

## Ideality in a Fiber-Taper-Coupled Microresonator System for Application to Cavity Quantum Electrodynamics

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The ability to achieve near lossless coupling between a waveguide and a resonator is fundamental to many quantum-optical studies as well as to practical applications of such structures. The nature of loss at the junction is described by a figure of merit called ideality. It is shown here that under appropriate conditions ideality in excess of 99.97% is possible using fiber-taper coupling to high- $Q$  silica microspheres. To verify this level of coupling, a technique is introduced that can both measure ideality over a range of coupling strengths and provide a practical diagnostic of parasitic coupling within the fiber-taper-waveguide junction.

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Microresonators have attracted considerable interest for investigation of fundamental processes ranging from cavity quantum electrodynamics (QED) [1,2] to nonlinear optics [3,4], and in more applied areas such as photonics [5,6] and chemical/biological sensing [7]. Resonators that feature modes with ultrahigh quality factors ( $Q$ ) and small modal volume, such as silica microspheres, can induce strong coupling between an atomic system and the cavity electromagnetic mode [8], as well as drastic reductions of the power necessary to observe nonlinear effects [9,10]. An important requirement for such studies (quantum-optical experiments in particular) is ultralow-loss field coupling both to and from the microresonator. Parasitic coupling loss in this process is quantified by defining a coupling “ideality” as the ratio of power coupled to a desired mode divided by power coupled or lost to all modes (including the desired mode). Ideality describes to what extent the coupling process behaves as single mode to single mode. Prism coupling, a popular technique for coupling to ultrahigh- $Q$  microspheres, does not typically possess a high ideality because of the large number of possible output modes (as evidenced by the observed spatial “fan” of output power [11,12]). In contrast, the small number of modes supported by fiber-taper-based coupler waveguides suggests that the situation could be improved over prism coupling. Tapered fibers have been shown to provide high coupling efficiency to microresonators with low nonresonant insertion loss, and this has prompted our proposal for their use in cavity quantum-optical studies [10,13]. Nonetheless, since neither resonator nor taper (in general) are single transverse mode devices, and since it is possible to couple to radiation modes by way of the taper, high ideality in this system is not necessarily expected. Here, the ideality of a fiber-taper coupler is measured using a novel method. It is shown that for appropriate conditions taper junctions offer near-unity ideality. Because the conditions for ideality can be generalized, it is expected that these results can apply in a variety of systems.

Typical fiber tapers are 1–4  $\mu\text{m}$  diameter air-clad silica cylinders, fabricated by flame heating and pulling standard single-mode fiber into a narrow thread [13,14]. Tapers are usually multimode waveguides. For example, six modes are supported in a 2.0  $\mu\text{m}$  diameter silica taper at an optical wavelength of 1550 nm. By control of taper adiabaticity [15], it is always possible to launch the fundamental  $\text{HE}_{11}$  taper mode. Likewise, excitation of a single resonator mode is possible through a combination of phase-matching and modal frequency selection (the power coupling coefficient to the next resonator mode assuming a mode linewidth of 1 MHz and separation between modes of 10 GHz is on the order of  $10^{-8}$ ). However, a resonator mode, once excited, can transfer power back to many taper modes (see Fig. 1). Additionally, the presence of the taper-waveguide can cause the resonator to couple power into the continuum of radiation modes (and induce scattering at the resonator-waveguide junction).

This waveguide/resonator system (Fig. 1) can be studied using a simple model [17], based on the assumption of coupling between the resonator and waveguide being weak, and intrinsic resonator loss being low. In such cases, the individual contributions to the cavity field decay are separable (these assumptions are valid for the fiber tapers and high- $Q$  microresonators used in the current work). The internal resonator field ( $a$ ) is determined by considering both excitation by the fundamental waveguide-mode (input wave amplitude  $s$  and coupling amplitude  $\kappa_0$ ), and decay due to intrinsic resonator loss (round-trip power loss coefficient  $\sigma_0^2$ ) and output coupling to all available waveguide/radiation modes (coupling amplitude to a higher-order waveguide-mode denoted by  $\kappa_{i>0}$  and to radiation modes denoted by  $\kappa_{\text{rad}}$ ). On resonance the internal field obeys the equation of motion [18],

$$\frac{da}{dt} = -\frac{1}{2} \left( \sum_i \kappa_i^2 + \kappa_{\text{rad}}^2 + \sigma_0^2 \right) a + i\kappa_0 s. \quad (1)$$

