THE MOMENTUM BALANCE METHOD IN EARTHQUAKE ENGINEERING

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1. INTRODUCTION. During an earthquake the shaking of the ground imparts movement to structures which are then stressed by the resulting inertia forces. When the structure is a dam, it experiences additional earthquake forces from the water in the reservoir. The horizontal motion of the ground does not impart movement to the water so the dam accelerates into and away from the water and experiences dynamic water pressures. The first analysis of this problem was by H. M. Westergaard, Professor of Theoretical and Applied Mechanics at the University of Illinois, his paper [1] appearing in the 1933 Transactions of the American Society of Civil Engineers. In the 1920's when Boulder Dam was being designed, Westergaard was a Visiting Scientist at the Bureau of Reclamation and became interested in this practical earthquake engineering problem. His paper elicited numerous discussions, also printed in the 1933 ASCE Transactions, and an especially interesting discussion was contributed by Theodore Von Karman, Professor of Aeronautics at the California Institute of Technology. Essentially the same result that Westergaard obtained by means of a series solution of a partial differential equation was deduced by Karman using a remarkably simple momentum-balance method. Although Karman's method was mathematically simple, some steps in the analysis appeared to be inexplicable. The method might be said to be an example

of Operational Mechanics which, like Operational Mathematics, involves operations that give the desired result without it being understood why they do so. It is the purpose of this paper to give a rational explanation of the momentum-balance method.

2. WESTERGAARD ANALYSIS. The two-dimensional problem solved by Westergaard was that of an accelerating rigid dam with rectangular, vertical face, retaining water of depth h in an infinitely long, rectangular reservoir, as shown in Fig. 1. A particle of water at position x, z has horizontal and vertical velocities $\dot{u}, \dot{v}$; and the dynamic water pressure is p. The physics of this problem is described by Newton's law and by the dilatational stress-strain relation:

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad ; \quad -\frac{\partial p}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \quad ; \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = -\frac{p}{k} \qquad (2.1)$$

These equations describe the behavior of an inviscid fluid of density ρ, and bulk modulus of elasticity k. Assuming u, v and their spatial deriviations to be small, the boundary conditions imposed by Westergaard were, no dynamic pressure at the water surface: $p = 0$ at $z = 0$; no vertical displacement of the reservoir bottom: $v = 0$ at $z = 0$ and the earthquake to have only uni-directional horizontal, harmonic oscillation, with amplitude a_o:

$$\frac{\partial^2 u}{\partial t^2} = a_o \cos \frac{2\pi}{T} t \qquad \text{at } x = 0$$

A sinusoidal earthquake is not realistic but, as shown by Karman, the result of the analysis can be applied to arbitrary horizontal ground acceleration. The solution given by Westergaard for the foregoing problem was

$$u = \frac{a_o T^2}{\pi^3} \sum_{1,3,5}^{\infty} \left(\frac{1}{n} e^{-\beta_n x} \sin \frac{n\pi z}{2h} \right) \cos \frac{2\pi t}{T} \tag{2.2}$$

$$v = \frac{a_o T^2}{\pi^3} \sum_{1,3,5}^{\infty} \left(\frac{1}{n c_n} e^{-\beta_n x} \cos \frac{n\pi z}{2h} \right) \cos \frac{2\pi t}{T} \tag{2.3}$$

$$p = -\frac{8\rho a_o h}{\pi^2} \sum_{1,3,5}^{\infty} \left(\frac{1}{n^2 c_n} e^{-\beta_n x} \sin \frac{n\pi z}{2h} \right) \cos \frac{2\pi t}{T} \tag{2.4}$$

$$\beta_n = \frac{n\pi}{2h} c_n \quad ; \quad c_n = \left(1 - \frac{16\rho h^2}{kn^2 T^2} \right)^{\frac{1}{2}}$$

Westergaard calculated numerical values for a variety of situations, and he noted that for most practical considerations the parameter c_n had a value essentially equal to unity, though he pointed out that for certain special situations the value might differ significantly from unity. He also noted that there was a singularity in the solution at the surface of the water immediately adjacent to the face of the dam, that is, $\dot{v}, \ddot{v}$ are infinite at $x = 0$, $z = 0$. He concluded that the extent of the region in which v and its derivatives were large was so restricted as not to have a significant effect, but he did not explain why there was such a singularity in the solution when the physics of the problem clearly required there should be none.

