Remotely Pumped Optical Distribution Networks: A Distributed Amplifier Model

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***Abstract*—Optical distribution networks using remotely pumped erbium-doped fiber amplifiers (EDFAs) with a single pump source at the head end can conveniently provide signal gain without adding to the power-consumption cost and management complexity of having multiple locally pumped EDFAs in densely populated metropolitan areas. We introduce an analytical model for understanding the basic physical foundations of remotely pumped network design and for analyzing the number of users that can be supported using such a remote-pumping scheme.**

***Index Terms*—Communication systems, erbium-doped fiber amplifiers (EDFAs), optical fiber communication.**

I. Introduction

An OPTICAL distribution network [1] serves a smaller number of users than, e.g., the feeder network and the wide-area network (WAN), shown in the hierarchy of Fig. 1. Several distribution networks share the resources of a single feeder network. Distribution networks typically operate in a geographically compact area, and we address metropolitan-area network (MAN) architectures in this paper. Often, a tradeoff between bandwidth utilization and simplicity is necessary. We take the approach that whereas the feeder (and higher) levels in this architectural hierarchy can have active components, the distribution networks will consume no electrical power directly.

In [1], we studied distribution networks comprising of lumped amplifiers. Sections of erbium-doped fiber of appropriately chosen length and doping concentration are inserted at suitable locations along the length of undoped single-mode fiber. The pump sources can be maintained at the feeder network access nodes, simplifying deployment and network management-and-control and ensuring that no component physically in the distribution network needs an electrical power supply. The paper estimates the number of users that can be supported in a typical 10-Gb/s wavelength-division multiplexed (WDM) setting for various architectures (bus, ring, star, tree, two-level models, etc.) using remotely pumped erbium-doped fiber amplifiers (EDFAs). Since propagation losses cannot be ignored, a 1480-nm pump is clearly superior to a 980-nm pump for networks as long as 20–30 km. It is possible to write down closed-form expressions for the number of users allowable for a given input pump power, etc., but as discussed in [2], optimization arguments are complicated in this framework given the nature of the formulas, which involve nonlinear mathematical operations such as the floor function.

In this paper, we introduce a different conceptual model that permits analytical insight and obtains results that can be understood from a physical viewpoint. The entire network is modeled as a single distributed amplifier, with erbium-doped fiber serving as both the transmission and the gain medium. This is not a new model in the analysis of optical networks and has been used in a computational study of a different but related problem using similar physical principles [3]. This model can be thought of as a limiting case of the lumped-amplifier chain. The role of gain in such a network is to overcome propagation losses and to compensate for the signal power extracted by users for detection. One way to do this is by using WDM-type mux/demux devices. A typical signal propagation model is shown in Fig. 2: the fraction of signal power that is coupled out of the main channel at the kth WDM demux is called the "tap fraction" t_k.

In general, t_k supports a collection ("subnetwork") of users: in a discrete network design methodology, we could want the number of users $\hat{M}_k$ in the kth subnetwork to be as large as possible practically (in the hundreds) and t_k can then be as high as 90–99%: most of the signal power is tapped and then made up for by inserting an appropriate 15–20-dB gain stage immediately before the tap or afterward, depending on the signal power. In this paper, we focus on the other limiting case: t_k is enough to support only a single user. Correspondingly, t_k is now a small number, e.g., 0.1%, and the previous lumped amplifier model cannot yield an accurate analysis. The gain needed to overcome this small tap is also small. We model the entire bus network span as a single distributed amplifier, with periodic taps that model the signal-power-extracting effect of the users along the length of the fiber.

II. Rate Equations

Our goal is to identify, at their simplest level, the limitations on the number of users in such a setting. We begin with the two-level EDFA model with pumping at 1480 nm. In the nomenclature of Table I, we can describe the evolution of the normalized upper level population [4]

$$\frac{\partial N_2(z,\,t)}{\partial t} = -\frac{N_2(z,\,t)}{\tau} - \frac{1}{\rho S}\sum_{k=1}^{N}\frac{\partial P_k(z,\,t)}{\partial z} \qquad (1)$$