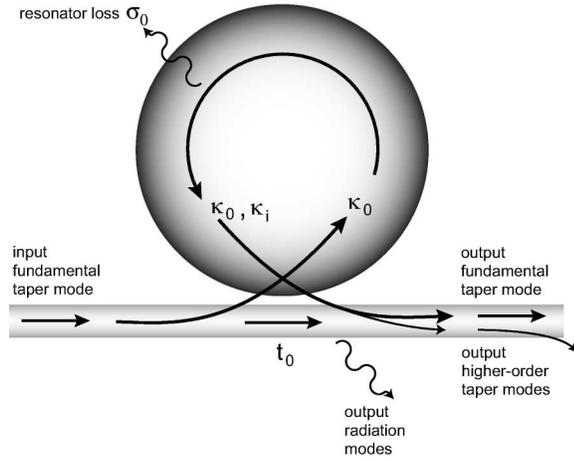


FIG. 1. Coupling and loss parameters in a taper-microresonator system. The input field is a fundamental taper mode which couples into the resonator with amplitude  $\kappa_0$  (transmission amplitude  $t_0$ ). The output field couples into the fundamental taper mode and higher-order taper modes with coupling constants  $\kappa_0$  and  $\kappa_i$ , respectively [16]. The presence of the waveguide can also result in a radiated field. The higher-order taper modes are radiated or coupled to cladding modes upon transition of the taper back to single-mode fiber. The round-trip resonator intrinsic power loss is given by  $\sigma_0^2$ .

The transmission through the waveguide consists of an interference of the partially transmitted input field with the field coupled from the resonator back into the fundamental  $\text{HE}_{11}$  taper mode (this assumes that power coupled into higher-order taper modes is lost upon transition to single-mode fiber), which is given by

$$T = |t_0 + i\kappa_0 a/s|^2. \quad (2)$$

In steady state, the transmission can be expressed as

$$T = \left( \frac{1 - K}{1 + K} \right)^2, \quad (3)$$

where the coupling parameter  $K$  is defined by

$$K \equiv \frac{\kappa_0^2}{\sum_{i \neq 0} \kappa_i^2 + \kappa_{\text{rad}}^2 + \sigma_0^2}. \quad (4)$$

$K$  is the ratio of the desired waveguide-mode power coupling to total system power loss.

The coupling factor  $K$  is composed of an intrinsic contribution  $K_I \equiv \kappa_0^2/\sigma_0^2$ , and a parasitic contribution  $K_P \equiv \kappa_0^2/(\sum_{i \neq 0} \kappa_i^2 + \kappa_{\text{rad}}^2)$ , such that  $K^{-1} = K_I^{-1} + K_P^{-1}$ . As noted earlier, ideality is defined as the amount of power coupled into the desired mode (in this case the fundamental  $\text{HE}_{11}$  mode) divided by the amount of power coupled into all modes, and is given by

$$I \equiv \frac{\kappa_0^2}{\sum_i \kappa_i^2 + \kappa_{\text{rad},T}^2} = \frac{1}{1 + K_P^{-1}}. \quad (5)$$

An ideal waveguide coupler ( $I = 1$ ) is characterized by coupling only between the intended resonator and waveguide modes. The degree of ideality is determined by the parasitic coupling factor  $K_P$ .  $K_P$  is, in general, a function of the relative position of the taper and resonator, and, as such, the deviation of  $K$  ( $K^{-1} = K_I^{-1} + K_P^{-1}$ ) from ideal behavior ( $K = K_I$ ) determines  $I$ .

$K$  can be obtained by measuring the dependence of coupling on waveguide-resonator gap and inverting Eq. (3) as follows:

$$\left( \frac{1 \pm \sqrt{T}}{1 \mp \sqrt{T}} \right) = K = \frac{\bar{\kappa}_0^2 e^{-\gamma_0 x}}{\bar{\kappa}_i^2 e^{-\gamma_i x} + \sigma_0^2}, \quad (6)$$

where the upper signs are taken for transmission values in the overcoupled regime, and the lower signs for the undercoupled regime. The second equality follows from Eq. (4) by noting that the coupling amplitudes  $\kappa_0$  and  $\kappa_i$  decrease exponentially with resonator/waveguide separation and by assuming that  $K_P$  is dominated by a single higher-order taper-waveguide mode (as shown below this assumption is valid for the data in this work).  $\gamma_0$  ( $\gamma_i$ ) are spatial decay rates (vs gap  $x$ ) such that  $\kappa_{0,i}^2 \equiv \bar{\kappa}_{0,i}^2 \exp(-\gamma_{0,i} x)$  with  $x = 0$  corresponding to zero gap. As demonstrated below, upon plotting  $K$  vs gap on a logarithmic scale,  $K_I$  and  $K_P$  can often be identified, as  $K_P$  (for higher-order taper mode parasitic coupling) is a line with slope less than that of  $K_I$ . In particular, if  $\bar{\kappa}_i^2 > \sigma_0^2$ , then the relation  $K^{-1} = K_I^{-1} + K_P^{-1}$  results in a roll-off of  $K$  for small gap distances due to parasitic coupling. In situations where  $\bar{\kappa}_i^2 < \sigma_0^2$ , the higher-order mode coupling is masked and a lower bound on ideality can be established.

