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Supporting Online Material for

Negative Refraction at Visible Frequencies

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This file includes:

SOM Text

Figs. S1 and S2

References

Correction (27 April 2007): The following typos have been corrected:

1) Section 1, first paragraph, line 3: $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$ replaces $\hat{\boldsymbol{k}} = |\boldsymbol{k}|/k$.
2) Section 2, line 13: $n_2 = \lambda_0 / P$ replaces $n_2 = 2\lambda_0 / P$.
3) Fig. S1, caption: $n_2 = \lambda_0 / P$ replaces $n_2 = 2\lambda_0 / P$.

1. Negative index and negative refraction

Consider an electromagnetic mode propagating with wavevector $\boldsymbol{k}$ in an isotropic medium. The phase and group velocity of this mode are collinear and given by, respectively, $\boldsymbol{v}_{\mathrm{p}} = (\omega / k)\hat{\boldsymbol{k}}$ and $\boldsymbol{v}_g = (d\omega / dk)\hat{\boldsymbol{k}}$, where $k = |\boldsymbol{k}|$, and $\hat{\boldsymbol{k}} = \boldsymbol{k} / k$. The phase velocity characterizes the speed and direction of propagation of the phase fronts, whereas the group velocity determines the direction of power flow $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H} = W\, \boldsymbol{v}_{\mathrm{g}}$, where $\boldsymbol{S}$ is the Poynting vector and W is the energy density of the electromagnetic field.

When $d\omega / dk > 0$, $\boldsymbol{v}_{\mathrm{p}}$ and $\boldsymbol{v}_{\mathrm{g}}$ are parallel and the standard forward-wave condition $\boldsymbol{v}_{\mathrm{p}} \cdot \boldsymbol{v}_{\mathrm{g}} > 0$ is obtained. $\boldsymbol{k}$ and $\boldsymbol{S}$ are parallel under such a condition and power propagates in the same direction as the phase-fronts. When $d\omega / dk < 0$, $\boldsymbol{v}_{\mathrm{p}}$ and $\boldsymbol{v}_{\mathrm{g}}$ are anti-parallel and the anomalous backward-wave condition $\boldsymbol{v}_{\mathrm{p}} \cdot \boldsymbol{v}_{\mathrm{g}} < 0$ is obtained. $\boldsymbol{k}$ and $\boldsymbol{S}$ are anti-parallel and power propagates in a direction opposite to that of the phase-fronts.

When a plane wave traveling in medium 1, characterized by phase and group velocities $\boldsymbol{v}_{\mathrm{p}}^1$ and $\boldsymbol{v}_{\mathrm{g}}^1$, is incident at an angle φ_1 upon a boundary with medium 2, characterized by phase and group velocities $\boldsymbol{v}_{\mathrm{p}}^2$ and $\boldsymbol{v}_{\mathrm{g}}^2$, the resulting refraction angle of the transmitted beam, φ_2, is dictated by conservation of the parallel component of $\boldsymbol{k}$ across the boundary as well as conservation of energy (see ref. S*1*):

$$\frac{\sin(\varphi_2)}{\sin(\varphi_1)} = \frac{\mathrm{sign}(\boldsymbol{v}_{\mathrm{p}}^1 \cdot \boldsymbol{v}_{\mathrm{g}}^1)}{\mathrm{sign}(\boldsymbol{v}_{\mathrm{p}}^2 \cdot \boldsymbol{v}_{\mathrm{g}}^2)} \frac{c / |\boldsymbol{v}_{\mathrm{p}}^1|}{c / |\boldsymbol{v}_{\mathrm{p}}^2|} \equiv \frac{n_1}{n_2}. \quad \text{(S1)}$$

By analogy with the classical form of the Snell-Descartes law of refraction $\sin(\varphi_2)/\sin(\varphi_1) = n_1 / n_2$, an effective refractive index for each medium can be identified based on Eq. S1:

$$n_{1,2} = \mathrm{sign}(\boldsymbol{v}_{\mathrm{p}}^{1,2} \cdot \boldsymbol{v}_{\mathrm{g}}^{1,2})c / |\boldsymbol{v}_{\mathrm{p}}^{1,2}|. \quad \text{(S2)}$$

If $\boldsymbol{v}_{\mathrm{p}}^{\mathrm{i}} \cdot \boldsymbol{v}_{\mathrm{g}}^{\mathrm{i}} < 0$, medium i acts as if it had a negative index of refraction. When light exits such a medium into a second medium characterized by a positive index of refraction, it will be refracted, according to Eq. (S1), to the same side of the normal, i.e. to a negative angle.

