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The frequency dependence of Q in the Earth and implications for mantle rheology and Chandler wobble

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Summary. For most solids the 'high temperature background' attenuation dominates at low frequencies and temperatures greater than about one-half the melting temperature. It is likely to be important in the mantle at seismic frequencies. The same mechanism also contributes to transient creep at low stresses and low total strains. A relaxation spectrum is found which satisfies the frequency dependence of laboratory Q and the time dependence of transient creep data. This makes it possible to provide a physical interpretation of the parameters in Jeffrey's modified Lomnitz creep function.

Q is predicted to increase as ω^{α} in the lower Q regions of the mantle. At high and low frequencies Q should increase as ω and ω^{-1}, respectively. The location of the ω^{α} band depends on temperature and therefore shifts with depth. At high temperatures, seismic waves are on the low-frequency side of the absorption band and Q decreases with frequency. Far from the melting point and at sufficiently high frequencies Q should increase linearly with frequency. We use Chandler wobble, tidal and free oscillation data to estimate that α is $\sim 1/5$ to $1/3$, consistent with laboratory measurements of transient creep and internal friction at high temperature. A preliminary attempt is made to estimate the transient creep response of the mantle from Q measurements. The inferred viscosity agrees well with direct measurements.

The effect of anelasticity is to lengthen the calculated period of the Chandler wobble by 5–20 days, depending on the Chandler wobble Q. A Q of 300 for the wobble, which is within the experimental uncertainty of recent determinations, gives the observed period after correcting for the effect of the oceans.

1 High temperature attenuation and transient creep

The Q of the Earth is usually assumed to be frequency independent but this is due more to the absence of pertinent information than to the presence of supporting evidence.

The dominant mechanism of attenuation in solids at temperatures of the order of one-half of the melting temperature and above is the high temperature background or HTB and this is likely to be important in the mantle (Anderson 1967; Jackson & Anderson 1970). The HTB increases exponentially with temperature and is a function of frequency and grain size but is independent of amplitude, at least to strains less than about 10^{-5}. The functional form is (e.g. Friedel 1964; DeBatist 1969)

$$Q^{-1}(\omega) = Q_0^{-1}(\omega\tau)^{-\alpha}$$
$$= Q_0^{-1}(\omega\tau_0)^{-\alpha} \exp(-\alpha E^*/RT) \tag{1}$$

where Q_0^{-1} and α are constants, E^* is the activation energy for diffusion and τ is a characteristic time. α is generally between ¼ and ½. When E^* is known from diffusion or creep experiments the parameter α can be determined from the temperature dependence of Q^{-1} as well as from its frequency dependence.

Attenuation is closely related to transient creep (Gross 1953), and specifically, equation (1) implies a transient creep response of the form

$$\epsilon(t) = \epsilon_0(t/\tau)^{\alpha}$$
$$= \epsilon_0(t/\tau_0)^{\alpha} \exp[-\alpha E^*/RT] \tag{2}$$

where t is the time.

This is in addition to the instantaneous elastic response. The transient response is appropriate for short term and small strain situations such as seismic wave attenuation, tidal friction, damping of the Chandler wobble and post-earthquake rebound. Note that the apparent activation energy, αE^*, is less than the true activation energy.

Many materials satisfy the transient creep law (2) with α between ⅓ and ½. Such values are obtained for example from transient creep experiments on rocks at high temperature (Goetze 1971; Goetze & Brace 1972; Murrell & Chakravarty 1973). Equations (1) and (2) are essentially equivalent. If attenuation is known as a function of frequency, transient creep can be determined and vice versa. In this paper we argue that such equivalence can be applied fruitfully to the study and interpretation of seismic wave attenuation and creep in the Earth.

When α = ⅓ equation (2) is the well-known Andrade creep equation. Some theoretical support exists for this value of the exponent (Mott 1953). This equation cannot hold for all time since it predicts infinite creep rate at $t = 0$ and a continuously decreasing strain rate as time increases. Real materials at high temperature with a constantly applied stress reach a steady-state creep rate at long time. As Jeffreys (1972) has pointed out, thermal convection is precluded for a material having the rheology of equation (2) and he has used this to argue against continental drift. This prohibition, however, only applies to relatively short times before steady state is established. For very small stresses the strain rate does approach zero at long times for some transient creep mechanisms such as dislocation bowing under stresses less than those required for dislocation multiplication. The critical stress is $\sigma = Gb/l$ where G is the shear modulus, b is the Burger's vector and l is the dislocation length. For dislocation lengths greater than 10^{-3} cm, multiplication, and therefore steady-state creep, will occur for stresses as low as 10 bar.