We experimentally investigate the ideality of a fiber-taper-microsphere system using the approach given above. The experimental setup consists of a silica microsphere coupled to a tapered optical fiber with the separation distance controlled by a closed-loop stage with a resolution of 20 nm. All data were taken for resonances near 1550 nm. The transmission data are obtained by normalizing the on-resonance power transmission with the power transmitted by the taper alone (i.e., infinite gap).

The dependence of ideality on fiber-taper diameter was investigated by varying the resonator location on the fiber taper. Figure 2 shows  $K$  vs gap curves for multiple fiber-taper diameters. The taper diameters, measured by a scanning electron microscope, are approximately 1.2  $\mu\text{m}$  (circles), 1.35  $\mu\text{m}$  (stars), and 1.65  $\mu\text{m}$  (triangles). For the smallest taper diameter measured, three waveguide modes are supported, the  $\text{HE}_{11}$ ,  $\text{TE}_{01}$ , and  $\text{TM}_{01}$  modes, although both higher-order modes are near cutoff. There are four modes supported (adding the  $\text{HE}_{21}$  mode) for the two larger taper diameters. The data show that for increasing taper diameter there is a deviation of  $K$  from the single-mode coupling regime (dashed lines) due to higher-order mode coupling. A fit using

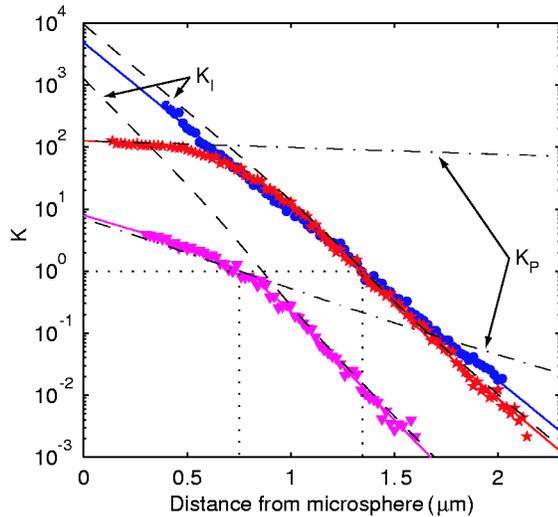


FIG. 2 (color online).  $K$  vs position for various taper diameters for a  $67\text{-}\mu\text{m}$ -diameter microsphere. The data represent taper diameters of approximately  $1.2\ \mu\text{m}$  (circles),  $1.35\ \mu\text{m}$  (stars), and  $1.65\ \mu\text{m}$  (triangles). Solid curves are fits using Eq. (6). The ideality at contact in the data (extrapolating fits to zero gap) is  $> 99.98\%$ ,  $99\%$ , and  $88\%$ , respectively. Data show that for increased taper diameter higher-order-mode coupling causes a deviation from the ideal case (dashed line). The dash-dotted line represents  $K_P$ , which is related to ideality through Eq. (5). Dotted lines mark the critical coupling point.

Eq. (6) shows excellent agreement (solid lines), suggesting that a single higher-order mode is responsible for the observed roll-off of  $K$  with decreasing gap distance. As the number of modes supported for the two largest taper sizes are identical, the strong variation of the coupling data suggests that phase matching is playing a significant role in determining the coupling behavior, resulting from the change of the taper-waveguide modes' propagation constant as taper diameter is varied.

The ideality of the coupler is determined by  $K_P$  through Eq. (5). Assuming the validity of the two-mode model, the dashed and dash-dotted lines in the figure give the  $K_I$  and  $K_P$  contributions to  $K$ , respectively. The  $1.65\ \mu\text{m}$  coupling data (triangles) show significant deviations from ideal coupling, with ideality ranging from  $88\%$  at microsphere-taper contact to  $13\%$  at a  $1.5\ \mu\text{m}$  gap. The  $1.35\ \mu\text{m}$  data (stars) exhibit less deviation, with ideality ranging from  $99\%$  at contact to  $98\%$  at a  $2\ \mu\text{m}$  gap. Finally, the data corresponding to the  $1.2\ \mu\text{m}$  taper diameter (circles) represent apparent ideal behavior over the range of separation gaps measured. The ideality at contact for this taper size is  $> 99.98\%$  and it is not possible to infer a dependence with gap from the data. The influence of nonideality is clearly illustrated at the critical point (dotted lines). For high ideality, the critical point in the data (given by the gap distance where  $K = 1$ ) is identical to the gap separation where  $K_I = 1$ . The data show that this condition holds for the two smaller taper diameters. However, the critical point for the  $1.65\ \mu\text{m}$

taper diameter data is shifted towards a lower separation than  $K_I$  ( $0.1\ \mu\text{m}$  shift of data from dashed line), as a result of the lower ideality (the large shift of  $0.5\ \mu\text{m}$  for  $K_I$  is mainly a result of phase matching).

