Nanomechanical Analog of a Laser: Amplification of Mechanical Oscillations by Stimulated Zeeman Transitions

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We propose a magnetomechanical device that exhibits many properties of a laser. The device is formed by a nanocantilever and dynamically polarized paramagnetic nuclei of a solid sample in a strong external magnetic field. The corresponding quantum oscillator and effective two-level systems are coupled by the magnetostatic dipole-dipole interaction between a permanent magnet on the cantilever tip and the magnetic moments of the spins, so that the entire system is effectively described by the Jaynes–Cummings model. We consider the possibility of observing transient and cw lasing in this system, and show how these processes can be used to improve the sensitivity of magnetic resonance force microscopy.

The invention of masers and lasers in the middle of the twentieth century [1] has engendered whole new fields of science and myriads of applications. This success of laser science and technology demonstrates the value of the basic principles of laser devices and encourages one to look for other systems in which these principles can be realized.

Regardless of the frequency range and other details of a practical implementation, all laser-like devices involve one or more quantum-mechanical oscillators resonantly interacting with a continuously pumped multilevel quantum system. In the ubiquitous optical laser, the oscillator is realized by a mode of a high-Q electromagnetic cavity, a mode resonant with optical transitions of bound electrons in the active medium. Masers use microwave transitions of gas molecules or electron spins of a paramagnetic solid in a strong magnetic field. Finally, the active medium of a free-electron laser is a relativistic electron beam, whose energy levels can be defined by a specially configured magnetic field [2].

This relative diversity of possible realizations of the active medium is not, however, matched by the demonstrated realizations of the other essential part of a lasing system, the oscillator. The authors are aware of only one laser-like device that used an oscillator different from a field mode of an electromagnetic cavity—the nuclear-magnetic-resonance (NMR) laser [3]. In that device, nuclear spins of a solid sample were inductively coupled to a resonant LC circuit. Although many properties of an LC circuit are strikingly different from those of a cavity resonator, one can argue that the underlying physics in the two cases is the same: The oscillations correspond to normal modes of a complex electromagnetic system, whether it consists of an electromagnetic field confined by reflecting walls or of coupled electric and magnetic fields of capacitive and inductive elements.

In this Letter, we propose a laser-like device in which the oscillator is realized by a fundamentally different kind of a device—a nanomechanical resonator, e.g., a nanoscale cantilever or doubly clamped beam. Recent advances in nanofabrication and detection techniques have pushed the fundamental-mode frequencies of nanomechanical oscillators to the microwave range [4], approaching the point where their properties begin to be limited by quantum effects [5]. In addition, micro- and nanoelectromechanical oscillators generally exhibit low noise and high quality factors, which naturally has led to applications for integrated high-frequency signal generation and processing [6]. These properties make it possible to use long-term coherent response of a high-frequency nanomechanical oscillator in a laser-like device.

Similarly to the case of NMR laser and solid-state masers, we propose to use nuclear or electron spins in a strong external magnetic field as the active medium of a "mechanical laser". A nanomechanical oscillator can be effectively coupled to the magnetic moments of such spins by incorporating a small ferromagnetic tip on its surface. At microscopic distances between the cantilever and the sample, the coupling between the tip and the spins is essentially magnetostatic.

Systems consisting of a micro- or nanomechanical cantilever coupled to resonant magnetic spins of a solid sample have been extensively studied in the context of magnetic resonance force microscopy (MRFM) [7]. However, in all MRFM experiments performed so far, the fundamental frequency of the cantilever was orders of magnitude below the Larmor frequency of the magnetic spins. Resonant transfer of energy quanta from spins to the mechanical oscillator is impossible in this case. Therefore, the resonance is achieved by using an RF or microwave field to modulate the sample magnetization at the cantilever frequency [7].

