Global Configuration Stabilization for the VTOL Aircraft With Strong Input Coupling

Reza Olfati-Saber

***Abstract*—Trajectory tracking and configuration stabilization for the vertical takeoff and landing (VTOL) aircraft has been so far considered in the literature only in the presence of a slight (or zero) input coupling (i.e., for a small ϵ). In this note, our main contribution is to address global configuration stabilization for the VTOL aircraft with a strong input coupling using a smooth static state feedback. In addition, the differentially flat outputs for the VTOL aircraft are automatically obtained as a by-product of applying a decoupling change of coordinates.**

***Index Terms*—Autonomous vehicles, backstepping, differential flatness, global stabilization, nonlinear control.**

I. Introduction

In recent years, trajectory tracking and configuration stabilization of the vertical takeoff and landing (VTOL) aircraft has been extensively studied by many researchers [1]–[3]. Here, we consider configuration stabilization of the VTOL aircraft, depicted in Fig. 1, from any arbitrary initial configuration and speed to any position with zero roll angle and zero speed. The simplified dynamics of the VTOL aircraft is given in [2] as the following

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -u_1\sin(\theta) + \epsilon u_2\cos(\theta) \\ \dot{y}_1 &= y_2 \\ \dot{y}_2 &= u_1\cos(\theta) + \epsilon u_2\sin(\theta) - g \\ \dot{\theta} &= \omega \\ \dot{\omega} &= u_2 \end{aligned} \tag{1}$$

where $\epsilon \neq 0$, θ is the roll angle and the VTOL moves in a vertical (x_1, y_1) plane. In [1], approximate linearization techniques were used which ignore the coupling between the first two second-order subsystems in (1) and the (θ, ω)-subsystem and then treat the system as a slightly nonminimum phase system. Under a similar assumption, for $\epsilon = 0$ and sufficiently small $|\epsilon|$, semiglobal stabilization of the origin for the VTOL aircraft is considered in [3].

Here, we are interested in the case where $\epsilon \neq 0$ with arbitrarily large $|\epsilon|$ or the *strong input coupling case*. This case is important due to the fact that it similarly appears in an accurate model of a helicopter where ϵ is no longer small [4, p. 168]. Since ϵ explicitly depends on the physical parameters of the aircraft that can be measured, the assumption that ϵ is known is justified. For maneuverable aerospace vehicles, the size of certain rotor-tilt angles or elevator deflection angles can be possibly large that leads to the strong input coupling case. From a theoretical point of view, the problem discussed in this note is an example of a nonlinear control system in the form $\dot{x} = f(x,u) + \epsilon g(x,u)$ that can be transformed into $\dot{z} = f(z,v)$ with no dependence on ϵ, after applying a globally invertible nonlinear change of variables

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The author is with the Department of Control and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125 USA (e-mail: olfati@cds.caltech.edu).

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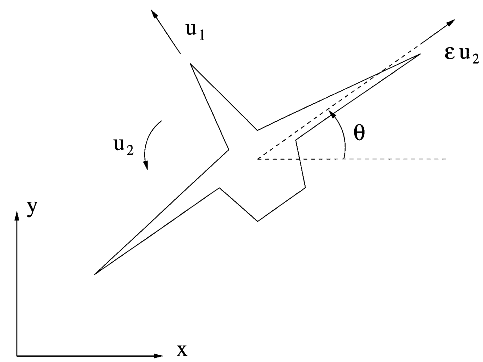

Fig. 1. The VTOL aircraft.

and control $z = \Phi_1(x,\epsilon)$, $v = \Phi_2(x,u,\epsilon)$ [i.e., $x = \Phi_1^{-1}(z,\epsilon)$], $u = \Phi_2^{-1}(x,v,\epsilon)$. Apparently, if the original system with $\epsilon = 0$ can be globally asymptotically stabilized to the origin using $u = K(x)$, the perturbed system can be globally asymptotically stabilized to $x = 0$ for any arbitrary ϵ with state feedback $v = K(z)$ which can be transformed into the original coordinates as

