# LETTERS TO THE EDITOR

*Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twenty-eighth of the preceding month; for the second issue, the thirteenth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.*

## One-Dimensional Problems in Quantum Mechanics

In studying the results to be obtained from the approximation of potential functions by step-functions a way of treating the one-dimensional problem has been found which seems somewhat unusual. That it is altogether new is doubtful, but I have not been able to locate it in any of the familiar treatises on differential equations. The type form of Schrödinger's equation is

$$(d^2\psi/dx^2) + 4\pi^2\alpha^2(x, E)\psi = 0 \qquad (1)$$

where $\alpha^2(x,E) = 2m[E - V(x)]/h^2$. If $\alpha = \text{const.}$ then $\psi$ is given by

$$\psi = c_1 e^{+i2\pi\alpha x} + c_2 e^{-i2\pi\alpha x} \qquad (2)$$

where $c_1$ and $c_2$ are constants. If $\alpha$ is not constant then we can try to solve (1) by treating $c_1$ and $c_2$ as well as $\alpha$ as functions of $x$. This is not quite the usual variation-of-constants method since we put a function of $x$ in the exponential as well as vary the $c$'s. The point to the scheme is that one can find differential equations for $c_1(x)$ and $c_2(x)$. These are

$$\begin{aligned}(dc_1/dx) &= A_1(x)c_1 + A_2(x)c_2\\ (dc_2/dx) &= A_2^*(x)c_1 + A_1^*(x)c_2\end{aligned} \qquad (3)$$

with

$$\begin{aligned}A_1(x) &= -[d \ln \alpha^{1/2}/dx](1 + i4\pi\alpha x)\\ A_2(x) &= [d \ln \alpha^{1/2}/dx]e^{-i4\pi\alpha x}.\end{aligned}$$

The exact method by which these formulas were derived will not be given here. Using (3) it can easily be verified that (2) satisfies (1), and since the solution of (3) requires just as many arbitrary constants as does that of (1), (2) will be the general solution of (1). Again using (3) one can readily show that

$$(d\psi/dx) = i2\pi\alpha[c_1 e^{+i2\pi\alpha x} - c_2 e^{-i2\pi\alpha x}].$$

More interesting than this is the fact that the current turns out to be

$$I = (h\alpha/m)[\,|c_1(x)|^2 - |c_2(x)|^2] = \text{constant}, \qquad (4)$$

exactly as though $\alpha$ were constant. Thus (2) can be interpreted, *in general* as representing currents travelling to the right and to the left with varying amplitudes $|c_1(x)|^2$ and $|c_2(x)|^2$ respectively. Eq. (4) shows that the current lost from one beam is gained by the other. It is this general separation of the two beams which seems to be new.

One of the original hopes was that some general differential equation could be found for the reflection coefficient $R$. This can be done, but only in an unsatisfactory manner. The natural definition of $R$ is $R(x) = |c_2(x)|^2 / |c_1(x)|^2$ so that $R(x)$ can be interpreted as the reflection coefficient for all of the potential curve lying to the right of $x$. This presupposes that $|c_1| \to 1$ and $|c_2| \to 0$ as $x \to +\infty$ so that the flow becomes unidirectional at $x = +\infty$. This $R$ satisfies the equation

$$[d \ln (1 - R)/dx] = [d \ln \alpha^{1/2}/dx]\cdot(z + z^*)$$

where $z = (c_2/c_1)e^{-i4\pi\alpha x}$ satisfies the differential equation

$$(dz/dx) = [d \ln \alpha^{1/2}/dx](1 - z^2) - i4\pi\alpha z.$$

This is unsatisfactory since $z$ involves $c_1$ and $c_2$ linearly while $R$ involves only $|c_1|^2$ and $|c_2|^2$.

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## Hyperfine Structure and Nuclear Moment of Rhenium

The Re I spectrum has been photographed in the range 3000A to 6800A with spectrographs of high resolving power at the Mount Wilson Observatory, and the majority of lines are found to be complex, having 2 to 6 components. The most striking lines are those which have "flag patterns," with both separations and intensities diminishing from one side of the pattern to the other. Such lines are undoubtedly transitions between two complex levels one of which has a much larger hyperfine structure than the other. For example the lines $5d^56s^2\,{}^6S-5d^56s6p\,{}^6P$ (3452, 3460, 3465A) and $5d^56s^2\,{}^6S-5d^56s6p\,{}^8P$ (4889, 5276A) are of this type and one would expect the interaction between the nuclear moment and the extranuclear electrons to be much larger for the $P$ states. Other lines for which the $j$-values of both levels are as high as $5\frac{1}{2}$ and which also have flag patterns, show only 6 components.

According to recent work of Aston, rhenium has two isotopes with mass numbers 185 and 187 which are present in the ratio 1 to 1.62. If either of these isotopes had zero moment one would expect a marked increase in intensity in the neighborhood of the center of gravity of each pattern. However, the flag patterns all decrease regularly in intensity with decreasing separation, so that one may conclude that both isotopes of rhenium have the same nuclear moment, $2\frac{1}{2}(h/2\pi)$.

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