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EXCITATION OF JOVIAN NORMAL MODES BY AN IMPACT SOURCE

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Abstract. We estimated the amplitudes of Jovian normal modes excited by a near-surface impact. The planet is modeled as a self-gravitating fluid sphere, and the impact source is approximated by a single force or a point pressure source. The effects of rotation are not included. An impact of a comet with a radius of 3 km and momentum Mv=6.8x10^{23} g-cm/s is modeled as a point source 100 km below the Jovian surface, assumed to be at 1 bar. The peak-to-peak amplitudes of displacement and velocity of disturbance with a period of 700 s are approximately 2.1 m and 1.9 cm/s, respectively, at a distance of 90° from the impact. An amplification of a factor of about 25 occurs near the antipode. This estimate is not sensitive to the assumptions on the density distribution above the surface. The amplitude in the higher altitude can be significantly higher. At shorter periods (100 to 200 s), the amplitudes depend on the source depth. For an impact with Mv=10^{23} g-cm/s, and for a source depth of about 20 to 50 km, the amplitude and the particle velocity at a distance of 90° are of the order of 100 m, and 3 m/s, respectively. The pressure change at a depth of 20 to 50 km is of the order of 20 dyne/cm², or about 2x10^{-6} of the ambient pressure at a depth of 40 km.

Introduction

In view of the expected impact of the Shoemaker-Levy 9 comet on the Jovian atmosphere in July, 1994 [Chapman, 1993], it is of interest to estimate the amplitude of seismic excitation of Jovian normal modes by an impact source. Although the periods of Jovian normal modes have been computed by many investigators [e.g. Vorontsov et al. 1976; Bercovici and Schubert, 1987; Mosser, 1990; Lee 1993], their excitation by an impact source has not been investigated.

Although normal-mode excitation in the solid planets like Earth is a straightforward problem, there is some question regarding how the excitation problem should be treated for the planet Jupiter because of the very rapid decrease in density, incompressibility, and sound velocity from the planet interior to the surface. Here, we treat the problem in essentially the same way as for Earth, except that the planet is modeled as a compressible self-gravitating fluid sphere. We assume, somewhat arbitrarily, the 1 bar isobar surface to be the surface of the planet, and compute the displacement on this surface. Although what type of kinematic source is appropriate for a comet impact is not obvious, here we consider excitation by a single force and an isotropic pressure source as a simple extension of the treatment for Earth.

Since the rotation period of Jupiter is about 10 hours, the effects of rotation on normal modes are more significant for Jupiter than Earth. Vorontsov and Zharkov [1981] used a perturbation method to investigate the effects of rotation and ellipticity on low order modes. Their results show that the spectral peaks of low order modes split as much as ±30 %. Mosser [1990] used an asymptotic theory to estimate the effects of rotation on high order modes. He showed that the effects on high order modes are mainly due to the oblateness caused by rapid rotation, rather than the effects of Coriolis force itself. Lee [1993] also investigated the effects of rotation and internal phase boundaries on normal modes. All these studies clearly demonstrated that rotation causes significant spectral splitting.

It is thus clear that the effects of rotation cannot be ignored if we are to use the normal-mode periods to determine the internal structure of Jupiter. However, since the main purpose of this paper is to obtain order-of-magnitude estimates of the normal-mode amplitudes due to an impact, we did not include the effects of rotation in this study. The amplitude of spectral singlets estimated for a non-rotating Jovian model is probably a good estimate of the representative amplitude of split multiplets.

However, we note that the rapid rotation may excite a suite of inertial normal-modes which are not included in computations for a non-rotating model.

Method

The excitation of free oscillations of a self gravitating fluid sphere can be easily formulated from the fully developed theories presented by Love [1911], Pekeris and Jarosch [1958], Alterman et al. [1959], Takeuchi et al. [1961], Bolt and Dorman [1961], Saito [1967] and Takeuchi and Saito [1972].