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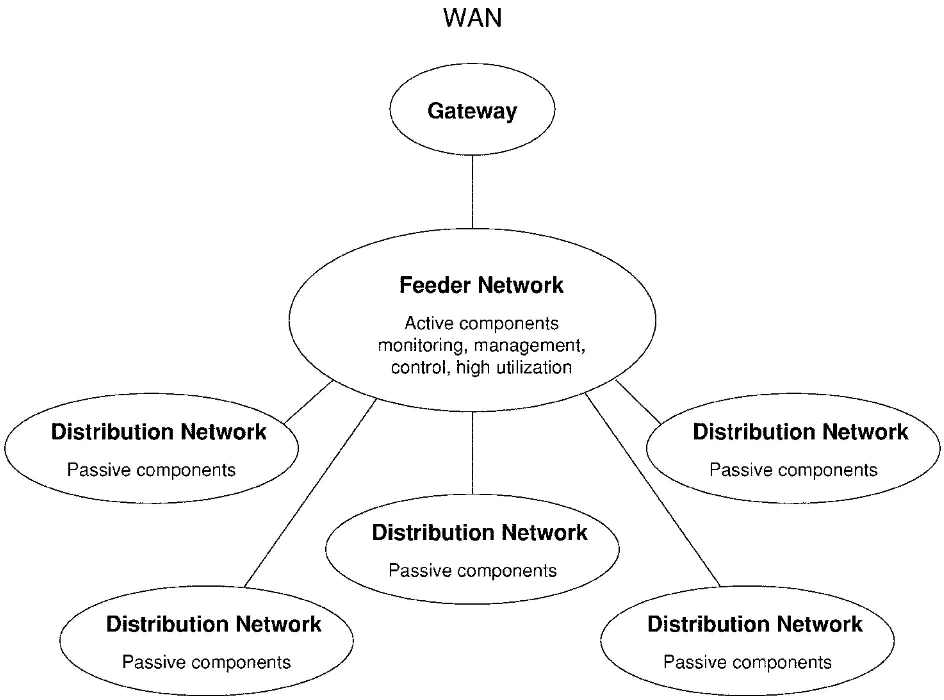

Fig. 1. Typical hierarchical structure for optical networks, showing the relationship between distribution networks and higher components.

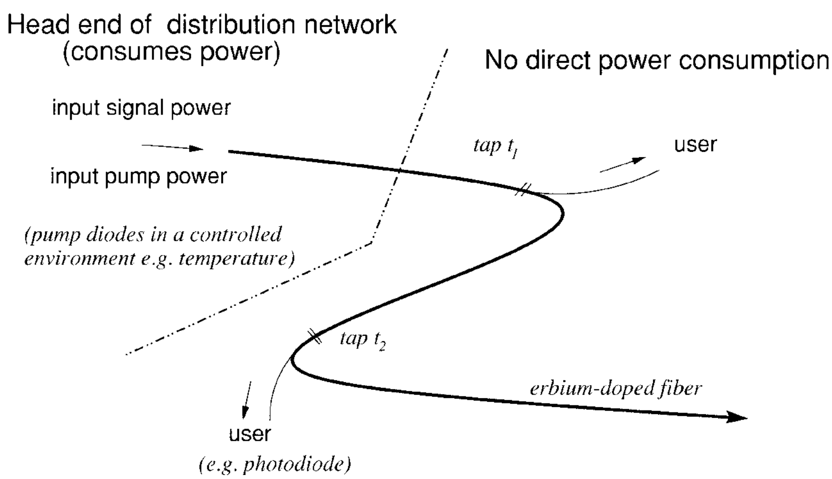

Fig. 2. Signal propagation model for distributed remotely pumped optical network. The only pump source is at the head end, where monitoring of the pump diode is not a problem. The remainder of the network (20–30 km) is passive.

where P_k indexes the power in the N optical channels [signal(s) and pump]. Similarly, the equation that describes the evolution of the (copropagating) optical channels is

$$\frac{\partial P_k(z, t)}{\partial z} = [(\gamma_k + \alpha_k)N_2(z, t) - \alpha_k - \alpha'_k - f_k(z)]\, P_k(z, t) \tag{2}$$

where

$$f_k(z) = \sum_{m=1}^{M} \delta(z - z_m)t_k(z) \tag{3}$$

represents the taps for M users along the entire erbium-doped fiber (EDF) length for channel k.