The calculated hydrodynamic pressure on the dam was found to have nearly an elliptical distribution over the height of the dam, that is, it had $p = 0$, $\partial p/\partial z = \infty$ at $z = 0$, and $\partial p/\partial z = 0$ at $z = h$, so Westergaard proposed the following simple expression, which gives an elliptical distribution for the pressure on the face of the dam

$$p_0 = 0.692\, \rho\, a_o \left[z(2h - z)\right]^{\frac{1}{2}} \qquad (2.5)$$

This gives the same total force on the face of the dam as results from equation 2.4. He pointed out that equation 2.5 can be taken as representing the effect of an "apparent mass" of water that accelerates back and forth as if it were a solid attached to the face of the dam.

3. KARMAN ANALYSIS. The approximate analysis by Karman was based on considerations of mechanics rather than mathematics, and went as follows. The effect of the compressibility of water is small, so take the water to be incompressible ($c_n = 1$), and consider only impulsive pressures, for the displacements of the water are too small to develop significant convective pressures. There is an "apparent mass" of water as shown in Fig. 2 and at time $t = 0$ the dam is given a horizontal acceleration a_o. At time $t = \Delta t$ the acceleration, velocity and displacement are

$$\frac{\partial^2 u_o}{\partial t^2} = a_o \quad ; \quad \frac{\partial u_o}{\partial t} = a_o \Delta t \quad ; \quad u_o = a_o \overline{\Delta t}^2/2 \qquad (3.1)$$

During this time the portion of the dam below the ordinate y has displaced a quantity of fluid $\left(y\ a_o \Delta t^2/2\right)$. (Note that Karman's y is Westergaard's (h - z)).

Continuity of flow, according to Karman, requires that the quantity passing the section BC in Fig. 2 must be equal to that displaced by the dam, that is

$$b\left(a_y \overline{\Delta t}^2/2\right) = y\left(a_o \overline{\Delta t}^2/2\right) \tag{3.2}$$

or

$$b a_y = y a_o \tag{3.3}$$

where b is the breadth of the apparent mass and a_y is the vertical acceleration of the element (b dy). Equation 3.3 is thus a continuity equation.

The element (ρb dy) is given a horizontal velocity ($a_o \Delta t$) by the impulse ($p\Delta t$) and, therefore, the impulse momentum equation gives the pressure at the face of the dam as

$$p = \rho b a_o \tag{3.4}$$

The element (ρb dy) is also given a vertical momentum by the difference in the pressure force (pb dy) at y and at y + dy; and the equation of impulse momentum is

$$-\frac{d}{dy}(bp\Delta t) = \rho b (a_y \Delta t) \tag{3.5}$$

Eliminating a_y and p by means of equations 3.3 and 3.4 gives the following equation for determining the breadth, b, of the apparent mass:

$$\frac{d(b^2)}{dy} = -y \tag{3.6}$$

Integrating and applying the boundary condition $b = 0$ at $y = h$, gives for the distribution of apparent mass

$$b = \left(\frac{h^2 - y^2}{2}\right)^{\frac{1}{2}} \qquad (3.7)$$

from which the pressure on the face of the dam is

$$p_0 = \rho a_o \left(\frac{h^2 - y^2}{2}\right)^{\frac{1}{2}} \qquad (3.8)$$

This elliptical pressure distribution agrees with Westergaard's proposed approximate distribution (equation 2.5), except that according to Westergaard's calculation the total force on the dam is

$$F_w = 0.543 \rho h^2 a_o \qquad (3.9)$$

whereas the integral of Karman's equation 3.8 over the height of the dam gives a value approximately 2% larger:

$$F_k = \frac{\pi}{4\sqrt{2}} \rho h^2 a_o = 0.555 \rho h^2 a_o \qquad (3.10)$$

It is seen that the foregoing analysis could be applied at any instant of time, even if there were convective fluid motions, providing there is an approximately uniform depth of water in the vicinity of the dam, then equation 3.8 will give the impulsive pressure on the dam proportional to the instantaneous value of the horizontal acceleration.