2. Derivation of mode index of thick Ag/Si_3N_4/Ag waveguide

To derive the actual value of the mode index of the thin Ag/Si_3N_4/Au waveguide, n_1, the mode index in the surrounding Ag/Si_3N_4/Ag waveguide, n_2, must be known. To circumvent the difficulty of estimating n_2 theoretically, given the predicted multimodal dispersion properties (Fig. 1A), we measure n_2 directly using an interferometric technique. To this end, Ag/Si_3N_4/Ag waveguides with 500-nm-thick dielectric cores were fabricated with semi-transparent Ag cladding layers of 150nm on each side; no additional Al cladding layer was added. The separation between input and output slit was varied from d = 1 µm to 3 µm in 25-nm increments. Each device was illuminated at normal incidence over its entire area and the intensity emitted from the output slit was monitored (Fig. S1 A). Due to interference at the output-slit, between light transmitted along the waveguide and light transmitted directly through the top Ag cladding layer, a periodic modulation in the emitted intensity is obtained as a function of d (Fig. S1 B). At a given wavelength λ_0, the effective mode index of the waveguide is given by $n_2 = \lambda_0 / P$, where P is the period of the modulation.

3. Effect of thickness and edge angle of Au/Si_3N_4/Ag prism

We characterize in detail the refractive properties of a Ag/Si_3N_4/Au waveguide within its frequency interval of negative refractive index .The result of refraction from a single prism of Ag/Si_3N_4/Au waveguide (W1), of dielectric-core thickness $t_1 = 75$ nm, into a thick Ag/Si_3N_4/Ag waveguide (W2), of dielectric core thickness $t_2 = 500$ nm, is shown in Fig. S2 A, at two closely-spaced wavelengths of $\lambda_0 = 501$ nm and $\lambda_0 = 488$ nm, respectively. Given an incident angle of $\varphi_1 = 7°$ at the W1/W2 interface, substantially different refraction angles $\varphi_2 = -31.2°$. and $-27.1°$. are obtained in each case. Applying Snell's law (Eq. S1) using these angles of refraction and measured values $n_2 = 0.65$ and 0.57, respectively (Fig. S1 B), we obtain effective refractive indices $n_1 = -2.72$ and -2.09, respectively. Weak transmission of a positively-refracted spot can also be observed, hinting that 75-nm-thick constrictions can support a strongly attenuated positive-index photonic mode in addition to the negative-index SPP mode. As the waveguide thickness is increased to $t_1 = 150$ nm, the positively refracted spot increases in intensity: photonic mode transmission competes with the SPP mode (Fig. S2 B). While the negatively-refracted spot size is slightly reduced with respect to the input slit length, the positively-refracted spot is enlarged. Such variations in spot size likely result from focusing or diffraction effects dependant on both the sign and the magnitude of n_1.

To confirm the applicability of Snell's law to the bimetallic waveguide structures, we vary the angle of incidence φ_1 by fabricating W1 prisms ($t_1 = 150$ nm) with edge angles of $\theta = 5°$ and $7°$ respectively. Refraction results at $\lambda_0 = 501$ nm are shown in Fig. S2 B. A distinct negatively-refracted spot is obtained in both cases, implying a