As discussed in [2], using the theory of dominated convergence, we can integrate the above equations to form path-averaged quantities: in fact, our intent was to define the effect of users in such a way as to permit this important analytical simplification. Carrying out the calculation yields, after some algebra, an ordinary differential equation for the time-evolution of the path-averaged upper level fraction $\overline{N}_2(t)$

$$\left(\frac{d}{dt} + \frac{1}{\tau}\right)\overline{N}_2(t) = -\frac{1}{\tau L \zeta}\sum_{k=1}^{N} P_{\text{in}}(\lambda_k, t) \times \left\{\exp\left[\left(\frac{\zeta}{P_{\text{sat}}(\lambda_k)}\,\overline{N}_2(t) - \tilde{\alpha_k}\right) L\right] - 1\right\} \tag{4}$$

TABLE I
DISTRIBUTED BUS NETWORK MODEL: NOMENCLATURE. ALL OPTICAL POWERS ARE NORMALIZED TO THE SATURATION VALUES AT THE CORRESPONDING WAVELENGTHS [5]

P_0	0 dBm single channel input power
q_0	200 (norm.) input pump power
l	10 m inter-tap (-user) spacing
$\alpha_{s,p}$	4.0, 1.6 dB/km (signal, pump) absorption coefficient
$\gamma_{s,p}$	4.15, 1.16 dB/km (signal, pump) emission coefficient
$\alpha'_{s,p}$	0.5 dB/km (signal and pump) background absorption coefficient
$\eta_{s,p}$	1.42, 0.37 (signal, pump) Ratio of σ_e to σ_a
L	total length of distribution network (propagation distance)
ρ	distributed erbium doping concentration
τ	upper level lifetime
S	fiber core cross-section area
t_k	-20 dB where assumed constant tap fraction (k-th)
$N_2(z,t)$	upper level population (fraction)
$N_1(z,t)$	lower level population (fraction) $= 1 - N_2$
$P_{in}(\lambda_k)$	input power in channel k
B	10 Gbits/s electronic data bit rate
B_o	optical bandwidth $= 2B$

where $\zeta = \rho S/\tau$ is a saturation parameter, $P_{\text{in}}(\lambda_k)$ and $P_{\text{sat}}(\lambda_k)$ are the input and saturation power, respectively, expressed in photons (for channel k)

$$P_{\text{sat}}(\lambda_k) = \frac{\rho S}{\tau(\gamma_k + \alpha_k)} = \frac{\zeta}{\gamma_k + \alpha_k} \tag{5}$$

and $\tilde{\alpha_k}$ is the redefined absorption coefficient

$$\tilde{\alpha_k} = \alpha_k + \alpha'_k + \frac{1}{L}\sum_{m=1}^{M} t_k(z_m) \tag{6}$$

modified by the path-averaged summation of the tap fractions for all the M users of that particular channel.

In practice [6], we overpump to compensate for the transient response of cascaded EDFAs. Under these circumstances, the EDFAs are well described by an unsaturated gain model. Then, the dominant contribution among the N terms on the right-hand side is usually from the pump, which reduces our focus to one term, indexed by p rather than k.

We focus first on the case of uniform and uniformly spaced taps

$$t_k\left(m\frac{L}{M}\right) \equiv t_{s,p} \tag{7}$$

which results in closed-form bounds on the number of users. In a later section, we also consider a simple recursive way of optimizing the tap fractions. Our method of analysis is simple to implement via a short computer program, which can easily account for unequal tap spacings, if so desired.

III. STEADY-STATE BOUNDS

We first investigate the limitations on the size of the distribution network, and therefore consider the steady-state conditions. Following the procedure outlined in [4], we can simplify (4) to

$$\overline{N}_2 = -\frac{1}{L\zeta}P_{\text{in}}(\lambda_p)\left\{\exp\left[\left(\frac{\zeta}{P_{\text{sat}}(\lambda_p)}\overline{N}_2 - \tilde{\alpha_p}\right)L\right] - 1\right\} \tag{8}$$

where, depending upon the physical geometry of the network

$$\tilde{\alpha_p} = \begin{cases} \alpha_p + \alpha'_p + \frac{M}{L}t_p, & \text{pump tapped along with signal;} \\ \alpha_p + \alpha'_p, & \text{otherwise.} \end{cases} \tag{9}$$

The first definition is applicable if power splitters with nonzero drop response at the pump wavelength (e.g., Y-junctions, or weakly coupled resonant structures) are used to tap a fraction of the signal power. Instead, if a bandpass WDM mux–demux is used to separate the pump before the tap, the second definition applies. The solution of the simple transcendental equation (8) enables characterization of each of the channel gains, using

$$\overline{g}_k = \frac{\zeta}{P_{\text{sat}}(\lambda_k)}\overline{N}_2 - \tilde{\alpha_k} = (\gamma_k + \alpha_k)\overline{N}_2 - \tilde{\alpha_k}. \tag{10}$$

An important concept in such a distribution network is signal transparency: we require that the signal power at the output of the chain of amplifiers and taps is the same as it was at the input to the network. We bound the size of network by requiring that the signal-to-noise ratio (SNR) has been degraded to such a level that no further amplification stages can be added. When we allow nonuniform tap fractions, we shall instead require that the final tap fraction be one, i.e., the last receiver can barely make the detection criterion by extracting all of the available signal power [because of the accumulated amplified spontaneous emission (ASE) from previously encountered amplifiers in the signal path].