For the device proposed here, it is essential that the motion of the mechanical oscillator be resonantly coupled to the free precession of magnetic spins, which means that the frequency of the used mechanical mode must be close to the Larmor frequency of the sample [8]. For conventional experiments with external magnetic fields of a few Tesla, the Larmor frequencies are of the order of tens of megahertz and tens of gigahertz for nuclear and electron magnetic resonance, respectively [10]. Given that the highest fundamental-mode frequency of nanomechan-
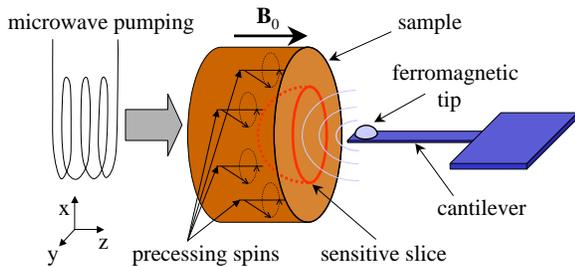
FIG. 1: Schematic of a mechanical laser device.

ical oscillators measured so far is slightly above 1 GHz [4], resonant coupling between mechanical oscillators and electron spins in strong magnetic fields seems unfeasible at this time. Operation in low magnetic fields, on the other hand, would prevent complete polarization of electrons and make the system more sensitive to ambient magnetic fields. In the rest of this Letter, we will therefore concentrate on the case involving nuclear spins.

Figure 1 shows a schematic of the proposed device. A nanomechanical oscillator—a cantilever in this case—is positioned near a sample that contains precessing nuclear spins, some of which are shown schematically in the figure. A ferromagnet on the cantilever tip creates a magnetic field, which can be approximated as the field of a magnetic dipole. When superposed on the uniform external field $\mathbf{B_0}$, this field modifies the total magnetic field seen by nuclear spins and, therefore, their Larmor frequency. As a result, only a certain slice of the sample, known as the sensitive slice, will have the Larmor frequency resonant with the frequency of the used mode of the cantilever [7].

The rotating transverse component of nuclear magnetization couples to the ferromagnetic tip via a dipolar magnetostatic interaction, with the resulting force driving cantilever oscillations. Conversely, a moving ferromagnet creates an AC magnetic field, oscillating at the frequency of the cantilever motion, inside the sample. This RF field can drive transitions between Zeeman levels of nuclear spins—stimulated transitions in the language of standard rate-equation laser theory. The resulting coupled interaction—spins driving the cantilever and the cantilever, in turn, driving the spins—leads to a kind of positive feedback that arises in all laser-like systems.

Although a variety of nanomechanical devices could be used—involving, for example, torsional or flexural modes—we focus here on nanocantilevers, which are especially convenient for scanning with small tip-sample separations. The device we propose can then be aptly termed a "cantilaser". To provide a concrete example, we will assume the following parameters for the cantilever: fundamental-mode frequency $\omega_c/2\pi = 20$ MHz, effective spring constant $k_c = 0.1$ N/m, quality factor $Q = 10^5$, the transverse magnetic field gradient (due to the ferromagnetic tip) $\frac{\partial \mathbf{B}_\perp(\mathbf{r})}{\partial x} = 1$ G/Å$= 10^6$ T/m within the sensitive slice. This magnetic field gradient can be created by a rare-earth-metal magnet at a distance of about

1 micron [11]. At such relatively large distances, one can usually neglect all nonmagnetic interactions between the cantilever and the sample. Note also that the intrinsic Q factor of a nanomechanical oscillator can be effectively increased by a few orders of magnitude using positive feedback [12] or parametric pumping [13].

For the parameters of the nuclear spin subsystem, we will take values representative of crystalline materials [10]: transverse relaxation time $T_2 = 50\,\mu s$ and nuclear gyromagnetic ratio $\gamma_n = 2\pi \times 10$ MHz/T. The bulk of the sample material will be resonant with the cantilever oscillations in an external field of $B_0 = \omega_c/\gamma_n = 2$ T.

