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On the convection velocity of turbulent structures in supersonic shear layers

by,

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Abstract

An ansatz, complemented by appropriate selection rules, is proposed to estimate the convection velocity U_c of turbulent vortical structures in supersonic shear layers. The proposed scheme assumes that, for supersonic convective Mach numbers, shocks will form in one of the two shear layer free streams. The strength of the shocks is estimated by assuming that the flow configuration is stationary with respect to perturbations in the mean flow quantities caused by the turbulent fluctuations. Given the shock strength, the convection velocity U_c and the associated convective Mach numbers are calculated by matching the estimated total pressure at the stagnation points in the convected frame. The data indicate a convection velocity U_c that is close to that of one of the free streams. That appears to be well accounted for by the proposed scheme, which also suggests that the flow can undergo large jumps in configuration with small changes in the flow parameters. This has important implications for supersonic mixing and hypersonic propulsion applications.

Introduction

Recent experiments in compressible flow shear layers indicate that the convection velocity U_c of turbulent structures in supersonic shear layers is much closer to the high or low speed free stream velocities, U_1 or U_2, respectively, than has been found to be the case in subsonic shear layers (Papamoschou 1989, Fourguette & Dibble 1990, Hall *et al.* 1990). Such asymmetric behavior suggests, in turn, that the entrainment rate of free stream fluid into the turbulent mixing region can potentially be expected to be greatly asymmetric.

An important attribute of the turbulent structure convection velocity is the proposed role it plays in the shear layer entrainment process. Specifically, it was proposed that E_v, the volumetric entrainment ratio in spatially growing mixing layers, *i.e.* the entrained volume (flux) of high speed fluid fluid per unit volume flux of low speed fluid*, can be estimated as

$$E_v \approx \frac{U_1 - U_c}{U_c - U_2}\left(1 + \frac{\ell}{x}\right) , \tag{1}$$

where ℓ/x is the local large scale structure streamwise extent to position ratio (Dimotakis 1984). The large structure streamwise extent ℓ is, in turn, expected to be of the order of the local transverse extent (visual thickness) of the layer δ, *i.e.*

$$\frac{\ell}{x} \approx C_\ell \frac{\delta}{x} , \tag{2a}$$

with subsonic experiments yielding a value for the constant of proportionality of

$$C_\ell \approx 2 . \tag{2b}$$

It was recently suggested (Clemens & Mungal 1990) that the fact that compressible shear layers do not appear to be characterized by two-dimensional, spanwise coherent structures may render the validity of the use of the expression in Eq. 1, for example, questionable. To address this issue, a brief review of the arguments that lead to this expression may be useful.

The first factor in Eq. 1 derives from the induction velocity ratio and scales the relative shear sustained between the turbulent structures and the corresponding free stream. No relative velocity, no shear, no entrainment**. Under supersonic flow conditions and the possible presence of shocks on one side of the turbulent structures or the other (but not both), the symmetry expected under subsonic flow conditions, with respect to the high and low speed stream in the vortical structure convecting frame, would be lost. Nevertheless, the relative velocity ($\approx$ shear) ratio should come close to scaling the volumetric entrainment ratio.

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* Note that, given the volumetric entrainment ratio E_v, the mass flux entrainment ratio would be given by $E_m = (\rho_1/\rho_2)\, E_v$, while the molar entrainment ratio would be given by $E_n = (T_2/T_1)\, E_v$.

** It should be recognized that this may not represent a consensus opinion. See, for example, discussion in Ferri *et al.* (1962) and Ferri (1973).

The second factor in Eq. 1 is a consequence of the geometry of the spatially growing layer and of the large scale structures that dominate the entrainment process. It should be noted that the large scale flow structures are assumed to be basically vortical, not necessarily two-dimensional, for the second factor to represent a reasonable estimate of this effect. The second factor plays an important role in subsonic shear layers and, indeed, accounts for the observed asymmetries in E_v for the case of equal free stream density ($\rho_1 = \rho_2$) subsonic shear layers, for which $U_1 - U_c \approx U_c - U_2$. Nevertheless, it is not expected to contribute to asymmetries that are as significant, as the convective Mach numbers of the flow rise. This is a consequence of the likely dependence on the ratio of the flow structure size to the streamwise coordinate and the decrease in the growth rate δ/x with increasing convective Mach number, as documented by Papamoschou & Roshko (1988) and others (*cf*. Eq. 2). See also discussion in Dimotakis (1989).

One may conclude that the considerable asymmetries in U_c with respect to the free stream velocities that have been documented should be expected to be responsible for corresponding asymmetries in the volumetric entrainment ratio E_v. This has important consequences for supersonic mixing and combustion applications, with the resulting stoichiometry of the mixed fluid potentially substantially different than what would be predicted on the basis of conventional models of turbulent entrainment and mixing.