2 A strain retardation model

Equations (1) and (2) are usually considered as merely phenomenological descriptions of the behaviour of solids at high temperatures. However, they can be constructed from a linear superposition of elastic and viscous elements and are thus a generalization of the standard linear solid (Zener 1948) or the standard anelastic solid (Nowick & Berry 1972).

The creep response of the standard linear solid with a single characteristic relaxation time (actually the strain retardation time), τ, is

$$\epsilon(t) = \sigma[J_u + \delta J(1 - \exp(-t/\tau))]$$

where σ is the stress, J_u the unrelaxed compliance (the reciprocal of the conventional high frequency elastic modulus) and δJ is the difference between the relaxed (i.e. long time or low frequency) and unrelaxed (i.e. short time or high frequency) compliances.

A more general model involves a spectrum of retardation times. For a distribution of characteristic times, $D(\tau)$, the creep is

$$\epsilon(t) = \sigma\left[J_u + \delta J\int_0^\infty D(\tau)[1 - \exp(-t/\tau)]\, d\tau\right]. \tag{3}$$

Although the stress relaxation spectrum must satisfy a variety of physical constraints (Gurtin & Herrera 1965; Chin & Thigpen 1979), the retardation spectrum used here need only yield physically realizable creep behaviour, in particular at short and long times. $D(\tau)$ will be assumed to be limited to a finite band $[\tau_1, \tau_2]$ – the cutoffs are determined by the physics of the problem, and depend on physical variables (e.g. diffusivity), geometric variables (e.g. diffusion paths), as well as the thermodynamic conditions.

The distribution function

$$D(\tau) = \begin{cases} \dfrac{\alpha}{\tau_2^\alpha - \tau_1^\alpha}\dfrac{1}{\tau^{1-\alpha}} & \text{for } \tau_1 \leqslant \tau \leqslant \tau_2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

yields a creep response which can be approximated by

$$\epsilon(t) \simeq \sigma J_u\left[1 + \frac{\delta J}{J_u}\Gamma(1-\alpha)\left(\frac{t}{\tau_2}\right)^\alpha\right], \qquad \tau_1 \ll t \ll \tau_2. \tag{5}$$

Comparison with equation (2) yields

$$\epsilon_0 = \frac{\delta J}{J_u}\Gamma(1-\alpha) = \frac{\delta M}{M_r}\Gamma(1-\alpha) \tag{6}$$

where M_r refers to the relaxed modulus. Note that the characteristic time appearing in equation (2) is actually the upper cutoff τ_2.

Furthermore, this model yields a finite initial creep rate

$$\dot{\epsilon} \simeq \sigma\delta J\frac{\alpha}{1-\alpha}\frac{\tau_1^{\alpha-1} - \tau_2^{\alpha-1}}{\tau_2^\alpha - \tau_1^\alpha}. \tag{7}$$

A finite creep rate at the origin was achieved differently by Jeffreys (1958) who proposed what he called the 'modified Lomnitz law'

$$\epsilon(t) = \frac{\sigma}{M_u}\left[1 + \frac{q}{\alpha}\left\{\left(1 + \frac{t}{\tau}\right)^\alpha - 1\right\}\right] \tag{8}$$

so that

$$\epsilon(t) \simeq \frac{\sigma}{M_u}\left[1 + \frac{q}{\alpha}\left(\frac{t}{\tau}\right)^\alpha\right] \qquad \text{for } t \gg \tau. \tag{9}$$

By identification with equation (5) and (7) we obtain the correspondance

$$\left.\begin{aligned} \tau &= [\Gamma(2-\alpha)]^{1/1-\alpha}\tau_1 \\ q &= [\Gamma(2-\alpha)]^{1/1-\alpha}\frac{\delta J}{J_u}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\tau_1}{\tau_2}\right)^{\alpha} \end{aligned}\right\}. \tag{10}$$

We note that the time constant in Jeffreys' equation is related to the shortest relaxation time in the spectrum, τ_1, which is less than 1 s for many materials at high temperature. The parameter q is related to the ratio τ_1/τ_2 which can be very small. For a dislocation bowing mechanism τ is proportional to the dislocation length squared (Minster & Anderson, in preparation). A spread of two orders of magnitude in dislocation lengths gives τ_1/τ_2 equal to 10^{-4}.