Like Westergaard's solution, Karman's solution also has a singularity on the water surface at the face of the dam, as can be seen by substituting from equation 3.7 into 3.3

$$a_y = \frac{y}{b} a_o = \frac{y\sqrt{2}}{\sqrt{h^2 - y^2}} \tag{3.11}$$

which gives an infinite value at $y = h$.

A straightforward analysis similar to the foregoing can be made also for a reservoir with sloping sidewalls. The case of a sloping dam face can also be analyzed by Karman's momentum-balance method, though this analysis is more subtle than that for a vertical dam face.

Karman's analysis has remarkable simplicity but at the cost of introducing an element of mystery. For example, the condition of continuity of flow as expressed by Eq. 3.2 seems to have no physical basis and it involves a contradiction. It is based on the assumption that the dotted line shown in Fig. 2 is fixed in space and the fluid is squeezed up between it and the moving dam face, whereas, in the analysis that follows Karman takes the fluid between the dam and the dotted line to move horizontally with the dam, as expressed in Eq. 3.4. The fact that he obtained satisfactory results, and that the procedure can be extended to other problems, provides motivation for developing a rational explanation.

4. ANALYSIS OF DAM WITH SLOPING FACE. A generalized version of Karman's momentum-balance approach was applied recently to dams with sloping faces by Chwang and Housner [2]. The rigid, sloping dam shown in Fig. 3 is given a horizontal acceleration a_o into an incompressible fluid. The increment of apparent mass ($\rho b\,dy$), shown in Fig. 3, is assumed to be given an increment of momentum $(\rho b\,dy)(a_n \Delta t)$ in the n-direction normal to the face of the dam, and a corresponding

increment of momentum in the parallel s-direction with respect to the face of the dam. The components of acceleration of the element in the x-direction and y-direction are

$$a_x = a_o - a_s \cos\theta \quad ; \quad a_y = a_s \sin\theta \tag{4.1}$$

Following Karman's approach, the continuity of flow is again described by the equation

$$b\, a_y = y\, a_o \tag{4.2}$$

with the boundary condition that at the dam face the normal component of fluid acceleration must be

$$a_n = a_o \sin\theta \tag{4.3}$$

Momentum balance of the increment of apparent mass, ($\rho b\, dy$), in the x-direction requires

$$p = \rho b\, a_x \tag{4.4}$$

and using Eq. 4.1, this can be written:

$$p = \rho\, a_o (b - \beta y) \tag{4.5}$$

where $\beta = \cot\theta$.

The equation of impulse momentum for the apparent mass ($\rho b\, dy$) in the y-direction is

$$- d(bp\Delta t) + (\beta p \Delta t) dy = (\rho b a_y \Delta t) dy \tag{4.6}$$

where ($\beta p\,dy$) represents the force acting on the sloping end of the element. Eliminating p from Eq. 4.5 by means of Eq. 4.4, and then eliminating a_y by means of Eq. 4.2, results in the governing differential equation for the breadth, b, of the apparent mass:

$$\frac{d}{dy}(b^2 - \beta yb) - \beta(b - \beta y) = -y \tag{4.7}$$

The appropriate boundary condition for this equation is, noting that $p = 0$ at $y = h$, from Eq. 4.5:

$$b = \beta h \quad \text{at} \quad y = h \tag{4.8}$$

Introducing the reduced breadth, $B = 2b - \beta y$, puts Eq. 4.7 in a form that can be integrated:

$$B\frac{dB}{dy} - \beta B = -2y \tag{4.9}$$

The closed-form solution of this equation, satisfying the end condition, is

$$\log_e\left(\frac{B^2 - \beta By + 2y^2}{2h^2}\right) = \frac{2\beta}{\sqrt{8-\beta^2}}\left\{\tan^{-1}\left(\frac{\beta}{\sqrt{8-\beta^2}}\right) - \tan^{-1}\left(\frac{2B - \beta y}{y\sqrt{8-\beta^2}}\right)\right\}$$

$$\text{for } \beta^2 < 8 \tag{4.10a}$$

$$\log_e\left(\frac{B^2 - \beta By + 2y^2}{2h^2}\right) = \frac{\beta}{\sqrt{\beta^2-8}}\left\{\log_e\left(\frac{\beta - \sqrt{\beta^2-8}}{\beta + \sqrt{\beta^2-8}}\right) - \log_e\left(\frac{2B - \beta y - y\sqrt{\beta^2-8}}{2B - \beta y + y\sqrt{\beta^2-8}}\right)\right\}$$

$$\text{for } \beta^2 > 8 \tag{4.10b}$$

When the face of the dam is vertical ($\beta = 0$), the foregoing solution reduces to that of Karman, as it should. The distribution of pressure, p_o, is shown in Fig. 4 and the total hydrodynamic force normal to the face of the dam, as determined by means of Eq. 4.10, is shown in Fig. 5. The total normal force agrees well with the exact value [3]. The maximum discrepancy is in the region of $\theta = 20°$ to $30°$ and deviates from the exact value by less than 5%. The momentum-balance method thus gives good results for dams with sloping faces despite the steps in the analysis being, perhaps, even more mysterious than for a dam with vertical face. It is of interest to note in Fig. 5 that the total normal force on the face of the dam, for most practical purposes, could be taken equal to $0.5 a_o \rho h^2$, independent of θ.

From Eqs. 4.2 and 4.8 the vertical acceleration at the surface of the water is

$$a_y = \frac{h}{b} a_o = a_o \tan\theta$$

thus a_y remains finite at $x = 0$, $y = h$ for all slopes $\theta < \frac{\pi}{2}$, but for a vertical dam face ($\theta = \frac{\pi}{2}$) there is a singularity, as in the case of the Westergaard and the Karman solutions.

5. EXPLANATION OF SINGULARITY. The singularity at $x = 0$, $y = h$ in the case of the vertical dam face should clearly not exist in the dam problem. It occurs here only because the Westergaard and Karman solutions are actually for a somewhat different fluid mechanics problem. This is shown in Fig. 6, where the two oppositely accelerated halves are precisely in line along $x = 0$ at time $t = 0$ and, therefore, from

symmetry considerations $p = 0$ along the center line $y = h$, which agrees precisely with the boundary condition imposed on the Westergaard solution. This latter condition is not satisfied in the dam problem for there the correct boundary condition is $p = \rho v(\ddot{v} + g)$ at $y = h$, and in the case of very strong earthquake ground shaking, $\ddot{v}$ at $y = h$, $x = 0$ can reach values that exceed g, the acceleration of gravity. Why the singularity exists in the foregoing solution (equation 4.7) for $\theta = \frac{\pi}{2}$ but does not exist for $\theta < \frac{\pi}{2}$ is best explained in terms of a different physical interpretation of the problem which is more easily visualized. The specification of the fluid dynamics problem of Fig. 6 in terms of particle accelerations $\ddot{u}$ and $\ddot{v}$, is

$$-\frac{\partial p}{\partial x} = \rho\ddot{u} \quad ; \quad -\frac{\partial p}{\partial y} = \rho\ddot{v} \quad ; \quad \frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y} = 0 \qquad (5.1)$$

The third of these equations is identically satisfied by defining a stream function φ:

$$\frac{\partial \varphi}{\partial y} = \ddot{u} \quad ; \quad -\frac{\partial \varphi}{\partial x} = \ddot{v} \qquad (5.2)$$

and eliminating p from the first two equations gives a Laplace equation that defines the problem:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad (5.3)$$