For subsonic shear layers, experimental data and computations support the proposal that the convection velocity can be estimated by matching the total pressures realized, from each of the free streams, on the stagnation points in between the large scale vortical structures in the convective frame. The experimental data also support the notion that the respective stagnation pressures can be estimated by applying the Bernoulli equation for each stream (Dimotakis 1984, Coles 1985), *i.e.*

$$p_1 + \frac{1}{2}\left[U_1 - U_c^{(i)}\right]^2 = p_2 + \frac{1}{2}\left[U_c^{(i)} - U_2\right]^2 . \tag{3a}$$

At higher, but still subsonic, convective Mach numbers, the convection velocity can be estimated by using the corresponding compressible isentropic pressure recovery relations (Bogdanoff 1983, Papamoschou & Roshko 1988, Dimotakis 1989), *i.e.*

$$p_1 \left[1 + \frac{\gamma_1 - 1}{2}\left[\frac{U_1 - U_c^{(i)}}{a_1}\right]^2\right]^{\frac{\gamma_1}{\gamma_1 - 1}} = p_2 \left[1 + \frac{\gamma_2 - 1}{2}\left[\frac{U_c^{(i)} - U_2}{a_2}\right]^2\right]^{\frac{\gamma_2}{\gamma_2 - 1}} . \tag{3b}$$

The quantity $U_c^{(i)}$ in these expressions denotes the convection velocity, estimated assuming matched free stream static pressures, *i.e.* $p_1 \approx p_2$, and an equal fraction of the isentropic total pressure recovered from each stream at the convected stagnation points, as above.

It may be interesting to ask for input on this issue from linear stability analyses of this problem, with the appreciation that finite amplitude wave effects, such as the loss in total pressure attendant on entropy production in shock waves, cannot properly be captured by linear stability analysis. Nevertheless, the very small entropy generation from weak oblique shocks, as would be expected under many flow conditions, might render linear stability analysis results useful for convective Mach numbers that are not too high.

Both temporal and spatial stability analyses have appeared recently, for both free (unbounded) flow (*e.g.* Jackson & Grosch 1988, 1989, 1990; Ragab & Wu 1989a, 1989b; Sandham & Reynolds 1989a, 1989b, 1990; Zhuang *et al.* 1988) and bounded flow (*e.g.* Tam & Hu 1988; Tam & Hu 1989; Zhuang *et al.* 1989). Unfortunately, no consensus exists as to how the convection velocity of the flow structures is to be estimated using linear stability analysis results. Some investigators have suggested that the phase velocity of the most unstable mode can be used to provide an estimate (*e.g.* Zhuang *et al.* 1988), while others have used the phase velocity at the neutral point of the solution branch of the most unstable mode (Sandham & Reynolds 1989)[†]. These analyses suggest that, under supersonic flow conditions, an unbounded shear layer can support more than one mode, *i.e.* a "fast mode" with a convection velocity U_c higher than the isentropically estimated value $U_c^{(i)}$ (Eq. 3b), and a "slow mode" with $U_c < U_c^{(i)}$. Sandham & Reynolds (1989, Fig. 2.25) have made a comparison of the convective Mach numbers estimated in this fashion with the data of Papamoschou (1989) for temporally growing, unbounded shear layers. The agreement at low convective Mach numbers is quite good. At higher convective Mach numbers, however, the linear stability analysis calculations underestimate the departure of the convection velocity from the isentropically computed values (Eq. 3b). This is as one would perhaps anticipate, *i.e.* in keeping with the caveat that entropy (and total pressure) losses cannot be ignored at high (convective) Mach numbers, where finite amplitude wave effects are expected.

The situation in bounded two-dimensional shear layers is more complicated, with many more unstable modes possible, some with $U_c > U_c^{(i)}$ and some with $U_c < U_c^{(i)}$. It is also not clear, in this case, how the convection velocity would be estimated using the neutral point phase velocity proposal of Sandham & Reynolds (1989), as the dispersion relation solution branch of the most unstable mode typically crosses those of many other modes before reaching neutral stability.

Finally, while an *adequate* numerical calculation of this flow is perhaps not beyond present computational means, the need to explore the possibility of a simple description is clear.

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† It should be noted, however, that, for unbounded flows and as long as the convective Mach numbers are low, the differences in the actual values derived using these different estimates are not large.

The convection velocity in the presence of shocks

To extend the estimation of the convection velocity to higher flow Mach numbers we must recognize that the (convective) Mach numbers, M_{c1} and M_{c2}, corresponding to the relative velocity of each of the free streams in the convective frame of the turbulent structures, *i.e.*

$$M_{c1} = \frac{U_1 - U_c}{a_1} \quad \text{and} \quad M_{c2} = \frac{U_c - U_2}{a_2} , \tag{4}$$

in which a_j denotes the speed of sound in the corresponding free stream (Papamoschou & Roshko 1988), can approach, or exceed, unity. Under those conditions, the flow can support shocks across which the loss in total pressure may no longer be negligible. It can then be expected that the isentropic assumptions that can be used to estimate the total pressure and, by extension, the convection velocity U_c at lower Mach numbers, will fail.