$$u = \Phi_2^{-1}(x, K(\Phi_1(x,\epsilon)), \epsilon). \tag{2}$$

The key point in control design for (1) is the decoupling of the first two second-order subsystems (x_1, x_2) and (y_1, y_2) and the third subsystem (θ, ω) with respect to the control input u_2. After, applying this decoupling global change of coordinates, the control design for the system in new coordinates is straightforward and can be done either using standard backstepping procedure, or by applying a second change of coordinates that transforms the system into a cascade nonlinear system with an exponentially stable linear subsystem. Here, we take the second approach to avoid the use of the second-order time derivatives of the Lyapunov function of the translational dynamics. As a by-product of applying this decoupling change of coordinates, we automatically obtain the differentially flat outputs for the VTOL aircraft. These outputs can be later used for trajectory generation/tracking [5]. We provide simulation results that suggest the settling time of the trajectories are relatively short.

Here is an outline of this note. In Section II, we explain our decoupling method and its connection to differentially flat outputs for the VTOL. In Section III, we provide a detailed control design for the VTOL. Finally, in Section IV, we give simulation results and concluding remarks.

II. Decoupling Method

To decouple three second-order subsystems of the VTOL aircraft in (1), we use a change of coordinates given in the following theorem [6], [7].

Theorem 1. (Decoupling Transformation): Consider the following system:

$$\begin{aligned} \dot{q}_1 &= p_1 \\ \dot{p}_1 &= f_1(q,p) + g_1(q_2)u \\ \dot{q}_2 &= p_2 \\ \dot{p}_2 &= f_2(q,p) + g_2(q_2)u \end{aligned} \tag{3}$$

where $q = (q_1, q_2) \in \mathbb{R}^2$, $p = (p_1, p_2) \in \mathbb{R}^2$, f_i's and g_i's are smooth functions, and $g_2(q_2) \neq 0, \forall q_2 \in \mathbb{R}$. Then, the following global change of coordinates:

$$\begin{aligned} z_1 &= q_1 - \int_0^{q_2} \frac{g_1(s)}{g_2(s)} ds \\ z_2 &= p_1 - \frac{g_1(q_2)}{g_2(q_2)} p_2 \\ \xi_1 &= q_2 \\ \xi_2 &= p_2 \end{aligned} \tag{4}$$

decouples (q_1, p_1) subsystem and (q_2, p_2) subsystem with respect to u and in new coordinates the dynamics of the system transforms into the normal form

$$\begin{aligned} \dot{z} &= f(z, \xi_1, \xi_2) \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= v \end{aligned} \tag{5}$$

where $v = f_2(q,p) + g_2(q_2)u$ and $z = (z_1, z_2)^T$.

After applying the change of coordinates

$$\begin{aligned} z_1 &= x_1 - \epsilon \sin(\theta) \\ z_2 &= x_2 - \epsilon \cos(\theta)\omega \\ w_1 &= y_1 + \epsilon(\cos(\theta) - 1) \\ w_2 &= y_2 - \epsilon \sin(\theta)\omega \\ \xi_1 &= \theta \\ \xi_2 &= \omega \end{aligned} \tag{6}$$

in new coordinates, we have

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\sin(\xi_1)\bar{u}_1 =: v_1 \\ \dot{w}_1 &= w_2 \\ \dot{w}_2 &= \cos(\xi_1)\bar{u}_1 - g =: v_2 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= u_2 \end{aligned} \tag{7}$$

where $\bar{u}_1 = u_1 - \epsilon \xi_2^2$ is a new control. The following result states an important property of the change of coordinates given in Theorem 1 for the purpose of trajectory generation and tracking for nonlinear systems [8], [5].

Corollary 1: The change of coordinates in (4) applied to the subsystems $(x_1, x_2, \theta, \omega)$ and $(y_1, y_2, \theta, \omega)$, respectively, (automatically) gives the two differentially flat outputs z_1, w_1 for the VTOL aircraft.

Proof: By direct calculation.

According to [5], the obtained flat outputs (z_1, w_1) can be now used for the real-time trajectory generation for the VTOL aircraft.

III. Control Design

In this section, we present a control design method for configuration stabilization of the VTOL aircraft. Our main result is as follows.

Theorem 2: There exists a smooth static-state feedback in explicit form that globally asymptotically and locally exponentially stabilizes any desired configuration of the VTOL aircraft in (1) with zero velocity.

