3)$ (e.g., [8]). Here $\mathbf{S}$ is electron spin, $\mathbf{I}$ is nuclear spin, a_0 is the Bohr radius, μ_B is the Bohr magneton, μ_N is the nuclear magneton, and g_N is the nuclear g factor ($g_p = 5.56$ for H; $g_{\rm D} = 0.857$ for D). The proton, with $I = 1/2$, splits the H ground state into a triplet with $F_+ = 1$ and a singlet with $F_- = 0$. The deuteron, with $I = 1$, splits the D ground state into a quartet with $F_+ = 3/2$ and a doublet with $F_- = 1/2$.

The population of atoms in the excited spin state relative to the ground state $n_+/n_- = (g_+/g_-)\exp\{-T_\star/T_s\}$ can be characterized by a spin temperature T_s. Here $g_+ = 2F_+ + 1$ and $g_- = 2F_- + 1$ are spin degeneracy factors and $T_\star = \Delta E/k_B$. For H and D, we have $T_\star^{\rm H} = 0.0682$ K, $T_\star^{\rm D} = 0.0157$ K, $(g_+^{\rm H}/g_-^{\rm H}) = 3$, and $(g_+^{\rm D}/g_-^{\rm D}) = 2$.

Three factors determine T_s: absorption of 21-cm CMB photons, absorption and reemission of Lyman-α photons [the Wouthuysen-Field (WF) effect [9,10]], and atomic spin-change collisions (free electrons are unimportant in these environments [10]). The first drives T_s toward T_γ, while the latter two drive it toward the gas temperature T_g. In equilibrium, the spin temperature of a species X is $T_s^X = (1+\chi^X)T_gT_\gamma/(T_g + \chi^X T_\gamma)$, where $\chi^X \equiv \chi_c^X + \chi_\alpha^X$ is the sum of the equilibrium threshold parameters for spin-change collisions and for radiative coupling through the

WF effect. Explicitly, $\chi_c^X = (C_{+-}^X T_\star^X)/(A_{+-}^X T_\gamma)$ and $\chi_\alpha^X = (P_{+-}^X T_\star^X)/(A_{+-}^X T_\gamma)$, where C_{+-}^X is the collisional deexcitation rate, A_{+-}^X is an Einstein coefficient, and $P_{+-}^X \propto P_\alpha$, where P_α is the total Lyman-α scattering rate. At $z \gg 10$, before the first galaxies formed, P_α is tiny and the WF effect can be neglected. However, it might have interesting consequences near $z \sim 10$.

H-H and D-H collision rates.—While the cross section for H-H spin-change collisions $\Sigma_{+-}^{\rm HH}$ is well-known [11–13], we could not locate the D-H spin-change cross section for the temperatures of interest [14] and computed $\Sigma_{+-}^{\rm DH}$ using standard partial-wave phase-shift methods. Our results agree with Refs. [11–13] for $\Sigma_{+-}^{\rm HH}$, with Ref. [13] for $\Sigma_{+-}^{\mu\rm H}$, and with Ref. [16] for $\Sigma_{+-}^{\rm DH}$ at 1 K (after accounting for our more recent molecular potentials).

The collisional deexcitation rate is $C_{+-}^X = \bar{v}_{X\rm H}\bar{\Sigma}_{+-}^{X\rm H} n_{\rm H}$, where $\bar{v}_{X\rm H} = \sqrt{8k_B T_g/\mu_{X\rm H}\pi}$ is the thermal velocity, $\bar{\Sigma}_{+-}^{X\rm H}$ is the thermally averaged spin-change cross section, and $n_{\rm H}$ is the number density of H atoms. In Fig. 1, we plot $\bar{\Sigma}_{+-}^{\rm HH}$, $\bar{\Sigma}_{+-}^{\rm DH}$, and $\bar{\Sigma}_{+-}^{\mu\rm H}$. While $\bar{\Sigma}_{+-}^{\rm HH}$ falls off for $T_g \lesssim 100$ K, $\bar{\Sigma}_{+-}^{\rm DH}$ continues to rise to a peak near $T_g \sim 1$ K. This occurs because of low-energy s-wave and p-wave contributions to D-H scattering. These do not appear for the H-H system due to its different reduced mass ($\mu_{\rm HH} \approx m_{\rm H}/2$ while $\mu_{\rm DH} \approx 2m_{\rm H}/3$). The discussion of D-H spin change in Ref. [10] did not account for this and incorrectly concluded that $\Sigma_{+-}^{\rm DH} \sim \Sigma_{+-}^{\rm HH}$.