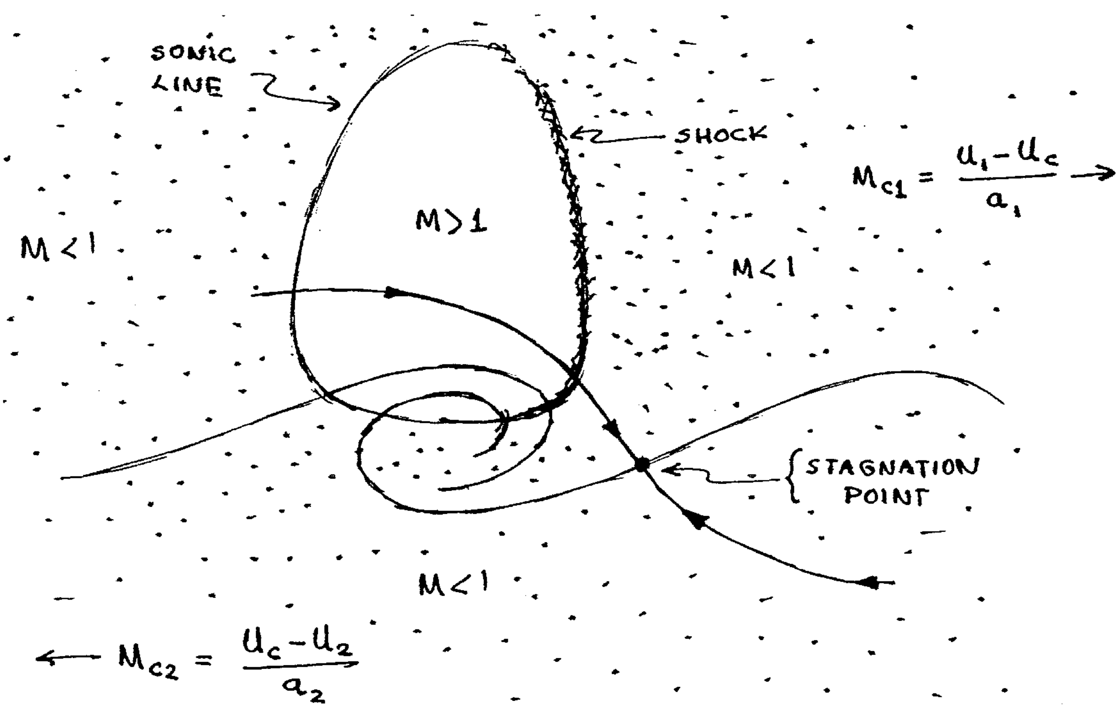

FIG. 1a Proposed vortex/shock configuration, sketched for a shock borne by the high speed stream and a transonic convective Mach number ($M_{c1} < 1$).

It has been argued that, to the extent that shocks can be borne on one side of the layer only, the large associated asymmetric losses in total pressure can be responsible for the large asymmetries in the observed behavior of the convection velocity U_c of the turbulent structures (Papamoschou 1989, Dimotakis 1989). Evidence for the formation of shocks can be found in the calculations of Lele (1989), for example, for transonic convective Mach numbers where one expects weak shocks (dubbed "shocklets") confined to the vicinity of the shear zone. See Fig. 1a. No experimental evidence for these transonic shocklets is available at this writing. For supersonic convective Mach numbers, experimental evidence for turbulent structure-generated shocks from the core region of supersonic jets has been documented by Lowson & Ollerhead (1964) and Tam (1971), and, in a two dimensional shear layer, more recently by Hall *et al.* (1990). In the Hall *et al.* experiments, a shock/expansion wave system extending into one of the free streams, as sketched in Fig. 1b, was documented.

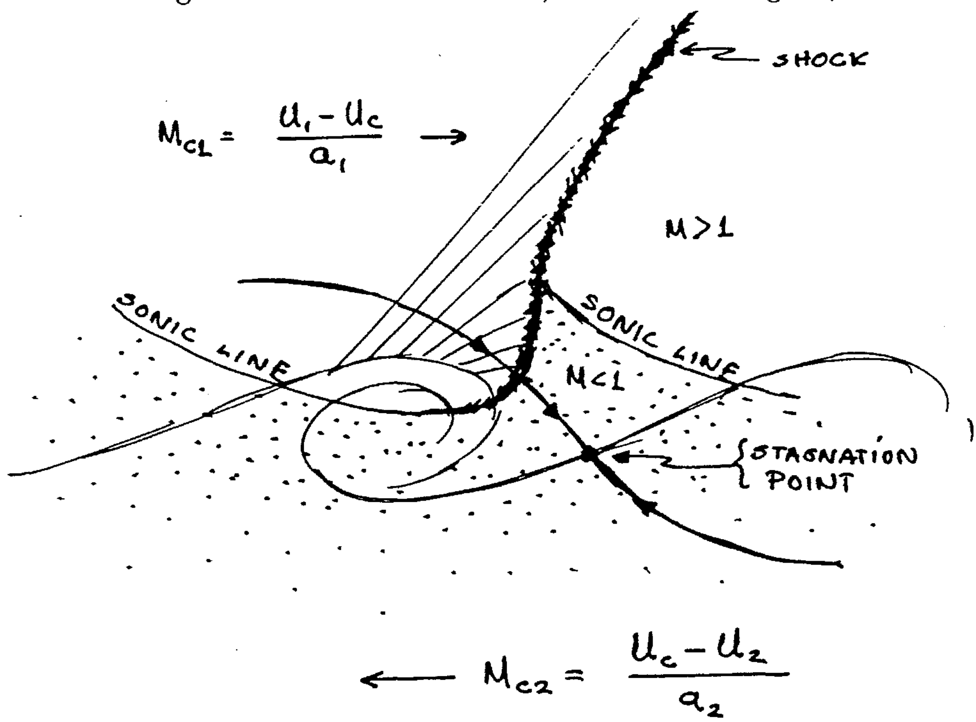

FIG. 1b Proposed supersonic vortex/shock configuration, sketched for a supersonic convective Mach number ($M_{c1} > 1$).

