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STRONGLY NONLINEAR WAVES IN POLYMER BASED PHONONIC CRYSTALS

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Abstract. One dimensional "sonic vacuum"-type phononic crystals were assembled from chains of polytetrafluoroethylene (PTFE) beads and Parylene coated spheres with different diameters. It was demonstrated for the first time that these polymer-based granular system, with exceptionally low elastic modulus of particles, support the propagation of strongly nonlinear solitary waves with a very low speed. They can be described using classical nonlinear Hertz law despite the viscoelastic nature of the polymers and the high strain rate deformation of the contact area. Trains of strongly nonlinear solitary waves excited by an impact were investigated experimentally and were found to be in reasonable agreement with numerical calculations. Tunability of the signal shape and velocity was achieved through a non-contact magnetically induced precompression of the chains. This applied prestress allowed an increase of up to two times the solitary waves speed and significant delayed the signal splitting. Anomalous reflection at the interface of two "sonic vacua"-type systems was reported.

Keywords: Strongly nonlinear, phononic crystal, polymers, wave propagation
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INTRODUCTION

One-dimensional chains of spherical beads have received increasing attention in the recent years for the study of a novel type of strongly nonlinear wave dynamics [1-6]. This system is the simplest example of phononic metamaterials with unique properties. The strong nonlinearity opens a new area of interest being a natural extension of the weakly nonlinear wave dynamics. The non-classical wave behavior appears if the granular chain is "weakly" compressed (i.e. when the wave amplitude is significantly higher than the forces caused by the initial precompression) [1]. The concept of "sonic vacuum" (SV) was introduced to emphasize that in such a chain with no initial prestress the sound speed is equal to zero. One of the distinguishing properties of SV is the existence of a qualitatively new solitary wave with a finite width that is independent of the wave amplitude. The solitary wave speed V_s in SV has a nonlinear dependence on the maximum strain ξ_m, the particle velocity υ_m and the force between particles F_m:

$$V_s = \frac{2}{\sqrt{5}} c \xi_m^{1/4} = \left(\frac{16}{25}\right)^{1/5} c^{4/5} \upsilon_m^{1/5} = 0.68 \left(\frac{2E}{a\rho^{3/2}\left(1-\nu^2\right)}\right)^{1/3} F_m^{1/6}. \qquad (1)$$

The solitary wave speed, V_s, can be tuned by applied static precompression (F_0) and can be written in terms of the normalized maximum force $f_r = F_m / F_0$ [4] as,

$$V_s = 0.9314 \left(\frac{4E^2 F_0}{a^2 \rho^3 \left(1-\nu^2\right)^2}\right)^{1/6} \frac{1}{\left(f_r^{2/3}-1\right)} \left\{\frac{4}{15}\left[3 + 2f_r^{5/3} - 5f_r^{2/3}\right]\right\}^{1/2} \qquad (2)$$

where Eq. (1) is a partial case of Eq. (2).

EXPERIMENTAL PROCEDURE

One dimensional phononic crystals were assembled filling a PTFE (polytetrafluoroethylene) tube (with inner diameter 5 mm) with chains of 21 PTFE (McMaster-Carr) and Parylene coated balls (AcraBall) with diameter a=4.76 mm and 4.86mm and mass 0.123 g and 0.44 g respectively (Fig. 1(a)). Details of the experimental set-up used for testing of 1-D chains are described elsewhere [4,5].

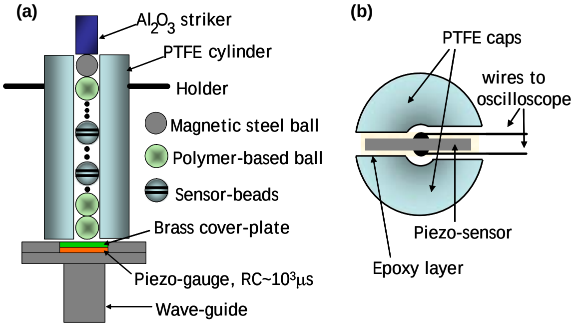

Figure 1. (a) Experimental set-up used for testing 1-D chains of polymer based (PTFE) beads. (b) Schematic drawing of a sensor embedded in a sphere [4].

Magnetically induced tunability

A peculiar characteristic of strongly nonlinear materials is the possibility of fine tuning the signal shape and speed under precompression. In the present study, the applied preload was achieved with a Neodymium-Iron-Boron ring magnet [5] placed around the PTFE cylinder containing the chains. It was held in place by the magnetic interaction with the steel ball placed on the top of the chain (Fig. 1(a)). This type of preloading allowed the application of a constant external force (2.38 N) to the end magnetic bead independently from its displacement, therefore maintaining constant boundary conditions of the system.