The data in Fig. 2 demonstrate that coupling (and ideality) vs position behavior is very sensitive to the size of the waveguide. In order to further investigate the influence of taper-waveguide diameter on the coupling behavior and ideality of the system, numerical calculations based on a modified coupled-mode theory [19] were performed. This model calculated the ideality based on coupling to the supported waveguide modes (i.e., it did not include radiation mode coupling). The results were in good agreement with the experimental data in Fig. 2 (values of  $K$  at contact and fundamental mode decay rates, i.e., slope of  $K_I$ , were within  $10\%$  of measured values). Finally, the degradation of ideality with increasing gap distance is a result of slower evanescent decay for higher-order taper modes.

Finally, by maximizing the value of  $K$  for near-contact gaps with a  $2\text{-}\mu\text{m}$ -diameter taper and using higher  $Q$ -factor microspheres ( $Q > 10^8$  by measurement of linewidth in a  $65\text{-}\mu\text{m}$ -diameter microsphere), it was possible to obtain even higher values for ideality in near-contact conditions (Fig. 3). The inset shows the transmission vs separation data for this system, with a maximum over-coupled transmission of  $99.95\%$  (determined by using the exponential fit to  $K$  at zero gap). Numerical calculations (described above) show that the data slope is consistent with  $K_I$ . This agreement, combined with a very low radiation mode coupling power loss  $< 0.05\%$  (using the

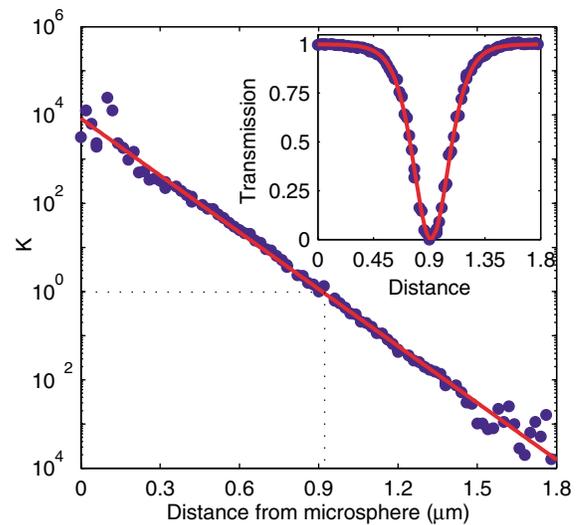


FIG. 3 (color online). Coupling parameter  $K$  vs taper-sphere separation for a  $65\text{-}\mu\text{m}$ -diameter microsphere. The data show a linear relation between  $\ln K$  and  $x$ , with a least squares fit (solid line). Ideality inferred at contact is greater than  $99.99\%$ . The dotted line marks the critical coupling point ( $K = 1$  at  $x = 0.91\ \mu\text{m}$ ). The inset shows transmission vs position data.

fact that the overcoupled transmission drop from unity is due to intrinsic resonator loss and all other coupling-induced losses), demonstrates that the taper behaves as a nearly ideal coupler (higher-order mode coupling is not observable over the range of gaps measured). A lower bound of ideality of 99.97% is obtained if the lowest data point at contact is used. However, using a fit to the entire coupling data set (solid line) establishes a lower bound on ideality at contact of 99.99%.

The observation that a fiber taper can obtain high ideality in ultrahigh- $Q$  systems shows that this form of coupling will be useful for the study of processes requiring very low loss. This includes quantum-optical studies involving cavities in general, with specific examples being the application of strong-coupling cavity QED to quantum information studies [2,20] or of weak-coupling cavity QED to new quantum sources [21,22]. In such examples, coupling quantum states to and transport over optical fiber has been proposed [23], making optical tapers an excellent coupling interface. As ideality is dominated by the mode spectrum of the coupler, alternate resonator geometries (e.g., microdisks and recently demonstrated ultrahigh- $Q$  microtoroids on a chip [24]) should exhibit the same high ideality confirmed in this work when used with tapers. Furthermore, an additional observation that coupling to radiation modes was negligible for taper diameters near single-mode operation (as given by the magnitude of off-resonance power loss when the resonator is in contact with the taper) suggests that ideal behavior can be obtained, if necessary, by operation at the taper size giving single-mode operation (although it is desirable to operate at the phase-matching point for optimal coupling efficiency [13,25]). Finally, the transformation described by Eq. (6) provides a sensitive method of determining the ideality of a coupler and more generally uses the resonant system to diagnose properties of the coupling.

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