The difficulty with performing *ab initio* calculations of the convection velocity, including the effects of shocks, is that the results depend on the shock Mach number, M_s, corresponding to the normal velocity component before the shock, which cannot be estimated *a priori* (Papamoschou 1989). It was suggested (Dimotakis 1989) that useful estimates could be made, at least of the qualitative behavior, by assuming that the normal shock Mach number could be approximated by the convective Mach number with respect to the corresponding free stream. Unfortunately, this assumption does not, in fact, explain the observed behavior‡. In the case of shocks, the problem of estimating the ratio X_j of the normal shock Mach number to the corresponding free stream Mach number in the j^{th} free stream, *i.e.* $X_j = M_{sj}/M_{cj}$, where the subscript j denotes the stream that carries the shock(s), requires additional information.

‡ A coding error in the implementation of that proposal (Dimotakis 1989) masked the actual consequences of that assumption, yielding estimates for the convection velocity which happened to be qualitatively close to observations.

Estimating the shock Mach number

In what follows, it will be assumed that the fundamental turbulent structure in supersonic shear layers remains basically vortical. The presently available evidence suggests that the two-dimensional (spanwise coherent) structures of Brown & Roshko (1974) are not the prevalent mode under supersonic flow conditions. See, for example, discussion in Clemens *et al.* (1990) and Clemens & Mungal (1990), but also in Tam & Hu (1989) and Zhuang *et al.* (1990). Nevertheless, there is also evidence that the structure that is there is not small scale, with a typical streamwise extent that is of the order of the local shear layer width (Clemens *et al.* 1990, Fourguette & Dibble 1990, Hall *et al.* 1990). The proposed model will also be implemented assuming that the flow can be treated as unbounded, ignoring, in other words, any influence on the convection velocity of the turbulent structures exerted by the presence of the guidewalls employed to confine the supersonic shear flow.

In the case where the flow can support a shock in one of the two free streams, the flow ahead of the shock would be turned *via* an (almost) isentropic expansion before crossing the shock to enter the subsonic region in the neighborhood of the stagnation point ahead of the vortical structure. Two possibilities arise. For transonic convective Mach numbers, a supersonic bubble can exist in the vicinity of the vortical structure, as on the lifting side of a transonic airfoil, with a shock wave ahead of the stagnation point joining the sonic line to enclose the bubble. See Fig. 1a. For supersonic Mach numbers, the region of supersonic flow — and the shock/expansion wave system — will extend to the far field, as noted in Fig. 1b.

As the velocities behind the shock(s) must be low — in fact zero at the stagnation point in the convective frame — the total pressure loss should be well approximated by that of a normal shock. In that case, it should be possible to estimate the total pressure realized at the stagnation point, using the Rayleigh pitot tube formula (*e.g.* Liepmann & Roshko 1957, p. 149). With these assumptions, the strength of the shock can be estimated if the angle $\Delta\theta$ through which the flow has been expanded is known. The turning angle $\Delta\theta_j$ in the j^{th} stream can be estimated, in turn, as the difference of the corresponding Prandtl-Meyer angles between the flow just ahead of the shock and the free stream (or sonic conditions), *i.e.*

$$\Delta\theta_j = \theta_{\text{PM}}(M_{sj}) - \theta_{\text{PM}}(M_{cj}) , \qquad \text{for} \qquad M_{cj} \geq 1 , \tag{5a}$$

where,

$$\theta_{\text{PM}}(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\,\text{atan}\left[\sqrt{\frac{\gamma-1}{\gamma+1}}\,\left(M^2-1\right)\right] - \text{atan}\sqrt{M^2-1} \tag{5b}$$

defined for $M > 1$, is the Prandtl-Meyer angle function (*e.g.* Liepmann & Roshko 1957, p. 99). If the convective Mach number M_{cj} in the j^{th} stream is close to, but less than, unity (transonic M_{cj}), the turning angle $\Delta\theta_j$ will be computed using

$$\Delta\theta_j = \theta_{\text{PM}}(M_{sj}) \ , \qquad \text{for} \qquad M_{cj} < 1 \ . \tag{5c}$$

The latter is equivalent to starting the calculation at the location where the streamline crosses the sonic line to enter the supersonic bubble. See Fig. 1a.

Depending on the flow parameters, the pressure matching condition can lead to several solution branches. Given the free stream that carries the shock and the shock strength, several branches will typically exist, with a continuum of solutions for the convection velocity U_c as a function of the shock strength parameter X. The ansatz proposed here is that *the convection velocity of the large scale structures is such as to render the flow stationary.* One can argue for this conjecture by noting that if the flow structure depicted in Figs. 1a,b is to represent a quasi-steady, convecting flow configuration, it must survive the small scale turbulent fluctuations which can be regarded as continuously disturbing it.

When the flow is computed as a function of the shock strength parameter $X_j = M_{sj}/M_{cj}$, corresponding to a shock in the j^{th} stream, one finds that the solution branches fall into two classes. In the first solution class, Type I flow, the turning angle $\Delta\theta$ can be computed by assuming that the flow chooses the stream j and the shock Mach number, *i.e.* shock strength parameter $X_j = M_{sj}/M_{cj}$, so as to render the turning angle $\Delta\theta_j$ stationary (a maximum). This corresponds to a stable flow configuration wherein small changes in the shock Mach number M_{sj} result in quadratically small changes in $\Delta\theta_j$. Alternatively, in Type II solutions, it is the shock strength parameter X_j that can be stationary with respect to small changes in the turning angle $\Delta\theta$, corresponding to the maximum admissible value for X_j that yields a solution for U_c.