Using the standard spherical coordinate (r, θ, ϕ), the displacement field (u_r, u_θ, u_ϕ) and the perturbation of the gravitational potential, ψ, in a fluid sphere can be written as follows.

$$u_r = y_1(r)S_l^m(\theta,\phi)e^{i\omega t},\quad u_\theta = y_3(r)\frac{\partial}{\partial\theta}S_l^m(\theta,\phi)e^{i\omega t},$$
$$u_\phi = \frac{y_3(r)}{\sin\theta}\frac{\partial}{\partial\phi}S_l^m(\theta,\phi)e^{i\omega t},\ \text{and}\ \psi = y_5(r)S_l^m(\theta,\phi)e^{i\omega t}, \tag{1}$$

where $y_1(r)$, $y_3(r)$ and $y_5(r)$ are the functions of r, and $S_l^m(\theta,\phi)$'s are the spherical harmonics with the angular and azimuthal order numbers l and m. We introduce two more functions $y_2(r)$ and $y_6(r)$ defined by

$$p = -k\Delta = -y_2(r)S_l^m(\theta,\phi)e^{i\omega t},\ \text{and}$$
$$\psi' - 4\pi G\rho_0 u_r = y_6(r)S_l^m(\theta,\phi)e^{i\omega t}, \tag{2}$$

where p is the pressure, k is the incompressibility, and Δ is dilatation; the symbol " $'$ " denotes differentiation by r.

The functions y_1, y_2, y_5, and y_6 satisfy the following set of differential equations:

$$\begin{aligned} y_1' &= -\frac{2y_1}{r} + \frac{y_2}{k} + \frac{l(l+1)}{r}y_3 \\ y_2' &= -(\omega^2\rho_0 + \frac{4\rho_0 g_0}{r})y_1 + \frac{l(l+1)\rho_0 g_0}{r}y_3 - \rho_0 y_6 \\ y_5' &= 4\pi G\rho_0 y_1 + y_6 \\ y_6' &= -\frac{4\pi G\rho_0 l(l+1)}{r}y_3 + \frac{l(l+1)}{r^2}y_5 - \frac{2y_6}{r}, \end{aligned} \tag{3}$$

where G, $\rho_0(r)$, and $g_0(r)$ are the universal constant of gravity, unperturbed density and gravity inside the planet, respectively.

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The function $y_3(r)$ above is given by

$$y_3 = \frac{1}{r\omega^2}(g_0 y_1 - \frac{y_2}{\rho_0} - y_5) \quad (4)$$

Solving (3) with the "free surface" boundary condition and the condition for continuity of gravitational potential at the surface $r=a$:

$$y_2=0, \quad \text{and} \quad y_6+[(l+1)/a]y_5=0 \quad (5)$$

results in a set of eigen values ${}_n\omega_l^m$ and eigen functions ${}_nu_l^m(r)$, each with radial factors y_1, y_2, ..., and y_6. The azimuthal order number m for ${}_n\omega_l^m$ can be dropped when rotation, ellipticity and lateral heterogeneity are ignored.

When a step function single force f is applied at (r_s, 0, 0) in the direction of $\theta=\pi/2+\delta$, and $\phi=\phi_f$, the displacement at $\vec{r}$ (r, θ, ϕ) is given by

$$\begin{aligned} u_r(\vec{r},t) &= \sum_l y_1(r)\frac{r_s}{y_3(r_s)}K_2[P_l^0 y_1(r_s)f\sin\delta - P_l^1 y_3(r_s)f \\ &\quad \cdot\cos\delta\cos(\phi_f-\phi)]\cos\omega_l t \\ u_\theta(\vec{r},t) &= \sum_l y_3(r)\frac{r_s}{y_3(r_s)}K_2[\frac{\partial P_l^0}{\partial\theta}y_1(r_s)f\sin\delta \\ &\quad -\frac{\partial P_l^1}{\partial\theta}y_3(r_s)f\cos\delta\cos(\phi_f-\phi)]\cos\omega_l t \\ u_\phi(\vec{r},t) &= -\sum_l y_3(r)r_s K_2\frac{1}{\sin\theta}P_l^1 f\cos\delta\sin(\phi_f-\phi)\cos\omega_l t \end{aligned} \quad (6)$$

where

$$K_2 = \frac{2l+1}{4\pi\omega_l^2(I_1+l(l+1)I_2)}\frac{y_3(r_s)}{r_s} \quad (7)$$

Similarly, for a step function isotropic source with a dipole moment of M_0, the displacement is given by

$$\begin{aligned} u_r(\vec{r},t) &= \sum_l M_0 y_1(r)N_0 P_l^0\cos\omega_l t \\ u_\theta(\vec{r},t) &= \sum_l M_0 y_3(r)N_0\frac{\partial P_l^0}{\partial\theta}\cos\omega_l t \\ u_\phi(\vec{r},t) &= 0, \end{aligned} \quad (8)$$

where

$$N_0 = -\frac{2l+1}{4\pi\omega_l^2(I_1+l(l+1)I_2)}\frac{y_2(r_s)}{k_s} \quad (9)$$

An isotropic source with a dipole moment M_0 is equivalent to a pressure source in a small cavity of radius ε with pressure $p=M_0/3\pi\varepsilon^3$.