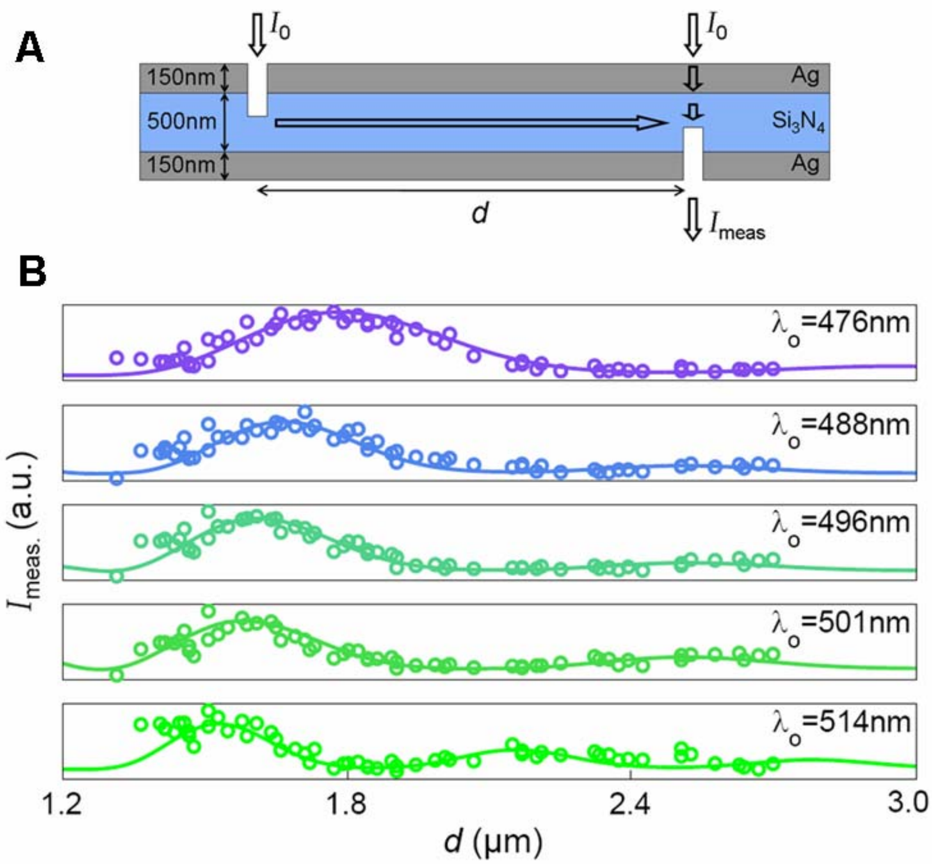

Fig. S1. Experimental determination of refractive index n_2 of thick Ag/Si_3N_4/Ag waveguide. (**A**) Cross-sectional diagram of interferometer used to determine n_2. The structure is a modified version of the device of Fig 2B, in which the optically opaque metal cladding on both sides of the dielectric core is replaced with a 150-nm-thick layer of Ag. This semi-transparent cladding allows for interference, at the output-slit position, between the transmitted waveguide mode and the partially transmitted incident illumination. Periodic modulation of the light emerging from the output slit as a function of slit-slit distance d yields the effective mode index of the waveguide (*S2*). (**B**) Measured output-slit intensity as a function of d for wavelengths within the predicted negative index-region. Distance d is varied from 1 to 3 μm in 25-nm increments. The waveguide index is given by $n_2 = \lambda_0 / P$, where P is the period of modulation. For free-space wavelengths $\lambda_0 = 476$, 488, 496, 501, and 514 nm, the resulting indices in the Ag/Si_3N_4/Ag waveguide are $n_2 = 0.40$, 0.57, 0.50, 0.65, and 0.82, respectively.