As is well known, this equation may also be interpreted as describing an idealized, stretched, unloaded membrane whose displacement normal to the x-y plane is $\varphi(x, y)$. The x, y slopes of the membrane, according to Eq. 5.2 are the analogs of the fluid accelerations $\ddot{u}$, $\ddot{v}$. Figure 7 shows the membrane problem that is the analog of Westergaard's hydrodynamics problem for a dam with sloping face. The flat membrane is stretched between the rigid bars AB, BC, AD, DE, which lie in the x-y plane. Apex A is then raised a small distance so that bar BA makes an angle Ψ with the horizontal, so its slope is $\tan \Psi$ and the slope of bar AD is $-\tan \Psi$. From Eq. 5.2, these slopes correspond to fluid accelerations normal to the sloping dam faces of $\tan \Psi$ and $-\tan \Psi$, respectively, so if $\tan \Psi$ is set equal to $(a_o \sin \theta)$, the analogy is complete. It can be seen from Fig. 7 that when $\theta < \frac{\pi}{2}$, the apex angle is acute and the slope of the membrane at this point is finite, its gradient being that of the plane defined by AB and AD. The vector gradient of φ, at the apex, lies in the vertical plane through A and F and is related to Ψ by $|\text{grad}\ \varphi| \cos \gamma = \tan \Psi$, where the angle γ is measured in the plane defined by AB and AD, and its direction is measured from the direction of grad φ. If, on the other hand, $\theta > \frac{\pi}{2}$, the apex angle in Fig. 7 is obtuse and the apex forms a re-entrant corner in the membrane and, as is well known, the slope is infinite at such a corner. When $\theta = \frac{\pi}{2}$ the angle γ also is equal to $\frac{\pi}{2}$, and $|\text{grad}\ \varphi|$ becomes infinite, which means that the fluid acceleration and velocity at $x = 0$, $y = h$ in the hydrodynamics problem are infinite. It appears that in the case of the fluid mechanics problem of Fig. 6, the singularity results from the discontinuity in fluid motion specified by the boundary conditions: $a_x = +a_o$ at $x = 0$, $0 \leq y < h$; and $a_x = -a_o$ at $x = 0$, $h < y \leq 2h$. This results in a shear flow at $x = 0$, $z = 0$ for which $\dot{\epsilon}_{xy} = \partial\dot{u}/\partial x + \partial\dot{v}/\partial z$ becomes infinite.

6. CLARIFICATION OF MOMENTUM-BALANCE METHOD. An explanation of Karman's momentum-balance method can be made in terms of mechanics, and this can be done for the case of the sloping dam, as follows. Figure 8 shows the rigid, sloping dam and the incompressible fluid in an infinite reservoir. The acceleration of a fluid particle has components a_x and a_y, both being functions of x and y. The dam is given a horizontal acceleration a_o and at time $t = \Delta t$ its velocity and displacement are $(a_o \Delta t)$ and $(a_o \overline{\Delta t}^2/2)$. The fluid in the strip of height (dy) has, at time Δt, a total vertical momentum given by the integral:

$$\int_0^\infty \rho\, dy (a_y \Delta t) dx$$

Letting b_y represent the following integral:

$$b_y = \int_0^\infty \frac{a_y dx}{a_{yo}} \tag{6.1}$$

The momentum of the strip (dy) can be written:

$$\text{Vertical momentum} = \rho b_y\, dy\, (a_{yo} \Delta t)$$

where a_{yo} is the acceleration at the dam face $(x = 0)$. The total horizontal momentum in the strip dy is given by the following integral

$$\int_0^\infty \rho\, dy (a_x \Delta t) dx$$

and letting b_x represent the following integral

$$b_x = \int_0^\infty \frac{a_x dx}{a_{xo}} \tag{6.2}$$

The momentum of strip dy can be written:

$$\text{Horizontal momentum} = \rho b_x dy(a_{xo}\Delta t)$$

where a_{xo} is the fluid acceleration at the face of the dam $(x = 0)$.