In (7) and (9), I_1 and I_2 are the energy integrals defined by

$$I_1 = \int_0^a \rho_0 r^2 y_1^2(r)dr, \quad \text{and} \quad I_2 = \int_0^a \rho_0 r^2 y_3^2(r)dr \quad (10)$$

The excitation functions K_2 and N_0 were used in Kanamori and Cipar [1974], Okal [1978] and Kanamori and Given [1982]. In (6), (7), (8), and (9), the order numbers n and m for eigen frequencies and eigen values are omitted. Expressions (6) to (9) hold for each radial overtone number n which is omitted for clarity.

The pressure $p(\vec{r},t)$ can be obtained by replacing $y_1(r)$ by $-y_2(r)$ in $u_r(\vec{r},t)$ in (6) and (8).

To compute short-period motions, l must be increased. However, to compute the motion at a period of 100 s, normal modes with l up to 10,000 must be computed, and the mode summation is impractical. It is more convenient, then, to use high-frequency asymptotic forms of equations (6) and (8) using the asymptotic form of the Legendre functions. Then (6) can be written as

$$u_r(\vec{r},t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{C}(\omega)\exp(i\omega t)d\omega \quad (11)$$

where

$$\hat{C}(\omega) = \sqrt{\frac{1}{\sin\theta}}\exp(-i\omega a\theta/C+\pi i/4)y_1(r)\frac{r_s}{l}\sqrt{\frac{\pi}{2l}}\frac{a}{U}(-K_2 l^2)f \cdot\left[-\frac{y_1(r_s)}{l y_3(r_s)}\sin\delta + i\cos\delta\cos(\phi-\phi_f)\right],$$

where C and U are the phase and group velocities respectively.

Results

Vorontsov et al. [1976] used equation (3) to compute the periods of the Jovian normal modes. We used the structure given in Vorontsov et al. [1976] to compute the normal modes. We smoothed the density and pressure curves given in Vorontsov et al. [1976], and computed incompressibility by $k=\rho_0(\Delta p_0/\Delta\rho_0)$. At the surface we used the relationship for ideal gas, $k=\gamma p_0$ to compute k.

The eigen frequencies, f, thus computed for the fundamental mode (n=0) and the first overtone (n=1) are shown in Figure 1 as a function of the angular order number l. We computed all the modes for $l\leq200$, but only at selected order numbers for $l\geq200$. The skin depths (the depth where the amplitude decays to $1/e$ of that at the surface) of the eigen function y_1 are 380, 150, 75, 40, 25, 15, and 7 km for periods of 729, 462, 328, 231,189, 146, and 103 s, respectively. The corresponding phase velocity C is given by $2\pi af/(l+0.5)$. These results are in good agreement with those obtained by Vorontsov et al. [1976].

Long-period oscillation

We used only fundamental modes with l=2 to 200, and computed the displacement by summing modes (equation 6); we assumed Q to be 10^5, which means that the medium is virtually non-attenuating. We took the derivative of the displacement given by (6) to obtain the displacement for a delta function source with a magnitude of 10^{23} dyne/s. Although

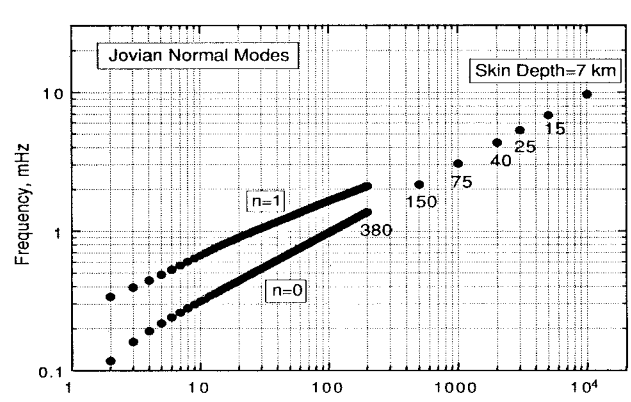

Fig. 1. Jovian normal modes. Eigen frequencies are shown as a function of the angular order number l, for the fundamental mode (n=0) and the first overtone (n=1). The skin depths are shown for a few modes.