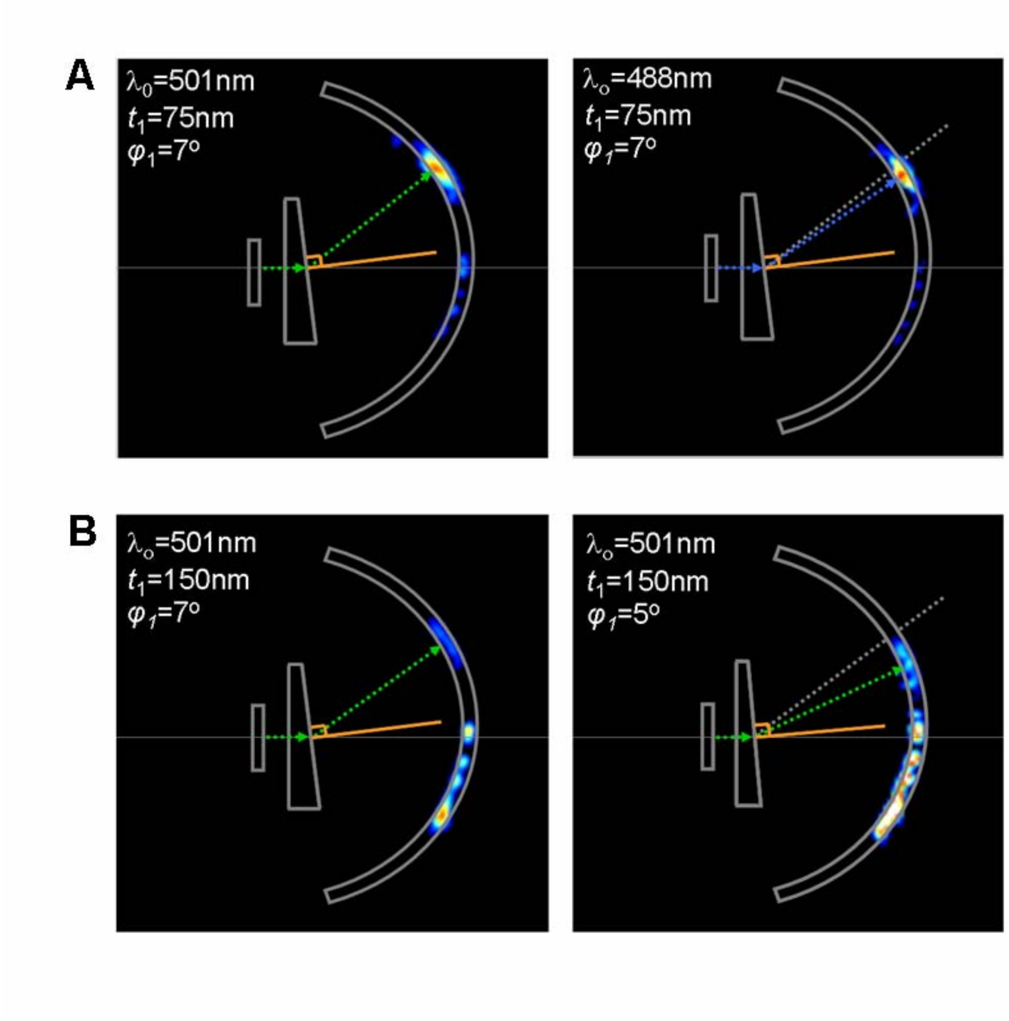

Fig. S2. Detailed exploration of refractive properties of Au/Si3N4/Ag waveguide in the negative index region. (**A**) Output-spot position for the single prism geometry of Fig. 2C as a function of wavelength. The core thickness and interface angle of the prism are held constant at $t = 75\,\text{nm}$ and $\theta = \varphi_1 = 7°$, respectively. The angle of refraction φ_2 varies from $-31.2°$ to $-27.1°$ as λ_0 is decreased from 501 to 488 nm. (**B**) Output-spot position for prism angles of $\theta = \varphi_1 = 7°$ and 5°. Here, the core thickness and excitation wavelength are held constant at $t = 150\,\text{nm}$ and $\lambda_0 = 501\,\text{nm}$, respectively. The angle of the negatively-refracted spot shifts from $-28.8.°$ (for $\varphi_1 = 7°$) to $-20.0°$ (for $\varphi_1 = 5°$), corresponding to a constant prism index $n_1 = -2.5$ and confirming the applicability of Snell's Law to such waveguides operating in the negative index region. Note that in both **A** and **B**, bands of positive and negative refraction be observed, corresponding to transmission of a positive-index photonic mode and a negative-index plasmonic mode, respectively.

refraction angle of $\varphi_2 = -20.0°$ when $\varphi_1 = 5°$ and of $\varphi_2 = -28.8°$ when $\varphi_1 = 7°$. The corresponding ratio $\sin(\varphi_2)/\sin(\varphi_1) = n_1/n_2$ is -3.82 and -3.88, respectively. The constancy of this ratio as a function of edge angle implies that Snell's law indeed applies to the present structures operating in the negative index region.

References

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S2. J. A. Dionne, H. J. Lezec, H. A. Atwater, *Nano Lett.* **6**, 1928 (2006).