Satisfying the pressure matching condition as a function of the convection velocity U_c classifies the solutions as Type I or Type II. It is found, however, that both types of solutions can be admissible (in the same flow). In the latter case, one can argue for a selection rule which favors the Type I branch, over the Type II solution branch, as being the more robust configuration of the two. If more than one solution branch of the same type is possible, the proposed selection rule is that the branch that yields the *lower* total pressure is chosen by the flow. In other words, the flow will try to satisfy the pressure matching condition at the lowest stagnation pressure possible, generating the shock with the requisite strength.

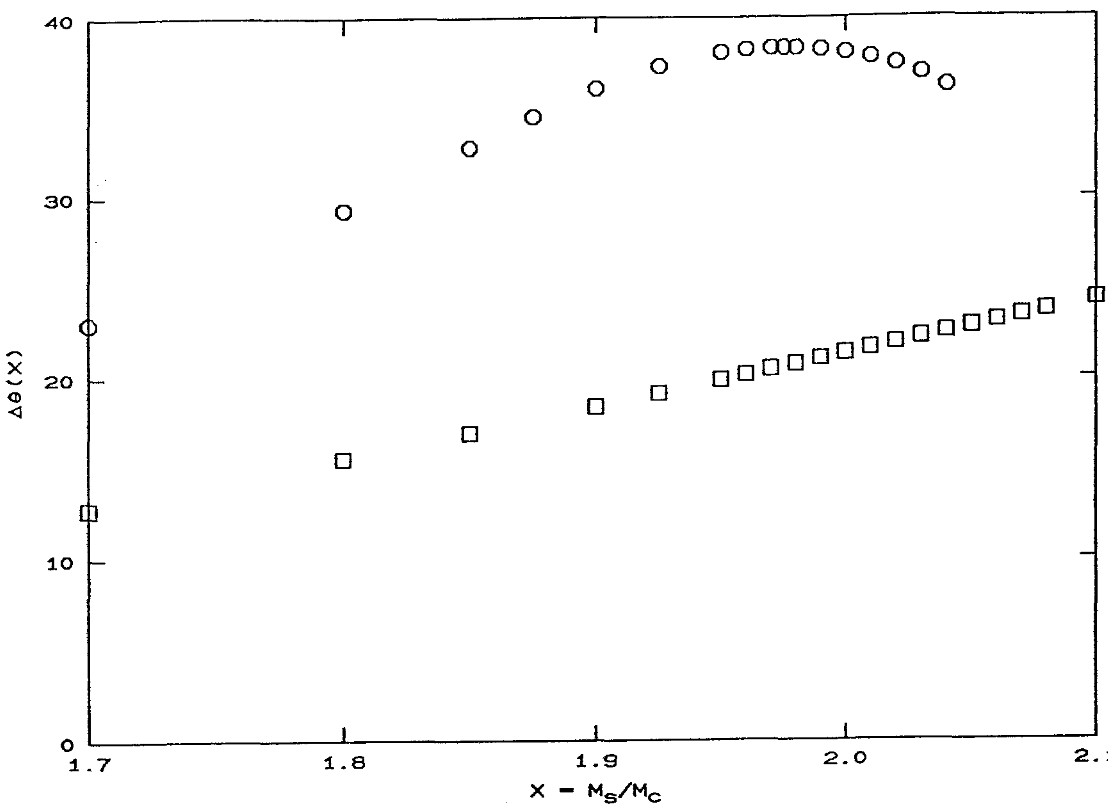

FIG. 2 Flow turning angle $\Delta\theta$, as a function of the shock strength parameter $X = M_s/M_c$, assuming the shock is borne either by the high speed stream (squares) and low speed stream (circles). Supersonic shear layer: $M_1 = 1.5\,[He]$, $M_2 = 0.3\,[N_2]$ (Hall *et al.* 1990). This can be seen to be Type I flow, yielding a stationary solution ($\max\{\Delta\theta\}$) with a shock in the low speed stream ($j = 2$) at a shock strength parameter value of $X_2 = 1.975$ (see text).

Figure 2 depicts the results of sample calculations of $\Delta\theta$ as a function of the shock strength parameter X, for a supersonic shear layer with $M_1 = 1.5\,[He]$ and $M_2 = 0.3\,[N_2]$ (Hall *et al.* 1990). In this figure, the squares were computed assuming that a shock is present in the high speed stream, while the circles were computed for a shock in the low speed stream. It can be seen that, for these flow parameters, the solution corresponds to a stationary point in which the turning angle is a maximum, *i.e.* Type I flow, with a shock borne by the low speed stream ($j = 2$) and a shock/convective Mach number ratio of $X_2 = 1.975$.

Figure 3 depicts the total pressure from each of the free streams, computed for this value of the shock strength parameter X_2. The dotted lines represent the isentropic pressure recovery from each free stream, while the solid lines represent the recovery pressure assuming shocks. The small vertical dashed line segments mark the free stream velocities $U_1 = 1160\,\mathrm{m/s}$ and $U_2 = 105\,\mathrm{m/s}$, which are the limits of the convection velocity U_c. The