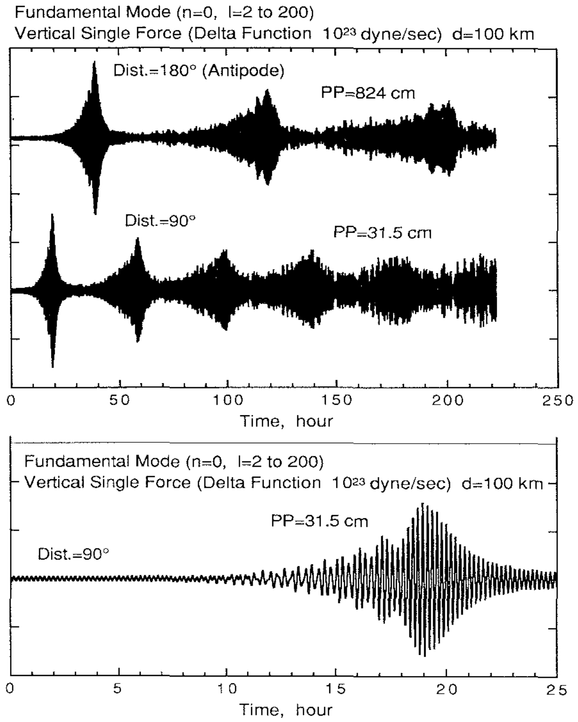

Fig. 2. Synthetic seismograms at Δ=90° and 180° computed for a vertical single force applied at a depth of 100 km. The detail of the first wavetrain is shown with an expanded time scale on the figure at the bottom.

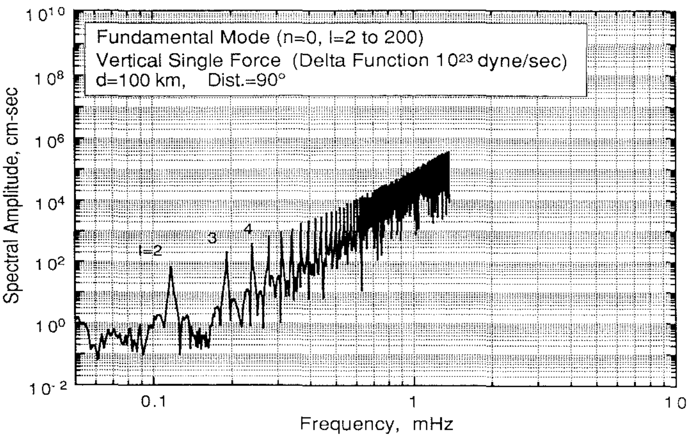

Fig. 3. The spectrum of the seismogram at Δ=90° computed for a single force shown in Figure 2.

the exact mechanism of seismic excitation by the impact of the Shoemaker-Levy 9 comet is uncertain, the time scale of the impact would be much shorter than the period of the normal modes, and the source can be approximated by a delta function. If the density and the entry velocity of the comet are 1 g/cm³ and 60 km/s, respectively, this force is equivalent to a momentum transfer, Mv, of a comet with a radius of 1.6 km.

Figure 2 shows the displacements at a distance of Δ=90° and 180° (antipode of the source) computed for a vertical single force applied at a depth of 100 km. The peak-to-peak amplitudes are 31.5 and 824 cm at Δ=90° and 180°, respectively. An amplification of a factor of about 25 occurs near the antipode. The results are summarized in Table 1, which includes the estimates for comets with larger sizes. Figure 3 shows the spectrum of the displacement at Δ=90°. The excitation amplitude increases rapidly with frequency; over the frequency range shown in Figure 3,

$$A(f) \propto f^4 \tag{13}$$

Since we included the modes only up to l=200 (period of about 700 s) in this computation, the waveform shown in Figure 2 is dominated by the wave with a period of about 700 s. Since the skin depth at this period is 380 km (Figure 1), the amplitude does not significantly vary for a range of source depth from 0 to 300 km. As the frequency increases beyond 2.5 mHz (400 sec), the amplitude of the eigen function decreases significantly at a depth of 100 km. As a result, excitation by a source at a depth of 100 km decreases and $A(f)$ rolls off beyond this frequency. However, for a very shallow source the trend given by (13) continues to higher frequencies.

For a source within a depth range of 0 to 300 km, the amplitude of the fundamental modes is about four times larger than that of the first overtone at a given period (not shown here). Hence a comet impact will excite primarily the fundamental modes.