In order to observe lasing in any system, one must introduce a pumping mechanism to compensate for the energy dissipated in both the oscillator and the active medium. For nuclear spins, such pumping can be produced by dynamic nuclear polarization (DNP) [10]. In this mechanism, microwave or optical radiation is used to saturate an electron transition, causing them to preferentially absorb photons of only one circular polarization. Some of the absorbed angular momentum is then transferred from the electrons to the nuclei of the sample through various equilibration processes. This technique has been successfully employed to pump the NMR laser at the liquid helium temperature (4.2 K) using a microwave source as in Fig. 1, with the effective pumping time as low as $T_p = 0.2$ s [3]. We will use this pumping rate and the equilibrium longitudinal polarization $M_{eq} = -0.3$ that is achievable in a 2-Tesla external field.

The dynamics of the cantilaser can be described by the Hamiltonian

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\gamma_n \sum_i \mathbf{B}_i \cdot \hat{\mathbf{S}}_i + \hbar(\hat{a}^\dagger + \hat{a}) \sum_i 2\mathbf{g}_i \cdot \hat{\mathbf{S}}_i + \hat{H}_r,$$

where $a^\dagger$ and a are the creation and annihilation operators of the cantilever mode, $\hat{\mathbf{S}}_i$ is the spin operator of ith nucleus, $\hat{\mathbf{B}}_i$ is the the external field at the site of the ith spin, $\mathbf{g}_i = \frac{\gamma_n}{2} \frac{\partial \mathbf{B}(\mathbf{r}_i)}{\partial x} \sqrt{\frac{\hbar\omega_c}{2k_c}}$ is the vector constant of the coupling between the ith spin and the cantilever, and $\hat{H}_r$ describes relaxation-inducing couplings to the environment. This Hamiltonian was first considered by Jaynes and Cummings, who used it describe quantum behavior of masers [14]. In the same paper, they also showed that the corresponding dynamics can usually be described by semiclassical equations, which treat the resonator classically and the spins quantum mechanically.

Considering the nuclear spins in their respective rotating frames (as defined by the local field $\mathbf{B}_i$ and field gradient $\frac{\partial \mathbf{B}(\mathbf{r}_i)}{\partial x}$ [8]) and using the slowly-varying-amplitude approximation for the cantilever, we can write such semiclassical equations in the form

$$\begin{aligned}
&\dot{A} + \kappa A = -gNM_-, \\
&\dot{M}_- + \Gamma_\perp M_- = gM_z A, \\
&\dot{M}_z + \Gamma_\parallel (M_z - M_{eq}) = -g(M_- A^* + M_-^* A)/2,
\end{aligned} \qquad (1)$$

where A is the (generally complex) amplitude of cantilever oscillations, normalized by the amplitude,

$\sqrt{\hbar\omega_c/(2k_c)} \approx 2.6 \cdot 10^{-13}$ m, of zero-energy quantum motion, $\kappa = \omega_c/2Q \approx 630\,\mathrm{s}^{-1}$ is the cantilever decay rate, N is the number of resonant spins in the sensitive slice, M_- and M_z are the normalized ($|M_-|^2 + M_z^2 \leq 1$) transverse and longitudinal (with respect to $\mathbf{B_0}$) components of nuclear polarization, $\Gamma_\perp = T_2^{-1} = 20 \cdot 10^3\,\mathrm{s}^{-1}$ and $\Gamma_\parallel = T_p^{-1} = 5\,\mathrm{s}^{-1}$ are the effective transverse and longitudinal polarization relaxation rates, and $g = \frac{\gamma_n}{2}\left|\frac{\partial \mathbf{B}_\perp(\mathbf{r})}{\partial z}\right|\sqrt{\frac{\hbar\omega_c}{2k_c}} \approx 8.0\,\mathrm{s}^{-1}$ is the scalar coupling constant of the interaction between the cantilever and the nuclear spins.