Spin-temperature evolution.—In Fig. 2, we plot T_γ, T_g (found using RECFAST [18]), $T_s^{\rm H}$, and $T_s^{\rm D}$ as a function of z. After the gas cools below T_γ, collisions keep $T_s^{\rm H}$ and $T_s^{\rm D}$ coupled to T_g. Near $z \sim 30$, collisions become inefficient for H and $T_s^{\rm H}$ returns to T_γ. $T_s^{\rm D}$ remains coupled to T_g down to significantly lower redshift both because the lifetime of the excited state of D is relatively long ($A_{+-}^{\rm H}/A_{+-}^{\rm D} = 61.35$) and because $\bar{\Sigma}_{+-}^{\rm DH} \gg \bar{\Sigma}_{+-}^{\rm HH}$ at low T_g.

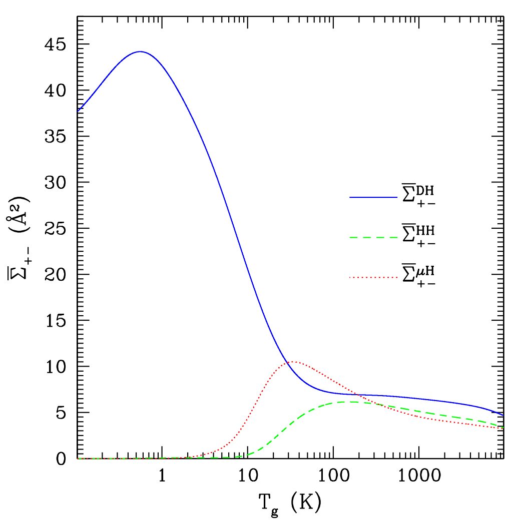

FIG. 1 (color online). The thermal spin-change cross sections that keep T_s collisionally coupled to T_g for D (solid line), H (dashed line), and muonium (dotted line). Although the potentials are identical, the peaks differ because of the reduced masses.