Requiring transparency may at first seem like a waste of resources—after all, can we not support a further number of users with this signal level? But over the span of a 20–30-km network supporting thousands of users, the signal has already suffered significant ASE accumulation. By definition, we cannot have any more gain stages—we cannot tolerate a further worsening of the SNR [7]. The number of users that can then be supported by a *passive* network with this low SNR is very small indeed—typically not more than a dozen or so [2].

A. Bounds Independent of ASE

Now consider a single signal channel, indexed by s rather than k. The transparency pump power defines unity net gain for the signal channel, or equivalently, the (steady-state) path-averaged exponential gain constant at the signal wavelength $\overline{g}_s = 0$. We can derive a simple condition on the required $\overline{N}_2$, using (10)

$$\begin{aligned}\overline{N}_2 &= \frac{\tilde{\alpha_s}}{\gamma_s + \alpha_s} \\ &= \frac{\alpha_s + \alpha'_s + Mt_s/L}{\gamma_s + \alpha_s}\end{aligned} \tag{11}$$

for uniform tap fractions represented by (7).

Since this represents the *fraction* (≤ 1) of the population in the excited state, the following inequality must be satisfied:

$$\frac{M}{L} \leq \frac{\gamma_s - \alpha_s'}{t_s} = \frac{\eta_s \alpha_s - \alpha_s'}{t_s} \tag{12}$$

which bounds the number of taps (and users) per unit length that can be supported for a given tap fraction.

Furthermore, the pump power required to achieve transparency can be found by substituting (11) into (8), and using (5)

$$P_{\text{in}}(\lambda_p)/P_{\text{sat}}(\lambda_p) = L\frac{(\gamma_p + \alpha_p)(\alpha_s + \alpha_s' + Mt_s/L)/(\gamma_s + \alpha_s)}{1 - \exp\left[\left(\frac{\gamma_p + \alpha_p}{\gamma_s + \alpha_s}\alpha_s - \alpha_p - \alpha_p' - I_{p-\text{tap}}\frac{Mt_p}{L}\right)L\right]} \tag{13}$$

where $I_{p-\text{tap}} \in \{1, 0\}$ is an indicator variable that takes on values depending on whether the pump is tapped along with the signal or not. This equation can also be used to define the maximum serviced length of EDF L for a given pump input power $P_{\text{in}}(\lambda_p)$.

Equations (12) and (13) can be combined to describe an upper bound on the number of receivers that can be supported. We assume the condition in (12) to be satisfied with equality, and substitute in (13) with the assumption that L is large so that the denominator of (13) $\approx$1

$$L^* = \frac{P_{\text{in}}(\lambda_p)/P_{\text{sat}}(\lambda_p)}{\gamma_p + \alpha_p} \tag{14}$$

and consequently

$$M^* = \frac{P_{\text{in}}(\lambda_p)/P_{\text{sat}}\lambda_p)}{t_s}\left(\frac{\gamma_s - \alpha_s'}{\gamma_p + \alpha_p}\right). \tag{15}$$

The validity of this approximation depends, of course, on the numerical values of the various parameters. We will see that for a representative set of numerical values, this is indeed valid. Using the same approximation in (13), if we are given L or M, we can solve for the other

$$L = \frac{1}{\alpha_s + \alpha_s'}\left\{\left(\frac{\gamma_s + \alpha_s}{\gamma_p + \alpha_p}\right)\frac{P_{\text{in}}(\lambda_p)}{P_{\text{sat}}(\lambda_p)} - Mt_s\right\} \tag{16}$$

$$M = \frac{1}{t_s}\left\{\left(\frac{\gamma_s + \alpha_s}{\gamma_p + \alpha_p}\right)\frac{P_{\text{in}}(\lambda_p)}{P_{\text{sat}}(\lambda_p)} - (\alpha_s + \alpha_s')L\right\}. \tag{17}$$

Since both L and M must be positive, we can derive another upper bound

$$\tilde{M} = \frac{P_{\text{in}}(\lambda_p)/P_{\text{sat}}(\lambda_p)}{t_s}\left(\frac{\gamma_s + \alpha_s}{\gamma_p + \alpha_p}\right) \tag{18}$$

which gives the maximum number of users that can be supported (we have not dealt with noise-related bounds yet), and correspondingly

$$\tilde{L} = \frac{P_{\text{in}}(\lambda_p)/P_{\text{sat}}(\lambda_p)}{\gamma_p + \alpha_p}\left(\frac{\gamma_s + \alpha_s}{\alpha_s + \alpha_s'}\right) \tag{19}$$

is the maximum length of EDF that this level of pump input power can support.