The Q corresponding to equation (5) for frequencies, ω, such that

$$\omega\tau_1 \ll 1 \ll \omega\tau_2$$

is given by

$$Q(\omega) = \cot\frac{\alpha\pi}{2} + \frac{2J_u}{\pi\alpha\delta J}[(\omega\tau_2)^{\alpha} - (\omega\tau_1)^{\alpha}]\cos\frac{\alpha\pi}{2}$$

$$\cong \frac{2J_u}{\pi\alpha\delta J}(\omega\tau_2)^{\alpha}\cos\frac{\alpha\pi}{2} \tag{11}$$

since $\tau_2 \gg \tau_1$ and $\cot \alpha\pi/2$ is small. These results are essentially equivalent to Macdonald (1961).

Thus, by comparison with equation (1)

$$Q_0 = \frac{2J_u}{\pi\alpha\delta J}\cos\frac{\alpha\pi}{2}. \tag{12}$$

Therefore Q is predicted to increase with frequency as ω^{α}. Jeffreys (1972) in an application of equation (8) to seismic and wobble data suggested $\alpha \sim 0.2$.

For high frequencies $1 \ll \omega\tau_1 \ll \omega\tau_2$

$$Q(\omega) = (1-\alpha)(J_u/\alpha\delta J)(\tau_2/\tau_1)^{\alpha}(\omega\tau_1). \tag{13a}$$

At low frequencies

$$Q(\omega) = (1+\alpha)(J_r/\alpha\delta J)(\omega\tau_2)^{-1}. \tag{13b}$$

For a thermally activated process the characteristic times depend on temperature

$$\tau = \tau_0 \exp(E^*/RT).$$

Inserting this into equations (5) and (11) yields the experimental equations. It is clear that at very low temperature the characteristic times are long and $Q \sim \omega$. At very high temperatures Q is predicted to increase as ω^{-1}. Equation (11) holds for intermediate temperatures.

3 Application to the Earth

Because of the importance of the frequency dependence of Q we will pursue this point further. It is important for several reasons. Since attenuation affects the periods of the natural oscillations of the Earth, including the normal modes and the wobble, it must be known in order to compare seismic data taken at different frequencies and to compute the theoretical period of the Chandler wobble. It must also be known in order to understand the damping of the wobble and tidal dissipation in the solid Earth. It is difficult, however, to separate the effects of the frequency dependence of Q from those stemming from its depth variation.

The fundamental spheroidal mode, ${}_0S_2$, the solid Earth tides and the Chandler wobble all sample the interior of the Earth in approximately the same way (Lagus & Anderson 1968; Jeffreys 1972). When corrected for ocean and rigid body rotation effects these modes offer the hope of shedding light on the frequency dependence of Q.

Recent estimates of Q_W are 30–35 (Mandelbrot & McCamy 1970), 50–400 (Wilson & Haubrich 1976), 52–92 (Currie 1974), 50–300 (Ooe 1978) and 40–60 (Yatskiv & Sasao 1975). Deformation of the Earth lengthens the theoretical period of the wobble from 305 to 404 days (Smith 1977), which means that only 24 per cent of the available energy is actually stored as deformational energy. For our purposes, the estimates of Q_W quoted above must therefore be divided by ~ 4.

A recent estimate of solid Earth tidal Q is 250 with limits between 160 and 480 (Lambeck 1977). The 54 min spheroidal mode ${}_0S_2$ has a Q of 589 ± 10 per cent (Sailor & Dziewonski 1978). These estimates are plotted in Fig. 1. Although there is uncertainty in the reduction of the data in all three cases there is an indication that Q increases with frequency and, in particular

$$Q = 4.72 \times 10^3 \omega^{1/3} \tag{14}$$

is consistent with the data. The data can accommodate α from about 0.2 to 0.4.

The mode ${}_1S_{11}$ was recently retrieved from a normal mode spectrum by Buland & Gilbert (1978). This mode has a period of 426 s and samples the mantle and core in roughly the same way as ${}_0S_2$. Its observed Q is > 900 and its theoretical Q relative to ${}_0S_2$, assuming no frequency dependence and model SL8 (Anderson & Hart 1978) is about 0.8. Therefore, it has an effective '${}_0S_2$ like' Q of > 1100. This point is also shown.

