Comparison of Gravity Interpretation Methods[1]

HIROO KANAMORI[2]

Division of Geological Sciences, California Institute of Technology, Pasadena

Applicability of the weighting function method to gravity problems is discussed as compared with the exact method. The merits of the weighting function method are its simplicity and rapidity of computation, particularly in three-dimensional problems. For local geological problems, the weighting function method can be used for the gravity calculation with sufficient accuracy when the structure is appropriately approximated by surface density distributions. The gravity inversion by use of the weighting function method is outlined and illustrated for the water layer correction of gravity. The method is useful in relating the gravity data, without introducing detailed structures, to other geophysical data such as seismic and heat flow data.

In geophysical applications of gravity data, two methods are usually used. In the first, gravitational attraction directly calculated for bodies of various shapes is compared with the observed gravity. *Talwani et al.* [1959] and *Talwani and Ewing* [1960] provided rapid computation methods for this purpose. In the second method, the actual mass distribution is approximated by an equivalent surface density distribution and the potential theory is applied. *Tsuboi and Fuchida* [1937] and *Bullard and Cooper* [1948] applied this method for the calculation of underlying mass distribution. *Tomoda and Aki* [1955] further developed the method and devised the $(\sin x)/x$ system which provides a method of simple and rapid gravity interpretation. These methods can generally be called the weighting function (WF) method. Uncertainties originating from the substitution of the mass distribution is inherent in the WF method, and in this respect Talwani's method has the advantage. In actual problems, however, the density and shape of the mass distribution are not exactly known, and the accuracy and spacing of the gravity measurement are usually limited. Uncertainties originating from these may be much larger than those inherent in the WF method.

The merit of the WF method lies in its simplicity and rapidity in computation, particularly in three-dimensional problems where the exact method is very time-consuming. In actual applications, it is important to know how closely the WF method can approximate the exact solution. What follows is a comparison of the WF method with the exact method for several problems of geophysical interest.

Gravity calculation. A typical geophysical example is treated by *Simmons* [1964]. For the interpretation of local gravity anomalies, he considered several anorthosite massifs (his Figure 7) 5 to 10 km in diameter, 7 km thick, and seated at a depth of about 8 km. As the models have similar dimensional ratios, we will consider spheres of radius R seated at the depth of $Z = R$, $Z = 2R$, and $Z = 3R$. The x axis is taken horizontally, the point on the surface above the center of the sphere being the origin (see Figures 1 and 2). For simplicity, the density ρ will be taken as numerically equal to $1/(2\pi GR)$ in the sphere problems, where G is the universal constant of gravity. Figure 1 shows a two-dimensional sphere (cylinder). In the $(\sin x)/x$ method, the mass distribution is first condensed on a plane at an appropriate level in such a way that the variation in surface density replaces the variation in thickness. The surface density m condensed at the depth Z is now given by

$$m(x) = \begin{cases} 2(R^2 - x^2)^{1/2}\rho & |x| \le R \\ 0 & |x| > R \end{cases}$$

This surface density distribution is approximated by the function

[1] Contribution 1415, Division of Geological Sciences, California Institute of Technology.

[2] On leave from Geophysical Institute, Tokyo University.

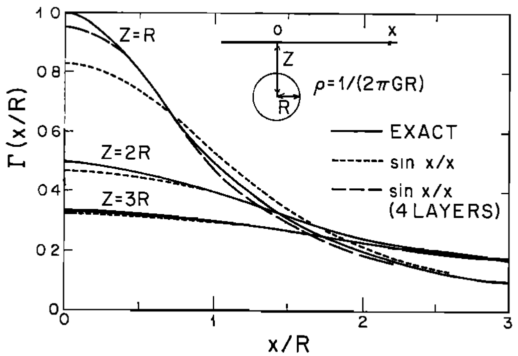

Fig. 1. Gravity for a two-dimensional sphere (cylinder).

$$M(x) = \sum_{i=-\infty}^{+\infty} m(ia) \frac{\sin \pi(x - ia)/a}{\pi(x - ia)/a}$$

where a is an equidistant grid spacing and i is an integer. $M(x)$ takes the same value as $m(x)$ at every grid point and is a superposition of the functions of the form $(\sin \pi x/a)/(\pi x/a)$. From the spectral point of view, $M(x)$ is derived from $m(x)$ when the components having wavelengths shorter than $2a$ are removed.