The given conditions will determine which form of the constraint is more applicable: if M/L is the starting point, then M^* and L^* are the appropriate bounds. Note that $M^* < \tilde{M}$ and $L^* < \tilde{L}$. However, if we are given either M or L and can trade off a lower receiver density for increased propagation length or number of users, as discussed later, then $\tilde{L}$ or $\tilde{M}$ is what we seek.

B. ASE Bound

Following Desurvire [5, pp. 76–77], the amplifier noise, related to the photon statistics master equation, is defined as

$$N(z) = G(z)\int_0^z \frac{a(z')}{G(z')}\,dz' \tag{20}$$

where

$$a(z) = \gamma_s N_2(z) \tag{21}$$

$$G(z) = \exp\left[\int_0^z \left\{\gamma_s N_2(z') - \alpha_s N_1(z') - \alpha_s' - \frac{M}{L}t_s\right\}dz'\right]. \tag{22}$$

In our analysis, we have dealt with path-averaged quantities, and so an evaluation of $N(L)$ is not possible, particularly in the case of transparency. However, if we assume uniform (but not necessarily complete) inversion because of our overpumping scheme and the subsequent negligible absorption of a high-power 1480-nm pump, we can simplify the above expression. This approximation may not be valid for all network geometries, but in the Massachusetts Institute of Technology (MIT) ONRAMP distribution network, we typically require [6] that the output pump power be much larger than strictly necessary (e.g., several tens of milliwatts). Under such circumstances, from the definition and analysis of the feedthrough ratio defined in [2], our uniform inversion model is quite a good one.

We now account for the nonzero N_1, the lower level population density, where

$$N_2|_{\max} = \frac{\tilde{\alpha_s}}{\gamma_s + \alpha_s} \tag{23}$$

because the gain coefficient is always negative at the pump wavelength. Because $N_1 + N_2 = 1$ and we have normalized the population densities by ρ, the doping concentration along the fiber (number density), the ASE noise power is [7]

$$P_N(L) \equiv N(L)h\nu B_o \tag{24}$$

$$= \left[\left\{\sigma_a(\lambda_s)\rho\Gamma_s\frac{\gamma_s + \alpha_s - \tilde{\alpha_s}}{\gamma_s + \alpha_s} + \alpha_s'\right\}L + Mt_s\right]h\nu B_o$$

$$= \frac{\gamma_s}{\gamma_s + \alpha_s}\left[(\alpha_s + \alpha_s')L + Mt_s\right]h\nu B_o \tag{25}$$

in terms of the optical bandwidth B_o and photon energy $h\nu$.

Now that we have an expression for the noise power, we can define the SNR in terms of the photocurrents generated by a photodiode in response to these incident optical powers. The signal power is, by definition of transparency, the same as at the input. One particular way of evaluating the SNR is due to Personick

[8]; another more sophisticated evaluation, which represents the filtered photodetector output in terms of components along an orthonormal basis over the pulse interval, is due to Humblet and Azizoglu [9]. Both methods are discussed in [2].

C. Numerical Example

We use the numerical values from Table I, and assume that the pump is not tapped along with the signal at each receiver along the fiber, so that the maximum possible utility is gained from a given pump input power. The first bound, given by (12), then implies that the number of receiver stations per kilometer $M/L \leq 244$.

Now we turn to the limit imposed by the limited input pump power available for amplification of the signal, as given by (14). First, we verify that the approximation we made in deriving that relationship, and the ones that followed it, is indeed valid. We want

$$1-\exp\left[\left(\frac{\gamma_p+\alpha_p}{\gamma_s+\alpha_s}\,\alpha_s-\alpha_p-\alpha_p'\right)L\right]\approx 1 \tag{26}$$

and upon substituting in numerical values, we want $1-\exp(-1.75\quad L)\approx 1$, which is satisfied with about 1% or less error if $L \geq 2.6$ km. Because the span of our distribution networks will turn out to be quite a bit longer than this, our approximation is self-consistent.