Short-period oscillation

We computed short-period waves using the asymptotic form (11) with l up to 10,000 (T=100 s). As suggested by the skin depths shown in Figure 1, the excitation at short period diminishes very rapidly as the source depth increases to 50 km. Also, at short period, the modes are strongly affected by the density and temperature distributions near the surface. Although detailed computations cannot be made, the effects can be expressed in terms of a cut-off frequency for normal modes. At frequencies higher than a certain frequency, the acoustic cut-off frequency, the mode energy propagates into the atmosphere, which prevents development of normal modes in the planet's interior. Bercovici and Schubert [1987] and Vorontsov et al. [1989] suggest that the acoustic cut-off frequency for Jupiter is about 3 mHz. Thus, the results at periods shorter than 300 s obtained here are subject to large uncertainties.

The pressure vanishes at the surface, increases gradually as the depth increases, and begins to decrease at a certain depth. The results are shown in Table 2, and Figure 4 shows the variation of pressure as a function of depth. The pressure change is about 2×10^{-6} of the ambient pressure at a depth of 40 km.

Table 1. Peak-to-peak amplitude of displacement and velocity at T=700 s.

Source	Δ=90°	Δ=180° (Antipode)
$Mv=10^{23}$ g-cm/s	32 cm	8.2 m
(a_c=1.6 km)	0.28 cm/s	16 cm/s
$Mv=6.8\times10^{23}$ g-cm/s	2.1 m	56 m
(a_c=3 km)	1.9 cm/s	.1 m/s
$Mv=2.5\times10^{25}$ g-cm/s	79 m	2.1 km
(a_c=10 km)	0.71 m/s	19 m/s

Table 2. Peak-to-peak amplitudes of displacement, velocity and pressure of short period (about 100 to 200 s) waves at Δ=90° for an impact with $Mv=10^{23}$ g-cm/s.

Depth (km)	Disp. (m)	Vel. (m/s)	Pressure (dyne/cm²) at 20 km	Pressure (dyne/cm²) at 50 km
0	890	15	51	81
20	210	4.1	22	37
50	98	1.2	6.7	15
100	12	0.21	2.1	7.1

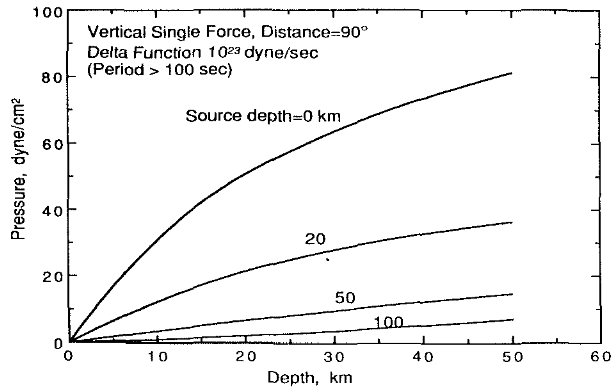

Fig. 4. Pressure changes as a function of depth. The source depth is varied from 0 to 100 km.

Discussion and Conclusion

In the above, we assumed the 1 bar surface to be the Jovian "free surface", and computed the displacements on this surface. In the real Jovian atmosphere, ρ_0 and k do not vanish completely above the 1 bar surface. If we assumed small ρ_0 and k at higher altitudes and computed the displacement there, the amplitude would be significantly larger. However, since the total normal-mode energy contained in the higher altitude is small because of the low density, the energy integrals I_1 and I_2 in the excitation function K_2 (equation 7) would remain approximately the same. As a result, $r_s K_2/y_3(r_s)$ in u_r and u_θ in (6) would also remain unchanged. Then it is evident from (6), that the displacement at r is determined by the values of the eigen functions at r and the source depth r_s. This means that the displacement at the 1 bar surface is insensitive to what we assume above this surface. Thus we can conclude that, despite the uncertainties in the details of the atmospheric structure near the Jovian surface, the amplitude at the 1 bar surface computed above is constrained well at periods longer than 700 s. At very short periods, however, the "free surface" boundary condition would be no longer valid.

The detectability of Jovian normal modes using Doppler shift of infrared and ultra-violet absorption lines has been discussed by Schmider et al. [1991] and Mosser et al. [1992]. According to these investigators, the detection threshold in velocity appears to be about 1 m/s for modes with a period of 5 to 20 min. Our results show that only for a comet with a radius larger than 6 km, particle velocities exceeding 1 m/s at a period of 700 s is expected, at least in the antipodal region where focussing of energy occurs (Table 1). For smaller comets, the excitation at long period would be probably too small to be detected. At periods around 100 to 300 s, the amplitude would be much larger, but the short wavelength of these modes and the complexities of the surface boundary conditions would make detection of such modes difficult.

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