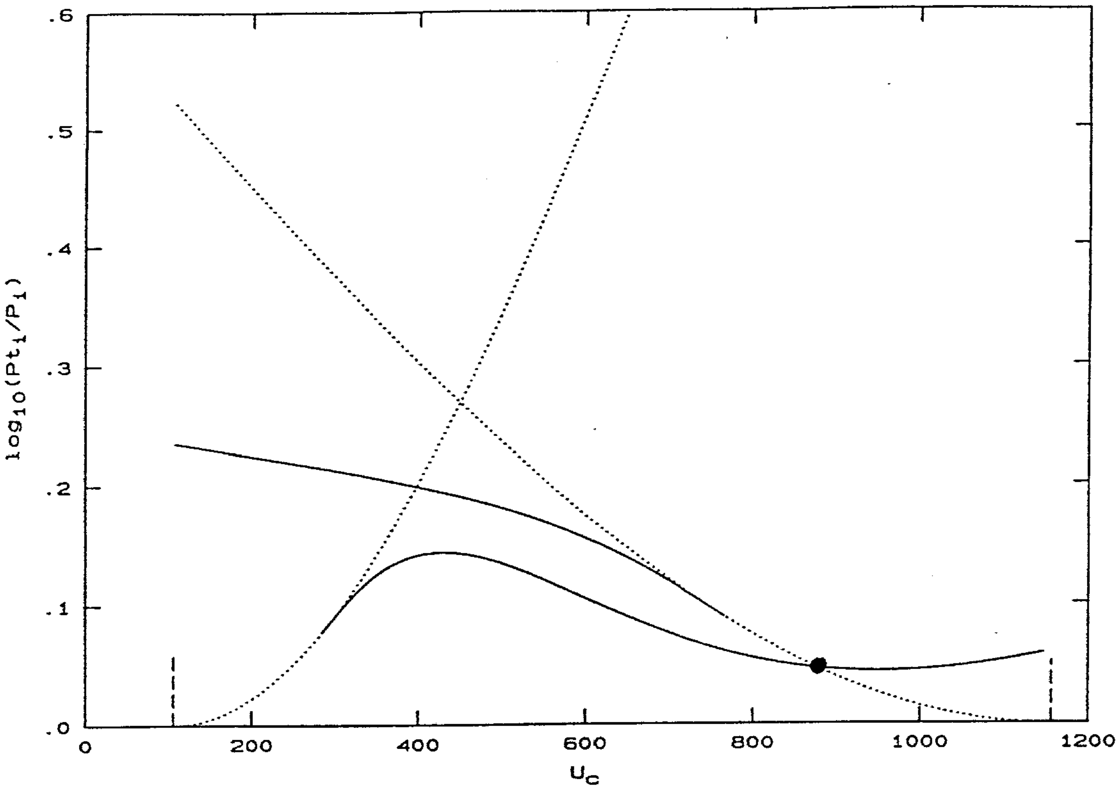

FIG. 3 Logarithm of total pressure from each stream as a function of the convection velocity U_c. Dotted lines represent isentropic recovery. Solid lines represent pressure recovery through shocks. Computed for flow conditions $U_2/U_1 = 105/1160\,\mathrm{m/s}$ and a low speed stream shock strength parameter value of $X_2 = 1.975$ (see Fig. 2). Solution of $U_c = 878\,\mathrm{m/s}$ is indicated by filled dot.

solution point is indicated by the filled dot, yielding an estimate for the convection velocity of $U_c = 878\,\mathrm{m/s}$, which is much closer to the high speed stream velocity, and convective Mach numbers of $M_{c1} = 0.36$ and $M_{c2} = 2.2$. These values are in good agreement with the Hall *et al.* (1990) observations of $2.1 \leq M_{c2} \leq 2.4$ for this flow, based on the shock angles in their schlieren flow visualization data. In contrast, the convection velocity, as estimated from the isentropic relation for this flow, is given by $U_c^{(i)} = 449\,\mathrm{m/s}$, corresponding to a pair of much more closely matched (isentropically estimated) values of the convective Mach numbers ($M_{c1}^{(i)} = 0.917$ and $M_{c2}^{(i)} = 0.983$).

Figure 4 is computed for a supersonic shear layer with a N_2, $M_1 = 2.8$, high speed stream and Ar, $M_2 = 2.6$, low speed stream (Papamoschou 1989), as a second example. It can be seen that, in this case, there are two stationary points possible, corresponding to maxima in the shock strength parameter with respect to the flow turning angle (Type II solutions). One derives from a shock wave in the high speed stream ($j = 1$) and a maximum in the shock strength parameter at $X_1 = 3.295$, while the other corresponds