Substituting in the appropriate numerical values, we see that by ignoring noise constraints, the maximum length $L^* = 0.505 \times P_{\rm in}(\lambda_p)/P_{\rm sat}(\lambda_p)$. If the normalized input pump power $q \equiv P_{\rm in}(\lambda_p)/P_{\rm sat}(\lambda_p) = 100$, we have $L^* = 50.5$ km, and, therefore, $M^* = 244 \times L^* \approx 12\,300$ receivers.

The upper bounds (18) and (19) can be evaluated for, e.g., $q = 100$, $t_s = 10^{-2}$, yielding $M < 30\,700$ and $L < 84$ km. Note that this exceeds the receiver density bound (M^*/L^*), and so the earlier bound is tighter.

A detailed evaluation of the ASE bound, including source code in MATLAB, is presented in [2]. We summarize the results: for two assumed receiver densities of 75 and 200 taps per kilometer (well within the bound of 244), we evaluate the required signal power for a bit error rate of $10^{-9} \approx Q(6)$ in terms of the Q-function.

Following [9], we assume that the photodiode generates electrons following an inhomogeneous Poisson process with rate equal to the square of the field envelope. The total number of electrons y generated over a bit time by the photodiode follows a Laguerre distribution, and the conditional error probabilities may then be evaluated explicitly. We may approximate y as Gaussian, and a target error probability $P_e = Q(\kappa)$ results in a necessary SNR [9]

$$E/N_0 \approx \kappa^2\left(1+\frac{1}{2N_0}\right)+\kappa\sqrt{\mathcal{M}\left(1+\frac{1}{N_0}\right)} \tag{27}$$

where $N_0/2 = P_N/2B_o$ is the power spectral density of the noise source modeled as additive white Gaussian noise and $2E$ is the signal of the "on" pulses in ON–OFF keyed modulation.

These results depend on the dimensionality of the space of finite energy signals with a bandwidth B_o and time spread T, which is about $2B_oT+1$ [10]. For convenience, we assume this is an even number $=2\mathcal{M}$. We have plotted results for two cases: $\mathcal{M} = 16$ in the family of continuous lines, and $\mathcal{M} = 1$ in the family of dotted lines.

As expected, the required signal power increases with L, as shown in Fig. 3 from about -10 dBm at $L = 4$ km to 0 dBm at $L = 20$ km or higher, depending on $\mathcal{M}$. We have also analyzed the effect of decreasing the receiver density from the theoretical maximum to M/L: we increase the maximum propagation length by

$$\Delta L = \frac{q}{\gamma_p+\alpha_p}\,\frac{\gamma_s-\alpha_s'-Mt_s/L}{\alpha_s+\alpha_s'+Mt_s/L}. \tag{28}$$

Using the above numerical values as an example, for an input $q = 100$ and $M/L = 75$ receivers per kilometer instead of the theoretical limit of 244, we have $\Delta L = 19.5$ km.

IV. Nonuniform Taps

The vector of tap locations $[z_m]$ is often a given parameter in a network design problem. Under the assumption of incomplete and uniform medium inversion, we can consider the noise power at the end of a section of EDF of length z

$$\begin{aligned} P_N(z)^{NC} &= \left[\left(\alpha_s'+\alpha_s\frac{\gamma_s-\alpha_s-\dfrac{1}{z}\displaystyle\sum_{m=1}^{M(z)} t_s(z_m)}{\gamma_s+\alpha_s}\right)z\right.\\ &\qquad \left. +\sum_{m=1}^{M(z)} t_s(z_m)\right] h\nu B_o\\ &= \frac{\gamma_s}{\gamma_s+\alpha_s}\left[(\alpha_s+\alpha_s')z+\sum_{m=1}^{M(z)} t_s(z_m)\right] h\nu B_o \end{aligned} \tag{29}$$

where $M(z)$ is the number of taps in $(0, z)$.

If we restrict ourselves to uniformly spaced taps

$$\begin{aligned} z_m &= \frac{m}{M}L, \qquad m = 1, 2, \ldots, M\\ M(z) &= \left\lfloor \frac{z}{L}M\right\rfloor. \end{aligned} \tag{30}$$

We can now obtain $t_s(z)$ by the following recursive process.

For $0 < z \leq z_1$, we can find the noise power from (29)

$$P_N^1(z) = \frac{\gamma_s}{\gamma_s+\alpha_s}\,(\alpha_s+\alpha_s')zh\nu B_o \tag{31}$$

since there are, by definition, no taps before z_1. Using this value in an appropriately chosen SNR constraint calculation such as (27) yields P_1^*, the minimum detectable signal power, when that the noise power is obtained from (31). Therefore, the tap fraction at $z = z_1$ from input signal power P_0 is $t_s(z_1) = P_1^*/P_0$.