Brightness-temperature fluctuations.—When the spin temperature of a given species is less than T_γ, it will absorb CMB photons. The brightness temperature is $T_b^X = a\tau_X(T_s^X - T_\gamma)$, where $\tau_X = g_+^X c\lambda^2 h A_{+-}^X n_X/[8(g_+^X + g_-^X)\pi k_B T_s^X \mathcal{H}(z)]$ is the optical depth of the spin-flip transition in question, $a = 1/(1+z)$, and $\mathcal{H}(z)$ is the Hubble parameter. We are interested in correlations between brightness-temperature fluctuations $\delta T_b^X(\hat{\mathbf{n}}, a) \equiv \beta^X(a) T_b^X(a)\delta(\hat{\mathbf{n}}, a)$ observed in a direction $\hat{\mathbf{n}}$ at wavelengths differing by a factor $\lambda_{92}/\lambda_{21}$. Here $\beta^X = 1 + \chi_c^X/\hat{\chi}^X + \Gamma\{T_\gamma/(T_g - T_\gamma) + (\chi_c^X/\hat{\chi}^X) d\ln(C_{+-}^X)/d\ln(T_g)\}$, $\delta(\hat{\mathbf{n}}, a)$ is the fractional density contrast, and $\hat{\chi}^X \equiv \chi^X(1 + \chi^X)$. At high z, when $T_g \approx T_\gamma$, $\Gamma \to 0$ due to residual Thomson scattering with free electrons [19] (fluctuations are isothermal), but, as the gas begins to cool adiabatically, $\Gamma \to 2/3$ [20]. We have neglected the contributions to δ_{T_b} from fluctuations in the neutral fraction (likely to be small at high z) and, for simplicity, fluctuations in the gradient of the radial velocity $\delta_{\partial_r v_r}$ [20,21]. The latter will enhance our signal by a factor of $\sim$1–2. In Fig. 3, we plot $T_b^{\rm H}$, $\tilde{T}_b^{\rm D}$, $\sigma_{T_b^{\rm H}}$, and $\tilde{\sigma}_{T_b^{\rm D}}$ (where $T_b^{\rm D} \equiv \epsilon\tilde{T}_b^{\rm D}$, $\sigma_{T_b^{\rm D}} \equiv \epsilon\tilde{\sigma}_{T_b^{\rm D}}$, and $\epsilon \equiv [{\rm D/H}]$). We see that $\tilde{T}_b^{\rm D}$ and $\tilde{\sigma}_{T_b^{\rm D}}$ peak at much lower z than their H counterparts because, as discussed above, $T_s^{\rm D}$ is coupled to T_g to lower z. The lower set of curves in Fig. 3 have $P_\alpha = 0$ throughout (appropriate for rapid late reionization), while the upper assume strong WF coupling and simultaneous heating at $z = 14$ (appropriate if gradual reionization begins at about that time).

D-H cross-correlations.—We now calculate the cross-correlation of brightness-temperature fluctuations across frequencies in the ratio $\lambda_{21}{:}\lambda_{92}$. We write the brightness-temperature fluctuation due to H or D as ${\rm H}(\hat{\mathbf{n}}, a) = \beta^{\rm H}(a) T_b^{\rm H}(a)\delta(\hat{\mathbf{n}}, a)$ and $\epsilon{\rm D}(\hat{\mathbf{n}}, a) = \epsilon\beta^{\rm D}(a)\tilde{T}_b^{\rm D}(a)\delta(\hat{\mathbf{n}}, a)$,

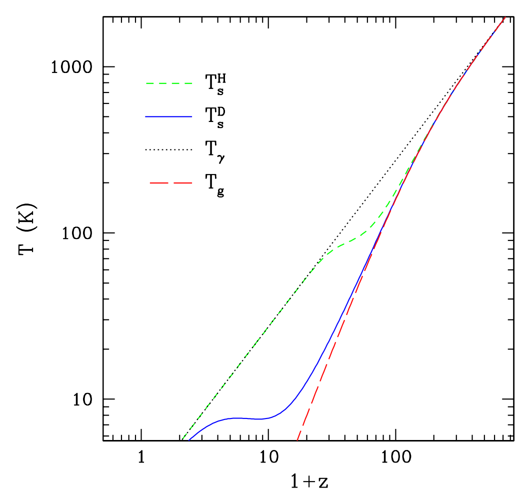

FIG. 2 (color online). The H and D spin temperatures as a function of z. Here we assume $P_\alpha = 0$ for all z.