In equations (1) we implicitly assumed that all resonant nuclei in the sensitive slice are spin-half and that they all see the same strength and gradient of the magnetic field. The latter is an obvious simplification since in MRFM experiments, the magnetic-resonance frequency and coupling strength varies continuously over the sensitive slice [9]. However, the same problem of inhomogeneous broadening and nonuniform coupling arises in most quantum optics and laser setups [15], and it was found experimentally [16, 17] that equations of the form (1) still correctly reproduce most features of the coupled spin–oscillator dynamics. In this Letter, we will therefore restrict our analysis to the simplest model of Eqs. (1)

It is easy to find the steady-state solutions of Eqs. (1). The nontrivial lasing solution may exist only in the case of population inversion, $M_{eq} < 0$, and is given by $A_{cw} = \sqrt{(N|M_{eq}| - N_t)\Gamma_\parallel/\kappa}$, where $N_t = \kappa\Gamma_\perp/g^2$ is the threshold population inversion. Substituting $M_{eq} = -0.3$ and other parameter values given above, we find that in order to support cw lasing, the number of atoms in the sensitive slice should be $N > N_{cw} = N_t/|M_{eq}| \approx 0.65 \cdot 10^6$. This may seem like a large number; however, even an atomically thin sensitive slice of a homogeneous sample contains of the order of 10^7 nuclei if the diameter of the sensitive slice is just $1\,\mu$m. Much larger sensitive slices have been used in nuclear MRFM experiments so far, so exceeding the lasing threshold seems quite feasible.

One of the more interesting transient phenomena predicted by the Jaynes–Cummings model is the coherent oscillation of population between the oscillator and spins [14], an effect similar to the oscillations of energy between two weakly coupled classical harmonic oscillators. Also known as ringing superradiance, this phenomenon has been observed in different quantum-optical systems [16, 17]. In order for the energy oscillations to be observable in a cantilaser, the effective frequency of the oscillations, equal to $\sqrt{|M_{eq}|N}g$ [14], should be larger than the fastest relaxation rate of the system, $\Gamma_\perp$. We can therefore roughly estimate the minimum number of atoms necessary to observe the oscillations as $N_{sr} = \Gamma_\perp^2/(|M_{eq}|g^2) \approx 20 \cdot 10^6$. As we show below, our numerical simulations support the validity of this estimate.

Another interesting transient predicted by the Jaynes-Cummings model is a solitary pulse that irreversibly depletes the energy stored in the active medium. Known
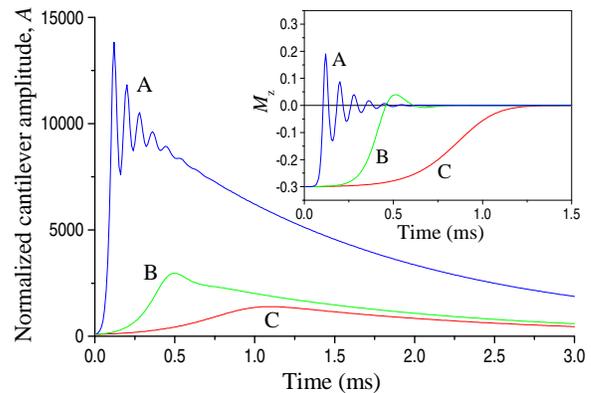

FIG. 2: Characteristic transients of a cantilaser for different number of resonant nuclei within the sensitive slice: (A) $N = 200 \cdot 10^6$, (B) $15 \cdot 10^6$, and (C) $5 \cdot 10^6$. The main panel shows the normalized amplitude of cantilever oscillations, $A(t)$, the inset shows the longitudinal nuclear polarization, $M_z(t)$. The initial conditions are $A(0) = 94$, $M_-(0) = 0$, $M_z(0) = -0.3$.

as giant pulses in the standard laser theory [15], these transients can appear if $\Gamma_\perp > |M_{eq}|Ng^2/\Gamma_\perp > \kappa$, which implies $N_{sr} > N > N_{cw}$. Such a giant pulse reduces the population inversion to zero and therefore consumes one half of the total potential energy of the active medium (i.e., the sensitive slice, which can in principle be adjusted to encompass most of the sample [9]). This is in contrast to the case of ringing superradiance, where all of the available energy oscillates back and forth between the cantilever and spins.