Heavy/Light interface testing

A chain of 20 nonmagnetic stainless steel (316) balls (plus a magnetic sphere on the top) was placed above 21 PTFE beads. The magnetically induced tunability was applied as described earlier to test the interface behavior under applied prestress. Further details of the experimental set-up can be found in [5].

RESULTS AND DISCUSSION

Pulses of different durations and amplitudes in the 1-D phononic crystals were generated by impacting an alumina (Al_2O_3) cylinder (0.47 g) or a PTFE ball with a diameter of 4.76 mm (mass 0.123 g) onto the top sphere of the chain. Single solitary waves were generated by an impactor with a mass equal to the mass of the beads in the system [1]. The results of these experiments are shown in Fig. 2. A very fast decomposition of the initial impulse was demonstrated at a distance comparable to the soliton's width and a clear tendency of signal splitting was noticed after only 10 particles.

Fig. 2 shows the strong dependence of the solitary wave's speed on the amplitude for PTFE chains. Here Equation (1) for SV and Equation (2) for a pre-compressed chain are plotted as common logarithmic values at different PTFE elastic constants together with the corresponding experimental data (solid dots) and the numerical calculations of the soliton speed for discrete chains.

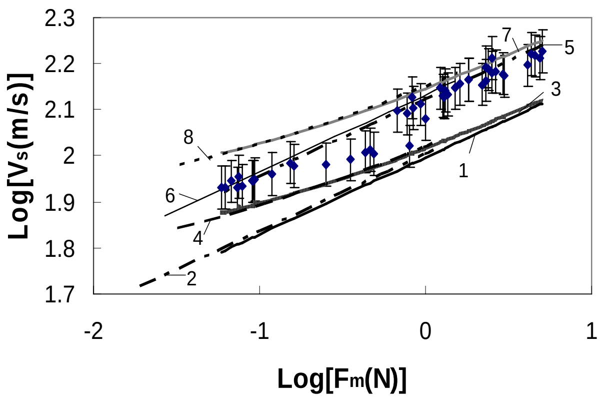

Figure 2. Dependence of the velocity of solitary wave on amplitude in a PTFE chain. Experimental values are shown by solid dots. Curves 1 and 5 are the theoretical curves based on Equation (1) with a Young's modulus equal to 600 MPa and 1460 MPa respectively. Curve number 2 and 6 represent the corresponding numerical calculations for these cases. Curve number 3 and 7 represent the long wave approximation for gravitationally pre-compressed systems (Equation (2)) at 600 MPa and 1460 MPa respectively; curves 4 and 8 represent the corresponding numerical calculation.

The values based on the long wave approximation and the numerically calculated data are basically indistinguishable. The wave speeds for different amplitudes were obtained via time-of-flight measurements between the peaks measured by the sensors in experiments (solid dots in Fig. 2).

PTFE and Parylene are polymeric viscoelastic materials with high strain rate sensitivities and low elastic modulus [7]. At normal conditions Young's (E) and flexural moduli for PTFE are in the range of 400 – 750 MPa and Poisson's ratio (ν) is 0.46 [4,5,8]. For Parylene these values are E = 2760 MPa, ρ = 1289 Kg/m^3, ν = 0.388 [9]. If proved to support strongly nonlinear behavior as in the case of chains made from typical linear elastic materials [1-3, 6], these properties can be very attractive for ensuring a very low solitary wave speed and an adequate tunability of the system.

We observed a significant difference between the solitary wave speeds obtained in experiments and derived from the theory, especially at large amplitudes of forces when using the Young's moduli found using ultrasonic measurements. It should be mentioned that in polymers, the bulk sound speed extrapolated from the Hugoniots in $u_s(u_p)$ coordinates results in significantly higher values than the sound speed measured using ultrasonic techniques [8]. This discrepancy indicates a rapidly varying change of compressibility at low values of shock amplitudes. For PTFE, using the value of bulk speed c_b extrapolated from Hugoniot (1.68 km/s, in comparison with 1.139 km/s from ultrasonic measurements) and Poisson ratio 0.46, we obtained a value of Young's modulus of 1.46 GPa based on the relations for elastic solids. The values of the solitary wave speed derived from Eq. 2 with this Young's modulus better matched our experimental results (Fig. 2). Smaller PTFE particles (2.38 mm diameter) also supported the SV type behavior, although in this case the effect of dissipation appeared to be more significant [4].

Chains of composite particles made of stainless steel beads (high density core) coated with a low elastic modulus ParyleneC layer 50 µm thick, with diameters 4.86 mm and 2.48 mm, also support strongly nonlinear solitary waves. Similar to the PTFE case, the effects of dissipation were significant; especially for the small diameter beads. Also, to match the experimental data, the Young's modulus used for Parylene in Fig. 3 (Eq. (2)) was significantly higher than its nominal value reported by the manufacturer [9] (15 GPa vs. 2.76 GPa).