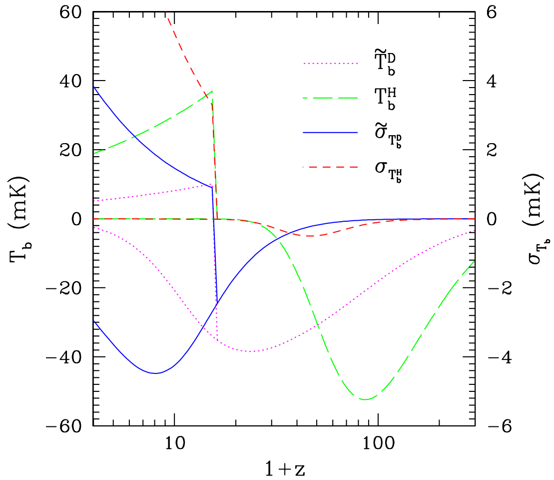

FIG. 3 (color online). The mean brightness temperatures T_b^{H} and $\tilde{T}_b^{\mathrm{D}}$ and the standard deviation of pixel-scale brightness-temperature fluctuations $\sigma_{T_b^{\mathrm{H}}}$ and $\tilde{\sigma}_{T_b^{\mathrm{D}}}$ as a function of redshift. The σ_{T_b} are shown for the benchmark experiment discussed in the text. The lower curves assume the gas remains cool, while the upper curves show the effect of heating the gas far above T_γ at $z = 14$.

respectively. A radio telescope observing at a frequency ν will measure the quantity $\mathcal{O}[\hat{\mathbf{n}}; \nu] = \mathrm{H}(\hat{\mathbf{n}}, \nu/\nu_{21}) + \epsilon \mathrm{D}(\hat{\mathbf{n}}, \nu/\nu_{92}) + N[\hat{\mathbf{n}}; \nu]$, where $N[\hat{\mathbf{n}}; \nu]$ is the noise. The product of observables at frequencies ν_h (the H band) and $\nu_d \equiv (\nu_{92}/\nu_{21})\nu_h$ (the D band) is $\mathcal{O}[\hat{\mathbf{n}}; \nu_h]\mathcal{O}[\hat{\mathbf{n}}; \nu_d] = \mathrm{H}_h\mathrm{H}_d + \epsilon\mathrm{H}_h\mathrm{D}_d + \mathrm{H}_h N_d + \epsilon\mathrm{D}_h\mathrm{H}_d + \epsilon^2\mathrm{D}_h\mathrm{D}_d + \epsilon\mathrm{D}_h N_d$, where we have introduced the shorthand $\mathrm{H}_x \equiv \mathrm{H}(\hat{\mathbf{n}}, \nu_x/\nu_{21})$, $\mathrm{D}_x \equiv \mathrm{D}(\hat{\mathbf{n}}, \nu_x/\nu_{92})$, and $N_x = N[\hat{\mathbf{n}}; \nu_x]$. Assuming that $\delta(\hat{\mathbf{n}}, a)$ is a zero-mean random field and uncorrelated noise, the expectation values (averaged over all the pixels observed on the sky) of most terms vanish as the ν_h-band and ν_d-band factors are uncorrelated. Only H_h and D_d are strongly correlated, and, thus, we have $\langle \mathcal{O}[\hat{\mathbf{n}}; \nu_h]\mathcal{O}[\hat{\mathbf{n}}; \nu_d]\rangle = \epsilon\langle \mathrm{H}_h\mathrm{D}_d\rangle$.

We now understand the crucial point of this Letter. The 21- and 92-cm fluctuations at these frequency separations must be correlated because they trace the same underlying patches of the Universe.

Note that we have neglected the relatively small intrinsic correlations $\langle \mathrm{H}_h\mathrm{H}_d\rangle$ that arise due to large-scale modes of the density field. These contribute $\lesssim 0.1\%$ of the D cross-correlation signal. If necessary, these correlations could be removed by correlating the fluctuations in the ν_h band with fluctuations at frequencies near but not equal to the corresponding ν_d band.

Signal estimate.—Our proposal is relatively straightforward: Brightness-temperature fluctuations in the same direction on the sky (the same angular pixel) at frequencies ν_h and $\nu_d = (\nu_{92}/\nu_{21})\nu_h$ should be cross-correlated. At a given redshift $z = \nu_{21}/\nu_h - 1 = \nu_{92}/\nu_d - 1$, the average signal contributed from a pixel in a frequency band $\Delta\nu_h$ about ν_h and $\Delta\nu_d = (\nu_{92}/\nu_{21})\Delta\nu_h$ about ν_d is just the product of the standard deviations of the H and D brightness-temperature fluctuations: $\epsilon\langle \mathrm{H}_h\mathrm{D}_d\rangle = \epsilon\sigma_{\mathrm{H}_h}\tilde{\sigma}_{\mathrm{D}_d} = \epsilon\sigma_\delta^2\beta^{\mathrm{H}_h}T_b^{\mathrm{H}_h}\beta^{\mathrm{D}_d}\tilde{T}_b^{\mathrm{D}_d}$. Here σ_δ^2 is the variance of density fluctuations averaged over the flat cylindrical volume of each pixel. Explicitly,