Figure 2 shows three characteristic transient outputs of a cantilaser, obtained by numerical integration of Eqs. (1). As the number of resonant nuclei in the sensitive slice decreases from $N = 200 \cdot 10^6 \gg N_{sr}$ to $N = 5 \cdot 10^6 \ll N_{sr}$, the frequency and amplitude of energy oscillations decreases until just one "giant" pulse is observed. If one further keeps decreasing the number of atoms, the single pulse becomes longer and smaller in amplitude until it disappears completely as the number of atoms goes below the cw lasing threshold N_{cw}. Note that the tails of the output transients always decay at the time scale of κ^{-1} because cantilever decay is the dominant mechanism of energy dissipation here. In contrast, the coherent oscillations, if present at all, decay at the time scale of $\Gamma_\perp^{-1} = T_2$ since spin-spin relaxation is the dominant mechanism for the loss of coherence.

The initial conditions for pulsed transients used in the simulations of Fig. 2 can be produced by a Q-switching technique [15]. Note that the initial nuclear polarization, $M_-(0) = 0$, $M_z(0) = M_{eq}$, is simply the equilibrium polarization achieved in the presence of dynamic nuclear pumping and negligible interaction with the cantilever. Also, the initial cantilever amplitude corresponds to thermal vibrations of the cantilever at the temperature $T = 4.2$ K: $A(0) = A_{th} = \sqrt{2k_B T/(\hbar\omega_c)} \approx 94$, where k_B is the Boltzmann constant.

Since a cantilaser shares so much in its design and

principles of operation with MRFM setups, it is natural to consider whether the effects describe above can be used to improve the sensitivity of nuclear MRFM. The single-shot sensitivity of the first nuclear MRFM experiment [18] was approximately 10^{13} thermally polarized nuclear spins at room temperature or about 10^{11} nuclear spins at 4.2 K (nuclear polarizability is inversely proportional to temperature). Since then, the sensitivity of MRFM experiments has been improved to about 100 Bohr magnetons [19], a magnetic moment that is created by roughly 10^8 nuclear spins at 4.2 K. The closest competing technology—scanning SQUID-based magnetometry—has so far demonstrated a sensitivity of 10^5 Bohr magnetons [20], or 10^{11} nuclear spins at 4.2 K.

To consider the MRFM sensitivity of a cantilaser, we will calculate the ratio of power spectral density of lasing outputs to the power spectral density of thermomechanical noise. Since the resonant frequency of the cantilever and the Zeeman transition frequency of spins coincide, we can express all energy quantities in terms of the number of energy quanta $\hbar\omega_c$. The power of the thermomechanical noise is then proportional to $n_{th} = A_{th}^2/2 = k_B T/(\hbar\omega_c) \approx 4400$, and its bandwidth is $\Delta\omega_{th} = \kappa$. Well above the lasing threshold, the power of the cw lasing signal is $n_{cw} = A_{cw}^2/2 \approx \Gamma_\| N|M_{eq}|/(2\kappa) \approx N/840$, and its bandwidth is $\Delta\omega_{cw} \approx \kappa(n_{th}+1/2)/n_{cw}$ [15]. The signal-to-noise ratio for the cw output is then $SNR_{cw} = \frac{n_{cw}/\Delta\omega_{cw}}{n_{th}/\Delta\omega_{th}} \approx (n_{cw}/n_{th})^2 \approx (N/3.7\cdot10^6)^2$. The quadratic increase in SNR_{cw} with the number of atoms reflects the spectral narrowing of the cw signal at high output power, a fact that is well known and used in optical lasers [15].

For both kinds of pulsed outputs we considered (ringing superradiance and single pulses), the efficiency of the energy transfer from spins to the cantilever mode is of the order of unity. The peak cantilever amplitude of a pulse in both cases then corresponds to mode population of about $n_{pulse} = N|M_{eq}|/2$. Since all pulsed outputs eventually decay at the time scale of κ^{-1}, their bandwidth can be taken to be $\Delta\omega_{pulse} \approx \kappa$. Proceeding as above, we find $SNR_{pulse} = N|M_{eq}|/(2n_{th}) \approx N/29000$. A cantilaser operating in the pulsed mode would therefore have a single-shot sensitivity of about $3\cdot10^4$ nuclear spins at 4.2 K, which is at least three orders of magnitude better than the sensitivity of any existing alternative. A large part of this improvement derives from the hyperpolarization of nuclei by DNP processes, but the near-unity efficiency of energy transfer between spins and cantilever in pulsed transients is also significant.