Proof: Without loss of generality, assume the desired configuration is $q = 0$ where $q = (x_1, y_1, \theta)$. Define

$$\begin{aligned} r_1 &= c_{11} z_1 + c_{12} z_2 \\ r_2 &= c_0 \sigma(c_{21} w_1 + c_{22} w_2) \end{aligned}$$

where c_{i1}, c_{i2} for $i = 1, 2$ are coefficients of a Hurwitz polynomial, $0 < c_0 < g$, and $\sigma(\cdot) = \tanh(\cdot)$. Given $v_1 = r_1$ and $v_2 = r_2$, $z = 0$ and $w = 0$ are globally asymptotically stable for the z and w subsystems, respectively (Later, we explain why r_2 should be bounded). This means that taking

$$\begin{aligned} \bar{u}_1 &= k_1(z, w) := \sqrt{r_1^2 + (r_2 + g)^2} \\ \xi_1 &= k_2(z, w) = \arctan\left(\frac{-r_1}{r_2 + g}\right) \end{aligned} \tag{8}$$

$(z, w) = (0, 0)$ is globally asymptotically and locally exponentially stable for the (z, w) subsystem of (7). To avoid the singularity of division-by-zero in the last equation, we use a bounded control r_2 with a bound $c_0 < g$. At this point, a straightforward use of the standard backstepping procedure proves that a globally stabilizing static feedback law exists for (7). However, we prefer to use a version of backstepping procedure which does not require the use of the Lyapunov function of the zero-dynamics associated with the output $\mu_1 = 0$ where μ_1 is defined in the following. This preference is due to simplicity of calculations. After applying the change of coordinates and control

$$\begin{aligned} \mu_1 &= \xi_1 - k_2(z, w) \\ \mu_2 &= \xi_2 - \dot{k}_2 \\ \bar{u}_2 &= u_2 - \ddot{k}_2 \end{aligned} \tag{9}$$

we get

$$\begin{aligned} \dot{\mu}_1 &= \mu_2 \\ \dot{\mu}_2 &= \bar{u}_2. \end{aligned} \tag{10}$$

Thus, applying $\bar{u}_2 = -d_1\mu_1 - d_2\mu_2$ with $d_1, d_2 > 0$ globally exponentially stabilizes $(\mu_1, \mu_2) = (0, 0)$ for the μ subsystem. The dynamics of the closed-loop system is in the form

$$\begin{aligned} \dot{\eta} &= f(\eta, \mu_1) \\ \dot{\mu} &= A\mu \end{aligned} \tag{11}$$

where A is a Hurwitz matrix, $\eta = (z_1, z_2, w_1, w_2)^T$, and

$$f(\eta, \mu_1) := \begin{bmatrix} z_2 \\ -\sin(k_2(z,w) + \mu_1)k_1(z,w) \\ w_2 \\ \cos(k_2(z,w) + \mu_1)k_1(z,w) - g \end{bmatrix}. \tag{12}$$

Given $\mu_1 = 0$ for $\dot{\eta} = f(\eta, 0)$, $\eta = 0$ is globally asymptotically and locally exponentially stable. Based on Theorem 3, for any solution of the μ subsystem the solution of the η subsystem is uniformly bounded and the asymptotic stability of $(\eta, \mu) = 0$ for the cascade system in (11) follows from a theorem in [9]. Therefore, global asymptotic stabilization and local exponential stabilization of the origin is achieved for the VTOL aircraft. To obtain an explicit expression for $\ddot{k}_2$, note that

$$\begin{aligned} \dot{k}_2 &= \frac{r_1\dot{r}_2 - (r_2+g)\dot{r}_1}{r_1^2 + (r_2+g)^2} \\ \ddot{k}_2 &= \frac{r_1\ddot{r}_2 - (r_2+g)\ddot{r}_1}{r_1^2 + (r_2+g)^2} \\ &\quad + \frac{2[r_1\dot{r}_1 + (r_2+g)\dot{r}_2][(r_2+g)\dot{r}_1 - r_1\dot{r}_2]}{(r_1^2 + (r_2+g)^2)^2} \end{aligned} \tag{13}$$