Gravity caused by the surface density distribution can be given by a weighted superposition of gravities resulting from the density distribution $(\sin \pi x/a)/(\pi x/a)$. The gravity at the surface $\Gamma(ia)$ at ith grid points $x = ia$ is therefore

$$\Gamma(ia) = 2\pi G \sum_{j=-L}^{L} \phi_j m((i + j)a) \qquad i = 1, 2, \cdots, N \qquad (1)$$

$$\phi_j = \phi_{-j} = d((-1)^j e^d - 1)/(d^2 + j^2\pi^2) \qquad j = 0, 1, 2, \cdots, L \qquad (2)$$

$$d = -\pi Z/a$$

where L is a number large enough to make $\phi_{\pm L}$ negligibly small and N is the number of grid points where the gravity is to be calculated. $2\pi G\ \phi_j$ $(j = 0, \pm 1, \ldots, \pm L)$ are gravity values at the grid points for the density distribution $(\sin \pi x/a)/(\pi x/a)$. The derivation of (2) is given by *Tomoda and Aki* [1955]. The results obtained with $a/R = 0.2$ and $L = 20$ are given by the dotted curves in Figure 1 and compared with the exact distribution given by the solid curves. Except when $Z = R$, the agreement is generally good. When $Z = R$, the effect of the near-surface mass is so large that the condensed distribution cannot give a good approximation. However, the discrepancy can be reduced by slicing the sphere and condensing the mass at several different levels. The broken curve in Figure 1 is obtained when four layers of condensation at levels $R/4$, $3R/4$, $5R/4$, and $7R/4$ are taken and the contribution from each layer is summed. The agreement with the exact solution becomes satisfactory.

Three-dimensional problems can be treated in a similar manner by using two-dimensional weighting coefficients $\phi_{i,j}$. $\phi_{i,j}$, based on the $(\sin x)/x$ method, cannot be given by a closed form as in (2), but results have been calculated by numerical integrations [*Tsuboi et al.*, 1958; *Shimazu*, 1962; *Saito et al.*, 1964]. Analytical expression of $\phi_{i,j}$ can be obtained by an alternative method which is a slight modification of the three-dimensional sin x/x method [*Kanamori*, 1963*a*]:

$$\phi_{0,0} = 2[e^d(d - 1) + 1]/d^2$$

$$\phi_{0,i} = \phi_{i,0} = d[(-1)^i e^d(d^2 + i^2\pi^2 - 2d) + 2d]/(d^2 + i^2\pi^2)^2 \qquad i = \pm 1, \pm 2, \cdots, \pm L \qquad (3)$$

$$\phi_{i,j} = \phi_{j,i} = 2d^2 \cdot [1 - (-1)^{i+j}e^d]/\{[d^2 + (i + j)^2\pi^2] \cdot [d^2 + (i - j)^2\pi^2]\}$$

$$i = \pm 1, \pm 2, \cdots, \pm L$$

$$j = \pm 1, \pm 2, \cdots, \pm L$$

These coefficients are used in the present computations. In case of a sphere, the surface mass distribution taken proportionally to the thickness is

$$m(x, y) = \begin{cases} 2\ [R^2 - (x^2 + y^2)]^{1/2}\ \rho & (x^2 + y^2)^{1/2} \le R \\ 0 & (x^2 + y^2)^{1/2} > R \end{cases}$$

Gravity $\Gamma(ia, ja)$ at (i, j) grid points can now be given simply by

$$\Gamma(ia, ja) = 2\pi G \sum_{k=-L}^{L} \sum_{l=-L}^{L} \phi_{k,l} \cdot [m(i + K)a, (j + l)a]$$

The results given in Figure 2 are obtained with $L = 20$, $a/R = 0.2$, and $d = -\pi Z/a$. Except when $Z = R$, the agreement is generally good. The calculation for four layers is made as before for $Z = R$, and the result is shown by a broken curve. The discrepancy has been satisfactorily reduced.

It should be noted here that, although two-dimensionality is often assumed in actual problems for the purpose of simplifying the calculation, the difference of the gravity for a two-dimensional body and the corresponding three-dimensional body is large, as illustrated in the sphere problems. The ratios of the maximum gravity of the two-dimensional sphere to that of the three-dimensional sphere are about 1.5, 3, and 4 for $Z = R$, $Z = 2R$, and $Z = 3R$, respectively. The uncertainties arising from the assumption of two-dimensionality are likely to be larger than those caused by the condensation of the mass, even though the actual mass distribution may have better two-dimensionality than spheres. Since the three-dimensional WF calculation is fairly simple and fast, it is

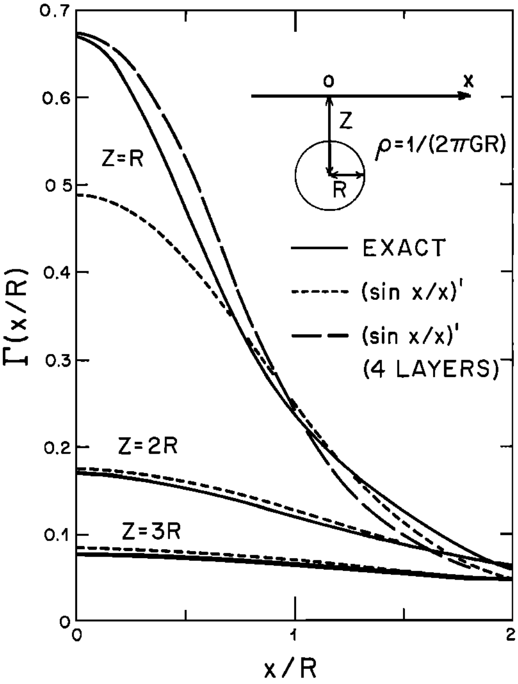

Fig. 2. Gravity for a three-dimensional sphere.