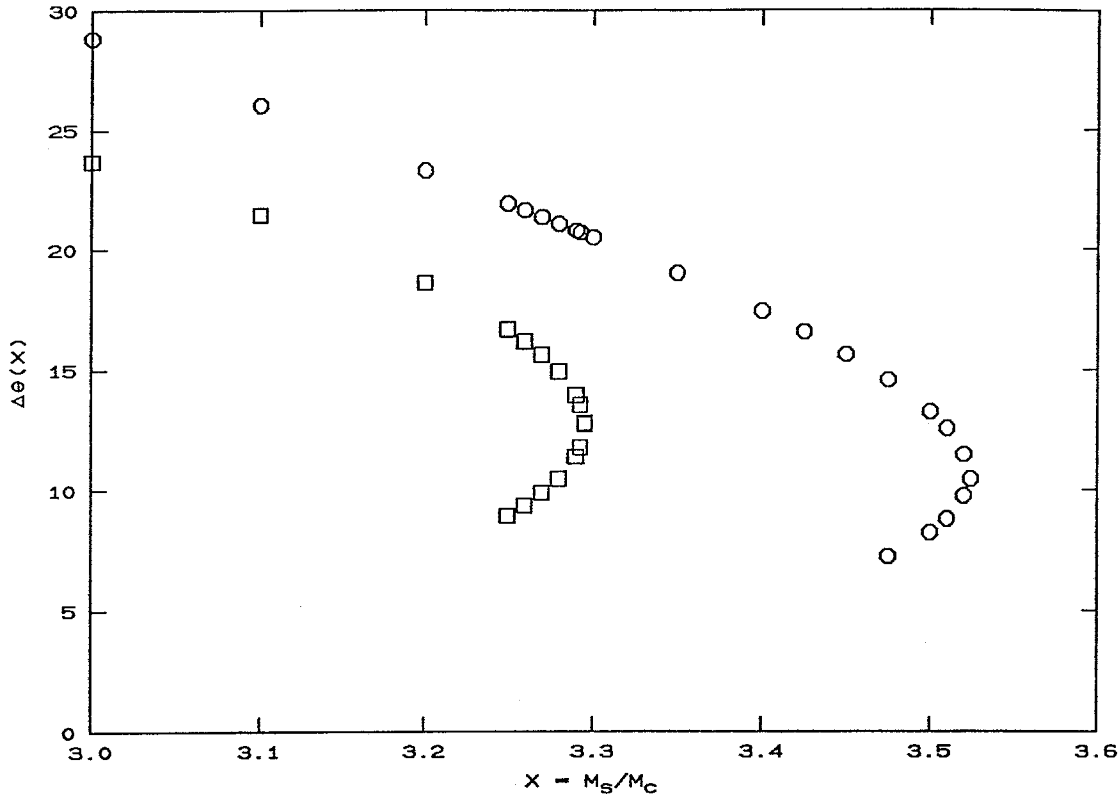

FIG. 4 Flow turning angle $\Delta\theta_j$, as a function of the shock strength parameter $X_j = M_{sj}/M_{cj}$. Supersonic shear layer: $M_1 = 2.8\,[N_2]$, $M_2 = 2.6\,[Ar]$. High speed stream shock denoted by squares and low speed stream shock by circles. Solution corresponds to the stationary point for a shock in the high speed stream ($j = 1$) at $X_1 = 3.295$ (Type II flow).

to a shock in the low speed stream ($j = 2$) and a shock strength parameter of $X_2 = 3.524$. Of these two, the solution with the shock in the high speed stream yields a lower stagnation (total) pressure ($p_t/p = 1.069$, *vs.* $p_t/p = 1.086$) and is the one accepted by the minimum stagnation pressure selection rule. This is also a transonic flow case (Fig. 1a), yielding values for the convective Mach numbers of $M_{c1} = 0.47$ and $M_{c2} = 0.28$ differing by almost a factor of two, in good agreement with the values of 0.48 and 0.26, respectively, reported by Papamoschou (1989). This represents an interesting flow. The isentropically estimated convective Mach numbers are, again, much closer to each other ($M_{c1}^{(i)} = 0.40$ and $M_{c2}^{(i)} = 0.36$) than the experimentally observed values. More significantly, they are rather low at these flow conditions. One might have argued that one should not expect any finite amplitude wave effects to speak of. Nevertheless, a Type II stationary solution exists with a rather large shock strength parameter ($X_1 = M_{s1}/M_{c1} \approx 3.3$), which the experiments suggest the flow has indeed availed itself of. Even so, the actual convective Mach numbers are in the transonic regime (Fig. 1a) with a shock Mach number estimated as $M_{s1} = X_1\, M_{c1} \approx 1.5$. The convective Mach numbers for this flow were also reasonably