where

$$\begin{aligned} v_1 &= -\sin(\xi_1)\bar{u}_1 \\ \dot{v}_1 &= -\cos(\xi_1)\xi_2\bar{u}_1 - \sin(\xi_1)\dot{\bar{u}}_1 \\ v_2 &= \cos(\xi_1)\bar{u}_1 - g \\ \dot{v}_2 &= -\sin(\xi_1)\xi_2\bar{u}_1 + \cos(\xi_1)\dot{\bar{u}}_1 \\ \dot{\bar{u}}_1 &= \frac{r_1\dot{r}_1 + (r_2+g)\dot{r}_2}{\bar{u}_1} \\ \dot{r}_1 &= c_{11}z_2 + c_{12}v_1 \\ \ddot{r}_1 &= c_{11}v_1 + c_{12}\dot{v}_1 \\ \dot{r}_2 &= c_0(c_{21}w_2 + c_{22}v_2)\sigma'(c_{21}w_1 + c_{22}w_2) \\ \ddot{r}_2 &= c_0(c_{21}v_2 + c_{22}\dot{v}_2)\sigma'(c_{21}w_1 + c_{22}w_2) \\ &\quad + c_0(c_{21}w_2 + c_{22}v_2)^2\sigma''(c_{21}w_1 + c_{22}w_2). \end{aligned}$$

($(')$ denotes the derivative). Note that in the equation of $\dot{\bar{u}}_1$, $\bar{u}_1$ appears in the denominator and could be a problem. However, due to the fact that $|r_2| < g$, for all time $\bar{u}_1(t) > 0$ and $\dot{\bar{u}}_1$ is well defined for all $t \geq 0$. This finishes the proof.

Theorem 3. (Boundedness of Solutions): Consider the nonlinear cascade system in (11). For any solution of the μ subsystem, the solution of the η subsystem remains bounded.

Proof: Define

$$\alpha = k_2(z, w),\ \beta = \mu_1.$$

Then, $f(\eta, \mu_1)$ in (12) can be rewritten as

$$f(\eta, \mu_1) = (z_2, -\sin(\alpha+\beta)k_1, w_2, \cos(\alpha+\beta)k_1 - g)^T.$$

Now, one can express $f(\eta, \mu_1)$ in the form

$$f(\eta, \mu_1) = f(\eta, 0) + h(\eta, \mu_1)\mu_1 \tag{14}$$

where $f(\eta, 0) = (z_2, r_1, w_2, r_2)^T$ and $h(\eta, \mu_1)$ is a continuous function that is explicitly determined in the sequel. The main objective is to construct a smooth, positive–definite, and proper Laypunov function $V(\eta)$ satisfying $\nabla V(\eta) \cdot f(\eta, 0) \leq 0, \forall \eta \in \mathbb{R}^4$ such that for all vanishing disturbances $\mu(\cdot)$ as the solutions of the μ subsystem, $V(\eta)$ remains bounded. Since $V(\eta)$ is proper, this implies η remains bounded in a compact set.

Step 1) Calculation of $h(\eta, \mu_1)$ in (14); define the following continuous function:

$$\rho(\beta) := \begin{cases} 1, & \beta = 0 \\ \frac{\sin(\beta)}{\beta}, & \beta \neq 0 \end{cases} \tag{15}$$

which takes its maximum at $\beta = 0$. Thus, $|\rho(\beta)| \leq 1, \forall \beta$. Based on elementary trigonometric properties of $\sin(x)$ and $\cos(x)$, we have

$$\begin{aligned} \sin(\alpha+\beta) &= \sin(\alpha) + h_1(\alpha,\beta)\beta \\ \cos(\alpha+\beta) &= \cos(\alpha) + h_2(\alpha,\beta)\beta \end{aligned} \tag{16}$$

where

$$\begin{aligned} h_1(\alpha,\beta) &= \rho\left(\frac{\beta}{2}\right)\cos\left(\alpha + \frac{\beta}{2}\right) \\ h_2(\alpha,\beta) &= -\rho\left(\frac{\beta}{2}\right)\sin\left(\alpha + \frac{\beta}{2}\right) \end{aligned} \tag{17}$$

and, therefore, $|h_i(\alpha,\beta)| \leq 1, \forall \alpha, \beta, i = 1, 2$. The function $h(\eta, \mu_1)$ can be expressed as

$$h(\eta, \mu_1) = \begin{bmatrix} 0 \\ -h_1(k_2(z,w),\mu_1) \cdot k_1(z,w) \\ 0 \\ h_2(k_2(z,w),\mu_1) \cdot k_1(z,w) \end{bmatrix}. \tag{18}$$