Therefore a frequency-dependent Q for the mantle of the same general type as found for a variety of materials at high temperature seems likely, at least in the period range

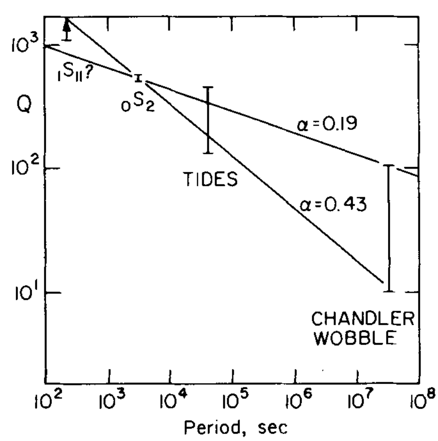

Figure 1. *Q* **versus period for normal modes, solid earth tides and Chandler wobble. The observed value for the Chandler wobble has been corrected for the non-dissipative part due to rigid body rotation.**

represented on Fig. 1. It is of interest to point out that transient creep experiments on a variety of igneous rocks at high temperature yield values for α generally between 0.33 and 0.44 (Goetze 1971; Goetze & Brace 1972; Murrell & Chakravarty 1973). These are for much higher stresses than the geophysical phenomena discussed here but they may provide a measure of the appropriate distribution function, for example, the distribution of dislocation lengths.

At frequencies of the order of 1 s and temperatures of about one-half the melting point or less a grain boundary relaxation peak becomes important in metals (e.g. Nowick & Berry 1972). This peak moves to higher temperatures at higher frequencies and may become important in the Earth at body wave periods. This would keep the Q from continuing to increase with frequency as predicted by equations (13a) and (14). The grain boundary peak and its possible relevance to seismic attenuation has been discussed by Jackson (1971), Solomon (1972) and Anderson & Hart (1978).

Kurita (1968) and Solomon & Toksöz (1970) have summarized the evidence for the frequency dependence of Q at relatively short periods. From about 1 to 15 Hz Q seems to be about proportional to frequency. This means equation (13a) applies and $\omega\tau_1 > 1$. Takano (1971) noted that Q for P waves doubled as the period decreased from about 3 to 0.3 s. Yoshida & Tsujiura (1975) found that the Q for *ScS* increased from 150 to 220 as the mean period decreased from 50 to 17 s. Sipkin & Jordan (1979) argue that the Q for *ScS* increases rapidly with frequency for frequencies higher than ~ 0.5 Hz, and suggest that this is consistent with a high-frequency cutoff of the relaxation spectrum $\tau_1 \sim 0.2$–1.0 s. This estimate agrees with the cutoff suggested by Minster (1978). Although the radial variation of τ must be taken into account in a more precise calculation it appears that Q may be on the high-frequency linear branch, equation (13a), in most regions of the Earth at frequencies greater than 1–10 Hz. Laboratory measurements of HTB on polycrystalline Al_2O_3 at high temperatures have a linear Q versus ω relation for frequencies as low as 0.2 Hz (Jackson 1969). This gives $\tau_1 > 8$ s.

4 Effect of absorption on Chandler wobble period

The Q of the wobble is so low that energy sinks have been sought outside the mantle which has too high a Q at seismic frequencies. It has been generally thought, except by Jeffreys (1958), Lomnitz (1962) and Macdonald (1961), that seismic Qs should be appropriate at Chandler periods. We have shown that a low Q is expected and the observed Q is actually consistent with seismic and tidal Qs when frequency dependence is taken into account.

The effects of elasticity, fluid core and oceans on the period of the Chandler wobble are now fairly well understood. Smith (1977) has calculated 403.6 sidereal days for the period for a realistic earth model including a neutrally stable fluid core and has shown that the period is insensitive to the earth model. A moderately stable core lengthens the period by about 1.5 days. An equilibrium theory for the oceanic tidal response lengthens the period by 27.6 days (Dahlen 1976). Therefore, the predicted Chandler wobble period is about 431 days. Observations are 434.2 ± 2 (Jeffreys 1968; Ooe 1978), 435.2 ± 2.6 (Wilson & Haubrich 1976) and 436.9 ± 0.7 (Guinot 1972). Gaposchkin (1972) obtained 437.20 days for the main Chandler peak and a smaller peak at 426.7 days. Thus, there is excellent agreement between theory and observation but the observed period may be longer by as much as 6 days.

It is therefore of interest to reassess the effect of attenuation on the period of the Chandler wobble. The elastic earth model used in Smith's calculations is based on free oscillations and therefore represents a high frequency, relative to the Wobble period, model.

Anderson *et al.* (1976) estimate that Q would lengthen the period by 1–2 days if Q is independent of frequency.