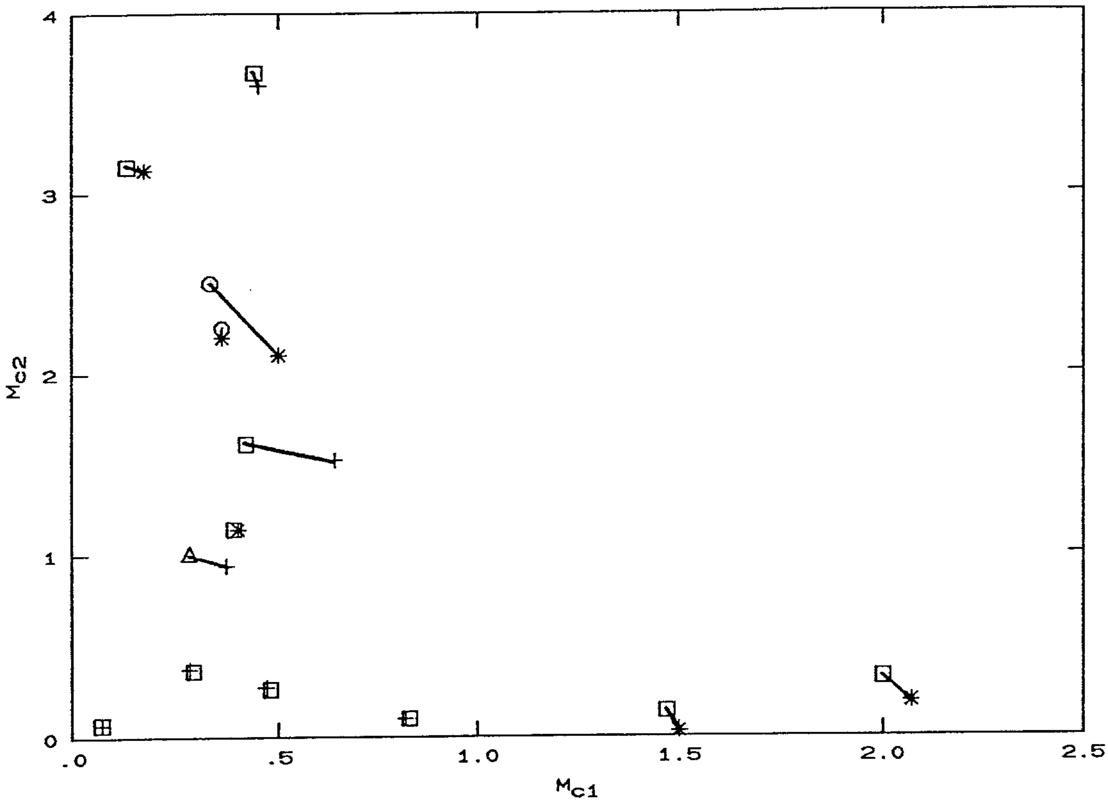

FIG. 5 Experimental data of (M_{c1}, M_{c2}) from Papamoschou (1989, squares), Hall *et al.* (1990, circles), and Fourguette & Dibble (1990, triangle). Computed points are joined to corresponding flow data points by straight lines, corresponding to Type I (asterisks) and Type II (crosses) flows.

well accounted for by the Sandham & Reynolds (1989, Fig. 2.25) stability analysis results.

The results of calculations based on the proposed scheme are summarized in Fig. 5, which is a composite plot of the (M_{c1}, M_{c2}) Papamoschou (1989) data (squares), the Hall *et al.* (1990) data (circles), and the data point (triangle) by Fourguette & Dibble (1990)[♯]. The estimates, based on the proposed scheme, for flows found to yield Type I solutions are denoted by asterisks, while those corresponding to Type II solutions are denoted by crosses. If the computed values are found to fall outside the extent of the experimental data point symbols, they are joined to the corresponding data points by straight lines.

♯ The point (M_{c1}, M_{c2}) derived from the Fourguette & Dibble data was computed using the quoted (directly measured) value for the convection velocity of $U_c = 352\,\mathrm{m/s}$.

Discussion and conclusions

It would appear that the proposed ansatz of a shock strength that renders the flow stationary, coupled with the two selection rules, *i.e.* of Type I over Type II branches (if both are present) and the choice of the solution that yields the lower stagnation pressure (if more than one solution is possible), correctly accounts for the stream that carries the shocks and provides reasonable quantitative estimates for the observed values of the convective Mach number. The proposal of stationarity for supersonic flow is also interesting in that it has no counterpart in subsonic, isentropic flow. The latter has no additional free parameter in satisfying the pressure matching condition.

It should be noted that it is more than likely that the flow may be characterized by hysteresis effects. As flow conditions may gradually change from one set of parameters to another, the flow configuration and when the jumps would occur is likely to depend on the history of these changes. If the shocks are borne by one of the two free streams, for example, it is likely that they will persist in the same stream beyond the point where a different solution, in which, say, the shocks are borne by the opposite free stream, may be indicated at the new flow conditions.

Some of the implications of these results can be appreciated in the context of the discussion on shear layer entrainment outlined in the introduction. These points will be illustrated using the values derived from the experimental data and the results of these calculations for the Hall *et al.* (1990) supersonic shear layer data ($M_1 = 1.5\,[He]$, $M_2 = 0.3\,[N_2]$), as an example. For this shear layer, we might have predicted a relative velocity ratio, based on isentropic estimates*, of

$$\frac{U_1 - U_c^{(i)}}{U_c^{(i)} - U_2} \approx 2.1 \; . \tag{6a}$$

Instead, we have

$$\frac{U_1 - U_c}{U_c - U_2} \approx 0.36 \; , \tag{6b}$$

using the convection velocity estimate of $U_c \approx 880\,\mathrm{m/s}$ that is suggested by the data and also derived using the scheme proposed here. Ignoring, for the moment, the near unity $(1 + \ell/x)$ factor in Eq. 1, stemming from the spatial growth of the layer, this implies that such a layer, rather than being *high speed fluid rich* with a mean volumetric mixture ratio of high speed fluid to low speed fluid of roughly 2:1, can be expected to be *low speed fluid rich* with a volumetric ratio of roughly 1:3. There is almost a factor of 6 difference between the two estimates. Restoring the spatial growth factor in these calculations would result in small changes in the individual estimates, but would not substantially alter their ratio.

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* Recall Eq. 1 and related discussion and that $M_{c1}^{(i)}/M_{c2}^{(i)} = 0.917/0.983$ for this flow (see p. 10).

Accepting the proposed scheme and the proposals on entrainment that were outlined in the introduction at face value leads to some interesting implications for practical applications. Specifically, in addition to the asymmetries in entrainment that the data have already suggested should be anticipated, the proposed model further suggests that even small changes in the free stream parameters may be responsible for changing the stream that carries the shocks, under some flow conditions. Under these circumstances, gradual changes in the flow parameters are predicted to be potentially responsible for jumps in the flow configuration. In some cases, such large changes (jumps) in the flow configuration are predicted to occur as a result of only small changes in flow velocity, composition, or stagnation temperature in one of the free streams. Such jumps would be responsible for correspondingly large changes in entrainment and, in turn, changes of the composition, chemical environment, chemical product formation and heat release in a combusting shear layer.

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