For $z_1 < z \leq z_2$, the definition of the noise power must now account for the tap at z_1, which we have just computed

$$P_N^2(z) = \frac{\gamma_s}{\gamma_s+\alpha_s}\,[(\alpha_s+\alpha_s')z+t_s(z_1)]\,h\nu B_o \tag{32}$$

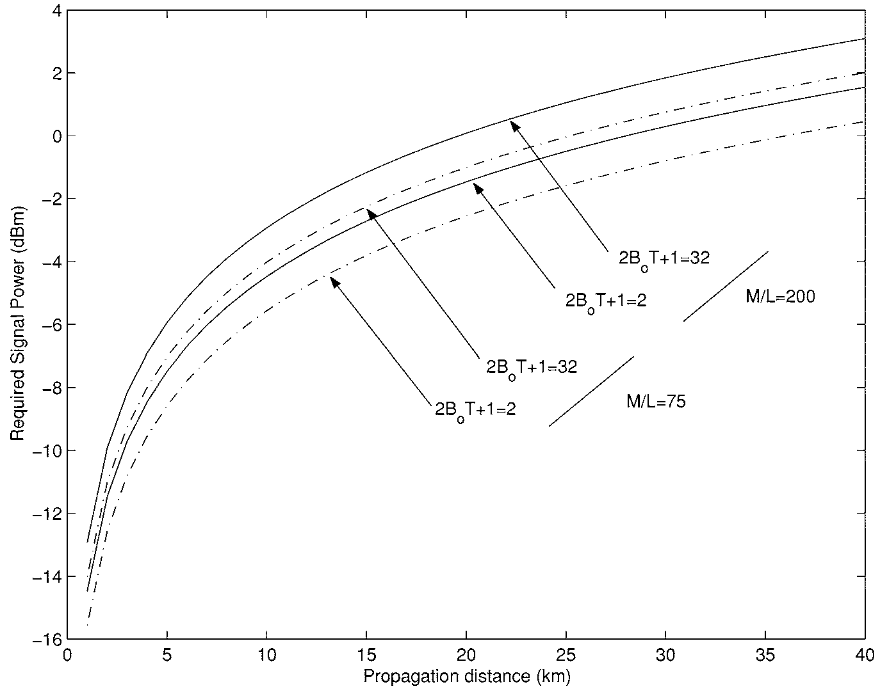

Fig. 3. The maximum length of EDF that can be supported by a given normalized input signal power (or *vice versa*), as dictated by the SNR constraint. Results of a numerical calculation are shown for $\mathcal{M} = 36$ and $\mathcal{M} = 2$, where space–time bandwidth product dimensionality factor $\mathcal{M}$ is defined in the text. The receiver densities are 200 and 75 users per kilometer, within the theoretical maximum allowed of 244 users per kilometer.

and the SNR constraint calculation yields P_2^*, the minimum detectable signal power at the second stage. Therefore, the tap fraction at $z = z_2$ is $t_s(z_2) = P_2^*/P_0$.

Each member of the increasing sequence (of tap fractions) t_s must be less than or equal to one. Therefore, the bound on the number of stations M^* that can be supported given a vector of tap locations $[z_m]$, $m = 1, 2, \ldots, M$ is

$$M^* = \min\left\{\min_{1\leq m\leq M} \{m | t_s(z_m) \geq 1\}\, M\right\} \tag{33}$$

where we assume that $\min[x] = \infty$ if x is an empty set.

We carry out a numerical evaluation of the above algorithm for receivers spaced apart by 10 m along the same EDF we have considered earlier. For a range of input signal powers varying from 0 to -10 dBm, we plot the tap fractions in Fig. 4, which range from a very small value, limited essentially by the receiver sensitivity dominated by thermal noise, to one in a domain where the receiver sensitivity is dominated by ASE-signal beat noise. In carrying out the above calculation, we have used Personick's Q-factor as representative of a SNR threshold. Obviously, a different likelihood ratio test will yield different numerical results, but our conclusions will remain qualitatively the same. Details of three different tests are presented in [2].