We conclude by considering different possible perspectives upon mechanical lasing—from the standpoints of quantum optics, NMR spectroscopy, and MRFM. From the perspective of quantum optics, the cantilaser is very similar to a cavity QED system [21], albeit one with weak but comparable coupling strength and longitudinal relaxation, $g \sim \Gamma_\| \ll (\Gamma_\perp, \kappa)$, and high thermal population, $n_{th} \gg 1$. Since such combinations of parameters are not available in quantum optical systems or NMR laser, this opens up new possibilities for studying coherent quantum phenomena in coupled oscillator–atom systems.

In conventional NMR studies, the possibility of a positive feedback between the sample and the detecting resonance circuit has been long recognized. Bloembergen first considered the back action of the detecting coil on the sample almost 50 years ago [22]. Unfortunately, such back action tends to shorten the signal pulses and therefore broaden spectral features. This explains why this positive-feedback effect, known to NMR practitioners as radiative damping, is generally undesirable in high-resolution NMR spectroscopy. In contrast, MRFM experimentalists are not interested in fine details of NMR spectra. Since the ultimate goal of MRFM is atom-by-atom 3D mapping of nanoscopic objects, the required spectral resolution should only be sufficient to distinguish between different nuclear species. MRFM practitioners are therefore willing to trade fine spectral resolution for signal strength and spatial resolution. This is exactly what a mechanical laser provides, by embracing and fully exploiting the positive feedback in the coupled oscillator–spin system.

[1] J.P. Gordon, H.J. Zeiger, C.H. Townes, Phys. Rev. **95** 282 (1954); T.H. Maiman, Nature **187** 493 (1960).
[2] T.C. Marshall, *Free electron lasers* (MacMillan, New York, 1985).
[3] R. Badii *et al.*, Rev. Mod. Phys. **66** 1389 (1994).
[4] X.M.H. Huang *et al.*, Nature **421** 496 (2003).
[5] A. Cho, Science **299** 36 (2003).
[6] H.J. De Los Santos, *RF MEMS circuit design for wireless communications* (Artech House, Boston, 2002).
[7] J.A. Sidles *et al.*, Rev. Mod. Phys. **65** 249 (1995).
[8] J.A. Sidles, J.L. Garbini, G.P. Drobny, Rev. Sci. Instr. **63** 3881 (1992).
[9] A. Suter *et al.* J. Magn. Res. **15** 210 (2002).
[10] C.P. Slichter, *Principles of magnetic resonance* (Springer, New York, 1990).
[11] K.J. Bruland *et al.*, Appl. Phys. Lett. **73** 3159-61 (1998).
[12] A. Mehta *et al.*, Appl. Phys. Lett. **78** 1637 (2001).
[13] D.A. Harrington and M.L. Roukes (unpublished).
[14] E.T. Jaynes, F.W. Cummings, Proc. IEEE **51** 89 (1963).
[15] H. Haken, *Laser theory* (Springer, New York, 1984).
[16] Y. Kaluzny *et al.*, Phys. Rev. Lett. **51** 1175 (1983).
[17] R.J. Brecha *et al.*, J. Opt. Soc. Am. B **12** 2329 (1995).
[18] D. Rugar *et al.*, Science **264** 1560 (1994).
[19] B.C. Stipe *et al.*, Phys. Rev. Lett. **87** 277602 (2001).
[20] B.W. Gardner *et al.*, Rev. Sci. Instr. **72** 4153 (2001).
[21] P.R. Berman (Ed.), *Cavity quantum electrodynamics* (Academic, Boston, 1994).
[22] N. Bloembergen and R.V. Pound, Phys. Rev. **95** 8 (1954).