For constant $Q \gg 1$ the seismic velocities at two frequencies are related by

$$\frac{C(\omega_2)}{C(\omega_1)} \equiv 1 + \frac{1}{\pi Q} \ln\left(\frac{\omega_2}{\omega_1}\right) \tag{15}$$

(Liu, Anderson & Kanamori 1976; Anderson *et al.* 1976; Kanamori & Anderson 1977). This can be used to calculate the effective rigidity, μ_1 at a period of 14 months from the rigidity μ_2 required to satisfy the spheroidal mode ${}_0S_2$ at a period of 54 min. For $Q = 589$, a value appropriate for ${}_0S_2$ (Sailor & Dziewonski 1978), the result, for constant Q is

$$\frac{\mu_1}{\mu_2} = 0.989.$$

The change in Love number, k, is (Jeffreys 1972; Dahlen 1979)

$$\frac{dk}{k} \sim -\frac{4}{5}\frac{d\mu}{\mu}.$$

The change in period is

$$\frac{dT}{T} = \frac{k}{k_f - k}\frac{dk}{k}$$

where k_f is the fluid Love number for a density stratified earth ($k_f \sim 0.937$, Stacey 1977). Using a value of $k = 0.301$ (Smith 1977), the calculated increase in period is 1.8 days for a constant Q model.

The dispersion appropriate for the frequency-dependent Q model considered in this paper gives, in the case $Q \gg 1$

$$\frac{\mu_1}{\mu_2} = 1 - \cot\left(\frac{\alpha\pi}{2}\right)(\omega_2/\omega_1)^{\alpha} Q_2^{-1} \tag{16}$$

where Q_2 is the Q at ω_2, in this case, for ${}_0S_2$.

We take 589 for $Q({}_0S_2)$ and $\alpha = 1/3$. This gives $Q(\text{cw}) = 108$ and a $Q_d(\text{cw})$ of 26, where Q_d is the deformational, or solid earth Q.

This gives an increase in Chandler period of 10.2 days. Using the extreme bounds of the range of available measurements quoted above we have

$$\begin{aligned} &35 \lesssim Q(\text{cw}) \lesssim 400 \\ &10 \lesssim Q_d \lesssim 100 \\ &0.19 \lesssim \alpha \lesssim 0.43 \\ &5.4 \lesssim dT \lesssim 19.5. \end{aligned} \tag{17}$$

The data slightly favour the higher values of Q and thus the lower values of α, in agreement with the conclusion of Jeffreys (e.g. Jeffreys 1972). On the other hand, with slightly different assumptions ($Q = 300$ at a period of 200 s) Dahlen (1979) concluded that a constant Q model is not inconsistent with the observed wobble period. In spite of the large uncertainties attached to the various estimates used in our agreement, it can be concluded that both the observed period and damping of the Chandler wobble are consistent with the model and the hypothesis that wobble energy is dissipated in the mantle.

5 Mantle rheology

By combining equations (5) and (11) we can write

$$\eta(t) = 10.8 \times 10^{15} t^{2/3} P (t \text{ in s}) \qquad \text{for } \tau_1 < t < \tau_2.$$

This gives a viscosity of $10^{21}P$ at the end of 1 yr and $10^{23}P$ at the end of 10^4 yr as estimates of the viscosity of the whole mantle. These values are upper bounds if $t > \tau_2$. The upper mantle has an average Q of about 1 order of magnitude less than the Q of the whole mantle. The corresponding viscosities are, therefore, $10^{20}P$ and $10^{22}P$, again upper bounds if $t > \tau_2$. In a subsequent paper we treat the high stress case and the transition to steady-state behaviour.

6 Conclusions

The relationship between Q and transient creep is well known. What we have shown is that the temperature and frequency dependence of the high temperature background (HTB) can be determined from transient creep equations of the type proposed by Andrade (1910), Jeffreys (1958) and Mott (1953). We have also shown that these equations can be derived from linear theory with an appropriate distribution of relaxation times. This makes it possible to attach a physical significance to the parameters in the mainly phenomological transient creep laws. In a later paper we will show how the characteristic times can be derived from diffusion and dislocation theory. In the present paper we were mainly concerned with relating Q to transient creep in the region between the shortest and longest relaxation times. In this case parameters such as temperature and pressure cancel out.

The effect of a frequency-dependent Q is to lengthen the wobble period by about 5 to 20 days. The observed period of the wobble is consistent with a Chandler wobble Q of about 300. With this value most of the damping of latitude variations can occur in the mantle and no resort need be made to electromagnetic or ocean effects. The correction of normal mode data to body wave period will be less than in the case of frequency-independent Q.

The transient viscosity of the mantle can be estimated from the Q data. Although data on the magnitude of the relaxation times are lacking, and we do not know, therefore, to how long a time we can apply the present analysis, the calculated viscosities are reasonable.

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