Physically, a sequence of nonuniform tap fractions implies that we have designed the couplers along the transmission channel with different coupling coefficients. The initial stages extract only a small fraction of the signal from the channel, and so we need, e.g., a highly asymmetric Y-branch with the receiver connected to the weaker arm. The situation reverses at the end of the network, where we once again need an asymmetric Y-branch but this time with the receiver connected to the stronger arm. At some location near the middle of the network, we need a Y-branch with a 50/50 splitting ratio. Obviously, designing Y-branches exactly according to the prescription of Fig. 4 is difficult, and one may resort to, e.g., a "staircase approximation" that combines practicality with the indications of this theoretical analysis.

V. Discussion

We have analyzed a distributed-amplifier model for remotely pumped bus distribution networks. As mentioned at the outset of this discussion, an analytic model is a useful counterpart to numerical simulations in understanding the capabilities of such networks. A remotely pumped chain of EDFAs offers an attractive, cost-effective solution to the problem of increasing the number of users without incurring severe penalties in terms of power consumption, ease of maintenance, and simplicity of design.

We start with a simple rate-equations model, modified to include the most important effect of users along the network: each user couples, or "taps," a fraction of the signal power out of the main transmission channel. Under some simplifying assumptions, we can obtain simple, closed-form expressions determining the usability of this remote-pumping scheme without blind recourse to computer simulations.

We can design the length of our network to suit a given number of users, or the other way around. Upper bounds on each of the parameters are given by simple relationships in terms of the input power and tap fraction. Also, we can use the receiver density M/L as the starting parameter instead, which may be more appropriate in some applications.

Nonuniform taps permit a lower input signal power to serve the same distribution network (i.e., same length and number of users). Similarly, the number of users, overall length, or receiver

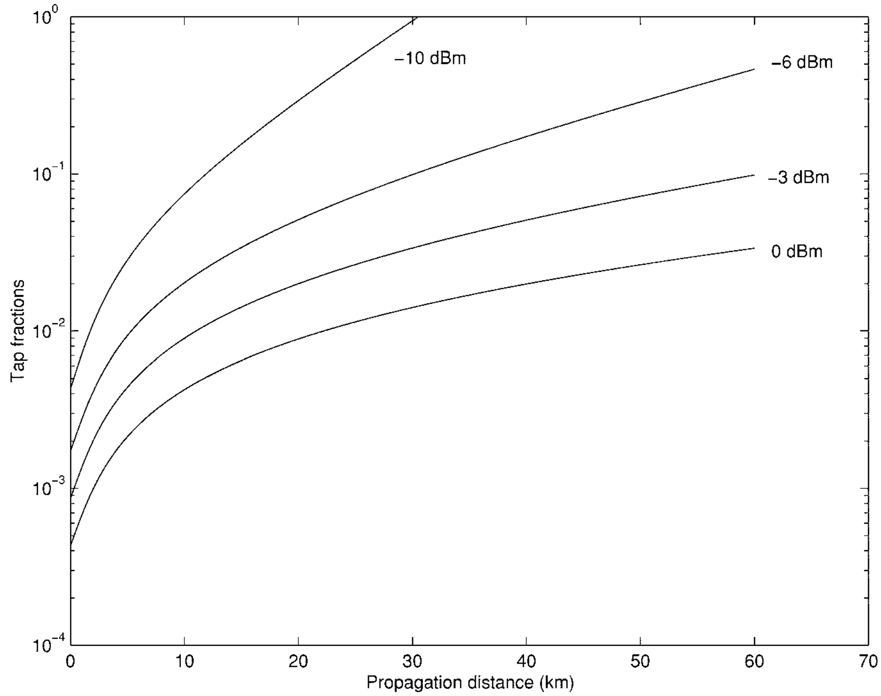

Fig. 4. Tap fractions along the EDF distribution network, for receivers spaced apart by 10 m (receiver density 100 per kilometer with a theoretical maximum of 244 as before), and input signal power $P_0 = -10$ dBm to $P_0 = 0$ dBm. The number of users $= 100\times$ propagation distance (in kilometers), e.g., $= 3000$ for input signal -10 dBm.

density can be increased for the same input signal power. As expected, the tap fractions form an increasing sequence, and we reach the limit on the size of our bus network when the last tap fraction reaches unity.

We have assumed, in our analysis so far, that the tap fraction represents the small fraction of signal power that is necessary for detection. In [2], we show that the same sequence of mathematical steps can be applied to a different interpretation: t_k now represents a division of the signal (and pump) power into two or more equal parts. The physical structure that a sequence of such operations results in is called a distribution tree, which is analyzed in the same framework as the bus network, but with a higher order-of-magnitude scale for the tap fractions.

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