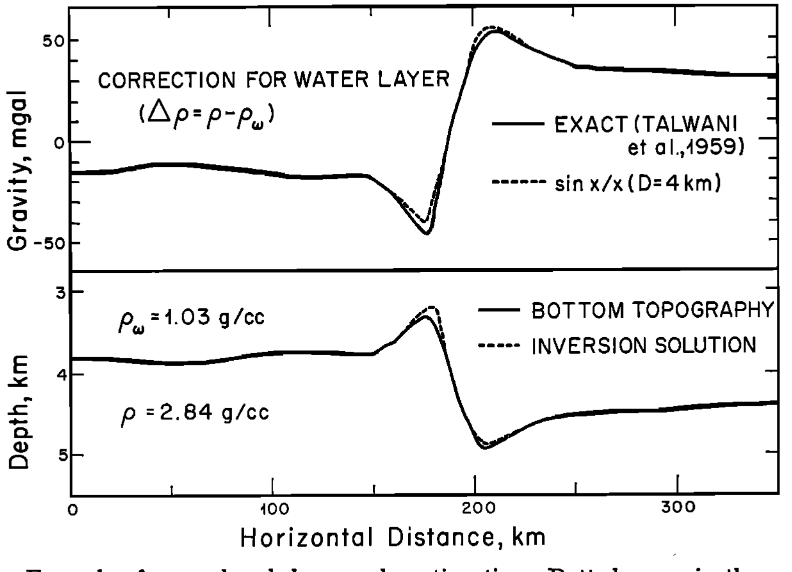

Fig. 3. Example of upward and downward continuations. Dotted curve in the upper half is calculated from the solid curve in the lower half, and dotted curve in the lower half is from the solid curve in the upper half using the (sin x)/x method. The gravity corresponding to a slab 4 km thick and of density $\Delta\rho$ has been subtracted.

advisable to replace the actual mass distribution by an appropriate set of plane density distributions and to apply the three-dimensional WF method.

As an example of more regional problems, the $(\sin x)/x$ method is applied to the correction for a water layer. Figure 3 shows an example taken from *Talwani et al.* [1959]. In the upper half of the figure the gravity of the rock layer ($\rho = 2.84$ g/cm^3) replacing the water layer ($\rho_w = 1.03$ g/cm^3) is given. The effective density is taken as $\Delta\rho = \rho - \rho_w$. The constant gravity corresponding to a slab 4 km thick and of density $\Delta\rho$ has beeen subtracted. In the calculation by the $(\sin x)/x$ method, the condensation level is placed at 4 km depth. The topography is read at every 10 km ($a = 10$) and L is 40. The solution given by the dotted curve can be regarded as essentially the same as the exact solution given by *Talwani et al.* [1959], allowing for the uncertainty originating from the assumption of two-dimensionality. When the actual topography is given by a contoured map, it can readily be used for the three-dimensional calculation by using (3). This kind of calculation, with negative d, is useful for the discussion of local geological structures and the terrain correction for gravity. Even in case of complicated multilayered structures having horizontal inhomogeneities caused by dikes and faults, the method is applicable as long as we introduce a sufficient number of condensation planes to represent the structure.

Inversion

Another advantage of using the coefficients given by (2) or (3) is that they can be used for the downward continuation of gravity merely by changing the sign of d. The downward continuation is illustrated as the inversion problem of the gravity distribution given in Figure 3. In the inversion problems the density distribution on the condensation plane is derived directly from the given gravity distribution by using (2) or (3) with positive d. The variations in density are then interpreted in terms of the variations in the vertical dimension of the mass distribution. The gravity distribution at the surface as given by the solid curve in Figure 3 is now read at equidistant grid points of appropriate spacing, a, to give a set of gravity data $g(ia)$. Assuming the condensation level at the depth of D, $g(ia)$ can be projected upon this level using (2) with $d = \pi D/a$ (≥ 0) to give gravity Γ (ia) at this level as

$$\Gamma(ia) = \sum_{j=-L}^{+L} \phi_j g[(i + j)a]$$

Undulation of topography can now be determined by dividing Γ (ia) by $2\pi G \Delta\rho$. The result obtained with $a = 10$ km, $D = 4$ km, and $L = 40$ is given by a dotted curve in the lower half of the figure. The agreement with the actual topography is good.