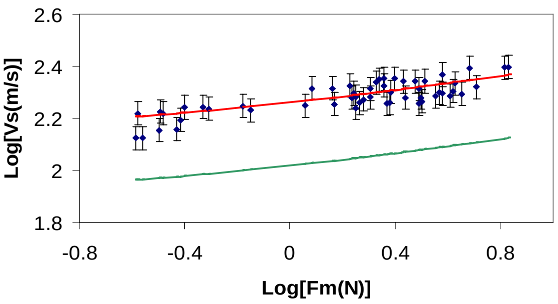

Figure 3. Dependence of the velocity of solitary wave on amplitude for ParyleneC coated steel chain only under weak gravitational precompression. The common log of the experimental values are shown by solid dots. The solid curves represent the common log of the theoretical values based on the analogue of Equation (2) for composite particles with a Young's modulus equal to 2.76 GPa (bottom) and 15 GPa (top).

The effect of the magnetically induced precompression on the solitary wave speed in PTFE chains is summarized in Fig. 4. The observed increase of the solitary wave speed in the PTFE based system under precompression was significant. The experimental data was consistently matched by the results obtained from the long wave approximation and the numerical analysis [5]. This increase of the speed resulted in a corresponding increase of the acoustic impedances.

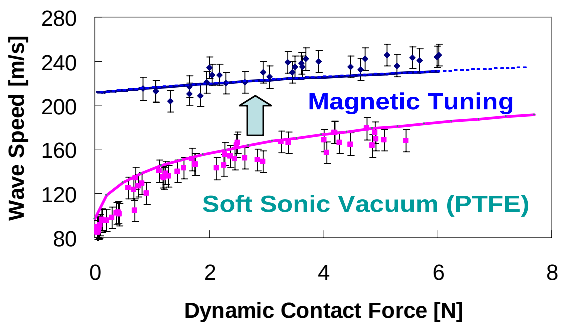

Figure 4. Dependence of the solitary wave speed on the amplitude of the dynamic contact force for gravitationally loaded and for magnetically tuned chains composed of PTFE beads. The experimental values for corresponding curves are shown by solid squares and dots. The solid curves represent the results for the long wave approximation with E= 1.46 GPa.

A delay of the splitting of the solitary waves under prestress was observed both in experiments and in numerical calculations.

The testing of the heavy/light interface [6] of the two strongly nonlinear granular media under the magnetically induced precompression described earlier resulted in a dramatic change of reflectivity. Anomalous reflected compression waves and transmitted rarefaction waves were detected in experiments and numerical calculations (see Fig. 5). We named this phenomenon the "acoustic diode" effect because of the dramatic change of the reflectivity triggered by the precompression.

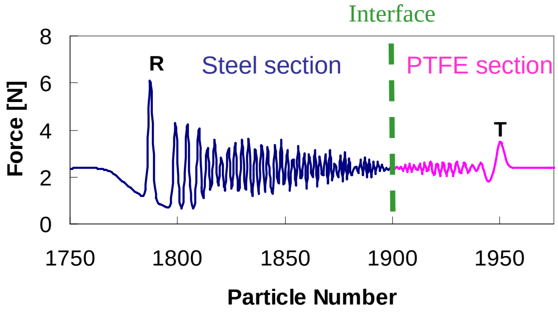

Figure 5. Typical profile obtained in numerical calculations showing a reflected rarefaction wave followed by the anomalous compression waves (R) and an oscillatory tail in the stainless steel chain and transmitted compression (T) rarefaction pulses and oscillatory tail in the PTFE chain [6].

The nonlinear phenomena described here can find useful application as tunable controllers of information flow through interfaces and in the design of novel types of tunable shock protection layers. The precompression can be employed for designing tunable information transportation lines with the unique possibility of manipulating the signal's delay, reflection and decompositions at will for security-related information.

CONCLUSIONS

Polymer-based strongly nonlinear one dimensional metamaterials (chains of PTFE and ParyleneC coated stainless steel beads) were assembled and tested. Both polymeric systems support strongly nonlinear solitary waves with very small amplitudes and speed. The wave speed of these solitary waves was less than the sound speed in air and lower than any other previously reported. The theory derived from the Hertzian model for contact interaction of linear elastic solids fit very well with the experimental results, despite the viscoelastic nature of the polymers. The elastic modulus of PTFE at higher signal amplitudes matched quite well with the elastic modulus extrapolated from the Hugoniot data. Tunability of the signal shape and speed was achieved through a magnetically induced precompression which also created anomalous reflections at the interface of steel/PTFE chains (novel "acoustic diode" behavior).

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