Step 2) Construction of $V(\eta)$: In this case, we designed the state feedback laws r_1 and r_2 so that the origin is globally asymptotically stable (GAS) for the (z_1, z_2) subsystem and the (w_1, w_2) subsystem, respectively. We take $V(\eta)$ to be the sum of the Lyapunov functions associated with these two closed-loop subsystems. To be specific, we have

$$\begin{aligned} S_1 &: \begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -a_1 z_1 - a_2 z_2 \end{cases} \\ S_2 &: \begin{cases} \dot{w}_1 &= w_2 \\ \dot{w}_2 &= -a_3\sigma(a_4 w_1 + a_5 w_2) \end{cases} \end{aligned} \tag{19}$$

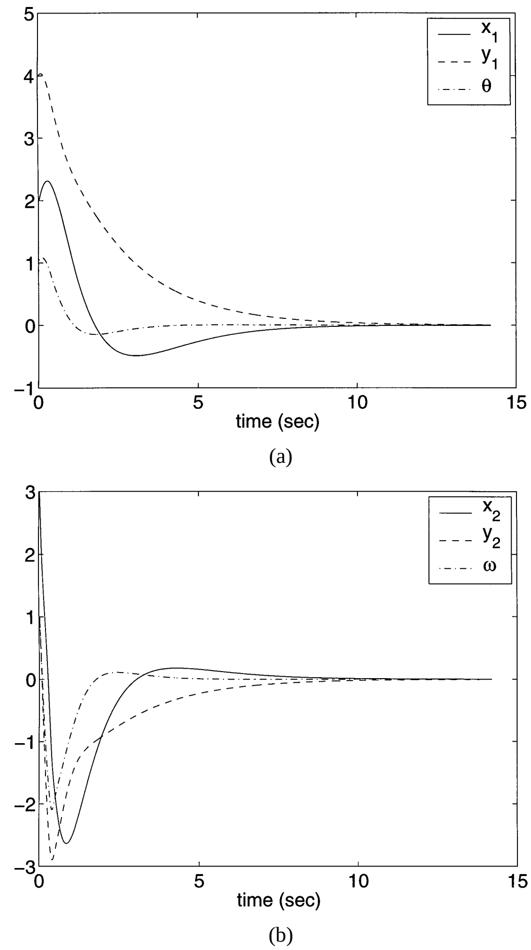

Fig. 2. The state trajectory of the VTOL aircraft. (a) Configuration. (b) Velocity.

where the $a_i > 0, i = 1, \ldots, 5$ are the parameters of the controllers of subsystems S_1, S_2 in (19) and $0 < a_3 < g$. Define the scalar function $\phi(x) = \int_0^x \sigma(s)ds$ and notice that $\phi(x)$ is a positive–definite function. Let

$$\begin{aligned} V_1(z) &= \frac{1}{2}a_1 z_1^2 + \frac{1}{2}z_2^2 \\ V_2(w) &= \frac{a_3}{a_4}\phi(a_4 w_1) + \frac{1}{2}w_2^2. \end{aligned} \tag{20}$$

Then, along the solutions of S_1 and S_2, we have $\dot{V}_1 \leq 0, \forall z \in \mathbb{R}^2$ and $\dot{V}_2 \leq 0, \forall w \in \mathbb{R}^2$. The former property is easy to see and the latter one can be verified as the following:

$$\begin{aligned} \dot{V}_2 &= \frac{a_3}{a_5}(a_5 w_2)[\sigma(a_4 w_1) - \sigma(a_4 w_1 + a_5 w_2)] \\ &= \frac{a_3}{a_5}\bar{w}_2(\sigma(\bar{w}_1) - \sigma(\bar{w}_1 + \bar{w}_2)) < 0, \forall \bar{w}_2 \neq 0 \end{aligned} \tag{21}$$