A downward continuation of this sort is useful in relating the gravity data to seismic data without introducing a detailed structure. A good example is given by *Press and Biehler* [1964], who discussed the two-dimensional continuation with $D/a = 1$ in relation to seismic velocities. Three-dimensional inversion with $D/a = 0.3$ was made in the discussion of the crustal structure of Japan [*Kanamori*, 1963*b*]. The inversion may also be useful in the interpretation of heat-flow data in terms of heat production, as recently discussed by *Simmons* [1966].

The inversion solution often becomes unstable, however, when D/a is large. The practical limit of D/a appears to be about 1. For this reason, for the local problems as discussed earlier, the inversion method is not suitable. A practical procedure would be to assume the mass distribution, calculate the resulting gravity using (2) or (3) with negative d, and compare it with the observed gravity.

Three-dimensional continuation using $\phi_{i,j}$ given by (3) has been discussed by *Hagiwara* [1965] and *Rikitake et al.* [1965].

Computation by this method is fast. The total computation time is roughly NL $(3\tau_a + \tau_m)$ for the two-dimensional case with N grid points and $(NL)^2$ $(5\tau_a + \tau_m)$ for the three-dimensional case with $N \times N$ grid points, where τ_a and τ_m are the times necessary for single addition and multiplication, respectively. By the IBM 7094 system, the time for a complete problem with $N = 31$ and $L = 20$ is about 0.1 second and 1 minute for two- and three-dimensional cases.

Acknowledgments. I am indebted to Dr. C. H. Dix for various discussions and for critically reviewing the manuscript.

References

Bullard, E. C., and R. I. B. Cooper, Determination of the masses required to produce a given gravitational field, *Proc. Roy. Soc. London, A, 194*, 332–347, 1948.

Hagiwara, Y., Analysis of the results of the aeromagnetic surveys over volcanoes in Japan, 1, *Bull. Earthquake Res. Inst., Tokyo Univ., 43*, 529–547, 1965.

Kanamori, H., A new method for downward continuation of two-dimensional gravity distribution, *Proc. Japan Acad., 39*, 469–473, 1963*a*. Also in *Geophys. Notes, Tokyo Univ., 16*, 1963*a*.

Kanamori, H., Study on the crust-mantle structure in Japan, 1, *Bull. Earthquake Res. Inst., Tokyo Univ., 41*, 743–759, 1963*b*.

Press, F., and S. Biehler, Inferences on crustal velocities and densities from *P* wave delays and gravity anomalies, *J. Geophys. Res., 69*, 2979–2995, 1964.

Rikitake, T., H. Tajima, S. Izutuya, Y. Hagiwara, K. Kawada, and Y. Sasai, Gravimetric and geomagnetic studies of Onikobe area, *Bull. Earthquake Res. Inst., Tokyo Univ., 43*, 241–267, 1965.

Saito, M., and H. Takeuchi, Gravity anomalies and the corresponding crustal structure, 1, *Bull. Earthquake Res. Inst., Tokyo Univ., 42*, 39–92, 1964.

Shimazu, Y., A study of the geophysical and geodetic implications of gravity data for Canada, *Publ. Dominion Obs., 26*, 323–354, 1962.

Simmons, G., Gravity survey and geological interpretation, northern New York, *Bull. Geol. Soc. Am., 75*, 81–98, 1964.

Simmons, G., The interpretation of heat flow anomalies due to contrasts in heat production (abstract), *Trans. Am. Geophys. Union, 47*, 183, 1966.

Talwani, M., and M. Ewing, Rapid computation of gravitational attraction of three-dimensional bodies of arbitrary shape, *Geophysics, 25*, 203–225, 1960.

Talwani, M., J. L. Worzel, and M. Landisman, Rapid gravity computations for two-dimensional bodies with application to the Mendocino submarine fracture zone, *J. Geophys. Res., 64*, 49–59, 1959.

Tomoda, Y., and K. Aki, Use of the function sin x/x in gravity problems, *Proc. Japan Acad., 31*, 443–448, 1955. Also in *Geophys. Notes, Tokyo Univ., 8*, 1955.

Tsuboi, C., and T. Fuchida, Relations between gravity values and corresponding subterranean mass distribution, *Bull. Earthquake Res. Inst., Tokyo Univ., 15*, 636–649, 1937.

Tsuboi, C., C. H. G. Oldham, and V. B. Waithman, Numerical tables facilitating three-dimensional gravity interpretations, *J. Phys. Earth, 6*, 7–13, 1958.

(Received July 16, 1966.)