$$\sigma_\delta^2 = \frac{2}{\pi^2}\int_0^\infty dk k P(k)\int_0^k dk_z j_0^2(\xi k_z \rho)\frac{J_1^2(\sqrt{k^2 - k_z^2}\rho)}{(k^2 - k_z^2)\rho^2}, \quad (1)$$

for a pixel at redshift z with comoving radius ρ and thickness $2\xi\rho$. The noise in each pixel is a combination in quadrature of the random noise $\langle N_x^2\rangle = \sigma_{N_x}^2$ and the confusion arising from the autocorrelation of the H and D fluctuations. In practice, since $\epsilon^2\langle \mathrm{D}_x^2\rangle \ll \langle \mathrm{H}_x^2\rangle$, we can always neglect the confusion due to D fluctuations in the noise. The average noise per pixel in the cross-correlation measurement is thus $\sqrt{\sigma_{\mathrm{H}_h}^2 + \sigma_{N_h}^2}\sqrt{\sigma_{\mathrm{H}_d}^2 + \sigma_{N_d}^2}$. Here $\sigma_{N_x}^2 = T_{\mathrm{sys}}^2/(f_{\mathrm{cov}}^2\Delta\nu_x t_{\mathrm{int}})$ (e.g., [22]), where $T_{\mathrm{sys}} \simeq 6500[\nu_\alpha/(30\ \mathrm{MHz})]^{-2}$ K is the noise temperature, f_{cov} is the covering fraction of the array, and t_{int} is the integration time. At redshift z, there are a total of $N_{\mathrm{pix}} = 16 f_{\mathrm{sky}}/(\theta_d^2)$ pixels for an instrument with maximum baseline L and angular resolution $\theta_d = \lambda_d/L = (1+z)\lambda_{92}/L$ in the D band that observes a fraction f_{sky} of the sky. The signal-to-noise contributed from the bands ν_h and ν_d at redshift z is thus

$$\frac{\mathcal{S}}{\mathcal{N}}(z) = \frac{\epsilon\sqrt{N_{\mathrm{pix}}}\sigma_{\mathrm{H}_h}\tilde{\sigma}_{\mathrm{D}_d}}{\sqrt{\sigma_{\mathrm{H}_h}^2 + \sigma_{N_h}^2}\sqrt{\sigma_{\mathrm{H}_d}^2 + \sigma_{N_d}^2}}. \quad (2)$$

If $\mathcal{S}/\mathcal{N}(z)$ did not depend on z and measurements were made over a total bandwidth B_h in the H band [or $B_d = (\nu_{92}/\nu_{21})B_h$ in the D band], the total signal-to-noise ratio would just be Eq. (2) multiplied by a factor $\sqrt{B_h/\Delta\nu_h}$. In practice, as $\mathcal{S}/\mathcal{N}(z)$ varies with z, we calculate $(\mathcal{S}/\mathcal{N})_{\mathrm{tot}}$ by summing $\mathcal{S}/\mathcal{N}(z)$ over all redshift bins in quadrature.