where $\bar{w}_1 = a_4 w_1$, $\bar{w}_2 = a_5 w_2$. The last property holds due to the fact that the sigmoidal function $\sigma(\cdot)$ is strictly increasing. Therefore, based on LaSalle's invariance principle, $(w_1, w_2) = 0$ is GAS for subsystem S_2 (the GAS property of $z = 0$ for subsystem S_1 is trivial). As a result, taking the following Lyapunov function:

$$V(\eta) = V_1(z) + V_2(w) \tag{22}$$

guarantees that for the overall η subsystem of the cascade system (11) the property $\nabla V(\eta) \cdot f(\eta, 0) \leq 0$ holds.

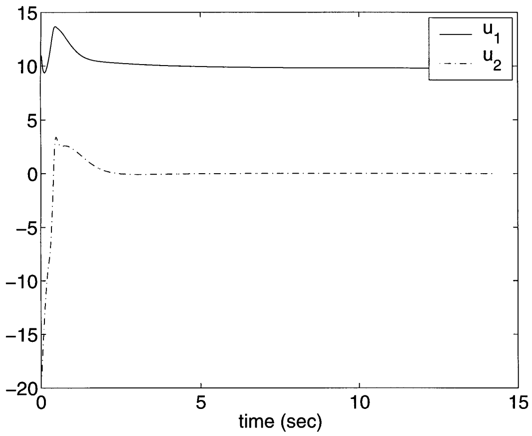

Fig. 3. The control input of the VTOL aircraft.

Step 3) Given the Lyapunov function $V(\eta)$ defined in (22), we prove the boundedness of the solutions of the η subsystem of the cascade system in (11) using some straightforward calculations that are omitted due to space limitations (see [4, Th. 7.3.4, p. 235] and [10] for a detailed proof). □

Figs. 2 and 3 show the state trajectory and control of the VTOL aircraft from initial condition (2, 3, 4, 1, $\pi/3$, 1) with $\epsilon = 1$. It can be observed that the control inputs in Fig. 3 are rather aggressive actions that occur in relatively short periods.

IV. Conclusion

In this note, we considered global configuration stabilization of the VTOL aircraft with arbitrary $\epsilon \neq 0$. We showed a key point in control of the VTOL aircraft is in decoupling of its three second-order subsystems using a global change of coordinates introduced in [6] and later generalized in [7]. Then, we gave a globally stabilizing smooth static state feedback law in explicit form for the VTOL aircraft. As a by-product of applying the decoupling change of coordinates, the differentially flat outputs for the VTOL aircraft are automatically obtained. Simulation results were presented for a difficult initial condition with initial roll angle of $\pi/3$ and strong input coupling. It is observed that the controller stabilizes the origin for the VTOL aircraft with an aggressive maneuver.

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Comment on "Order-Recursive Factorization of the Pseudoinverse of a Covariance Matrix"

Jie Zhou and Yunmin Zhu

***Abstract*—Numerical counterexamples and theoretical analysis of the aforementioned paper are presented, and they show that the main result, Theorem 2, in the paper is incorrect.**

***Index Terms*—Factorization, order-recursive, pseudoinverse.**

Consider the following covariance matrix throughout this paper:

$$\Sigma = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}. \tag{1}$$

Its pseudoinverse is

$$\Sigma^{\dagger} = \tfrac{1}{8} \begin{pmatrix} 5 & -4 & -2 & 5 \\ -4 & 8 & 0 & -4 \\ -2 & 0 & 4 & -2 \\ 5 & -4 & -2 & 5 \end{pmatrix}. \tag{2}$$

The matrix Σ can be partitioned as the following form:

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix}$$

where the submatrices Σ_{11}, Σ_{12}, and Σ_{22} are all 2×2 matrices, respectively. According to the notation and recursive algorithm given in Section VI of the above paper,[1] we can get

$$M = \begin{pmatrix} -0.3249 & -0.5257 \\ -1.3764 & 0.8507 \end{pmatrix}$$

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The authors are with the College of Mathematics, Sichuan University, Sichuan 610064, China (e-mail: zhoujie68@263.net; ymzhu@scu.edu.cn).

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