We note that our choice for T_{sys} is only an estimate, and the noise (ultimately due to Galactic synchrotron radiation) varies strongly across the sky. Potential systematic effects including instrument calibration and foreground removal will undoubtedly make a detection more challenging and will necessitate the use of more sophisticated statistical estimators of the cross-correlation signal than the simple estimator we describe here. For instance, as we propose to cross-correlate brightness-temperature fluctuations that vary rapidly as a function of frequency, most foreground signals (which are expected to vary smoothly in frequency space) can be removed by applying a high-pass filter in cofrequency space to multiband observations (which would reject any signal smooth in frequency space) [22,23]. In fact, as we are looking for a cross-correlation signal, this type of measurement should be robust to even more pathological foreground and systematic effects. For example, a foreground or instrumental effect which leads to correlations between the ν_h band and the ν_d band may also lead to related correlations between the ν_h band and another frequency band ν_o. However, a statistical estimator for $\epsilon = [\mathrm{D/H}]$ can be devised that rejects correlations between the ν_h band and ν_d band if the fluctuations in these bands are also significantly correlated with the fluctuations in the ν_o band or *any other band*. The extent to which these or other techniques will be needed will become clearer in the years to come as 21-cm experiments gain real-world experience with these issues.

If collisions dominate the coupling between T_s^{H} and T_g down to $z \sim 7$, then we estimate that a value $[\mathrm{D/H}] \sim 3 \times 10^{-5}$ could be detected at 1–2σ by a benchmark experiment with $L \sim 7.5$ km and $\Delta\nu_d \sim 100$ kHz in ~6 years. If, however, the first generation of stars created a flux of Lyman-α photons which coupled T_s^{H} to T_g until $z \sim 7$ through the WF effect (without heating the gas), a similar detection might be made by a smaller experiment with $L \sim 2.5$ km and $\Delta\nu_d \sim 1$ kHz. Although it strongly enhances the 21-cm signal, the WF effect does not improve the [D/H] measurement by the same margin once $\sigma^2_{\mathrm{H}_h} \gg \sigma^2_{N_h}$: In that regime, it only makes the H fluctuations a better template. Finally, we note that an experiment capable of mapping 21-cm brightness-temperature fluctuations out to $l_{\max} \sim 10^5$ (where it may be a powerful probe of the small-scale matter power spectrum [4]) could measure [D/H] to a precision as good as ~1%—or even ~0.1% if the WF effect coupling is efficient.

For these estimates, we have assumed a ΛCDM cosmology with $n_s = 1$ and that a significant fraction of the Universe remains neutral until $z \sim 7$. The largest contribution to the signal originates from $z \lesssim 10$, where the D signal peaks and the variance in the density fluctuations is largest. Complete and sudden reionization at $z_r = 10$ (14) would reduce the observable signal by a factor of 5 (20). However, a long phase of partial ionization would have much less severe effects (only reducing the number of available pixels by the filling factor of partially neutral gas). Moreover, any heating from these ionizing sources would boost the signal by strengthening the H template (upper curves in Fig. 3): If heating begins at $z = 14$, $(\mathcal{S}/\mathcal{N})_{\mathrm{tot}}$ decreases by only a factor of 2 relative to our fiducial case if $z_r = 10$—and it even *increases* the signal by a factor of 2 if $z_r = 7$.

Discussion.—Despite the obvious technical challenges in observing this signal, it has the virtue of providing the cleanest possible measurement of the primordial [D/H], free from contamination by structure formation processes at lower z. Via the window of BBN, this would allow radio telescopes to peer into the first few minutes of the Universe. We believe future searches for cosmic 21-cm fluctuations should bear this possibility in mind.

We also note that $^3\mathrm{He}^+$ has a hyperfine transition (with $\lambda = 3.46$ cm) that can be used in a similar fashion; it has the advantage of much lower foreground contamination at higher frequencies. This line will show a strong anticorrelation with the 21-cm signal during reionization. If the details of reionization can be understood well enough, the cross-correlation of this line with the 21-cm line could supplement the D-H experiment, especially if the Universe has a long period of partial ionization.

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*Electronic address: ksigurds@tapir.caltech.edu
†Electronic address: sfurlane@tapir.caltech.edu

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