Conservation of mass requires:

$$\int_0^\infty (a_y \Delta t)dx = y(a_o \Delta t)$$

which states that the amount of fluid that passes through the surface $y = \text{constant}$, must equal the corresponding volume displaced by the dam in moving with velocity $(a_o\Delta t)$ to the right. Making use of Eq. 6.1, the conservation of mass is expressed by

$$b_y a_{yo} = y a_o \tag{6.3}$$

This agrees with Karman's equation of continuity, Eq. 3, 3, if his b is interpreted as being b_y. There is, therefore, a rational explanation of his continuity equation.

A fluid particle adjacent to the face of the dam has horizontal acceleration $a_x = a_o - a_s \cos\theta$, and vertical acceleration $a_{yo} = a_s \sin\theta$, where a_s is the relative acceleration of the fluid particle at the dam face, parallel to the face; and it follows that:

$$a_{xo} = a_o - \beta a_{yo} \tag{6.4}$$

where $\beta = \cot\theta$. From the preceding two equations it follows that a fluid particle adjacent to the face of the dam has x, y components of acceleration expressed by

$$a_{xo} = a_o(b_y - \beta y)/b_y \quad ; \quad a_{yo} = a_o y/b_y \tag{6.5}$$

Momentum balance in the y-direction, for the strip (dy), is

$$-\int_0^\infty \left(\frac{\partial p}{\partial y}\,dy\Delta t\right)dx + \left(\beta p_o \Delta t\right)dy = \int_0^\infty \rho\,dy\left(a_y \Delta t\right)dx \tag{6.6}$$

where the term $(\beta p_o dy)$ represents the vertical force exerted upon the sloping end of the strip. The preceding equation, by means of Eq. 6.1, can be written

$$-\frac{d}{dy}(p_o b_y) + \beta p_o = \rho b_y a_{yo} \tag{6.7}$$

Momentum balance applied to the motion of the strip in the x-direction gives

$$(p_o \Delta t)dy = \int_0^\infty \rho\,dy(a_x \Delta t)dx \tag{6.8}$$

and combining this with Eq. 6.2 gives for the pressure, p_o, at the dam face:

$$p_o = \rho b_x a_{xo} \tag{6.9}$$

Using Eqs. 6.5 and 6.9 to rewrite Eq. 6.7 gives

$$\frac{d}{dy}\left[\frac{b_x}{b_y}\left(b_y^2 - \beta y b_y\right)\right] - \beta\frac{b_x}{b_y}\left(b_y - \beta y\right) = -y \tag{6.10}$$

Up to this point no simplifying approximations have been introduced in the analysis. In order to solve Eq. 6.10, however, there is required a relation between b_y and b_x. This might be expressed as

$$b_y = b_x(c_0 + c_1 y + c_2 y^2 + \dots\ ) \tag{6.11}$$

and the c_n's might be evaluated on the basis of a minimum principle. However, this would require a knowledge of the terms a_x^2 and a_y^2 for the fluid, and a_x and a_y have been integrated out of the foregoing analysis. But, as an approximation, all c_n's may be taken to be zero except c_0 which can then be set equal to unity, that is, b_y is assumed to be equal to b_x. Equation 6.10 then becomes

$$\frac{d}{dy}\left(b^2 - \beta y b\right) - \beta\left(b - \beta y\right) = -y \tag{6.12}$$

which is the same as Eq. 4.7 derived by the momentum-balance method.

In the case of Karman's problem, that is, a dam with vertical face, Eq. 6.10 reduces to

$$\frac{d}{dy}\left(b_x b_y\right) = -y \tag{6.13}$$

The integral of this equation satisfying condition $p = 0$ at $y = h$ is

$$b_x b_y = \left(h^2 - y^2\right)/2 \tag{6.14}$$

This equation is based solely on mechanics and involves no simplifying approximations. Taking $b_y = b_x$ gives Karman's solution:

$$b_x = \left(\frac{h^2 - y^2}{2}\right)^{\frac{1}{2}}$$

It is seen that b_x and b_y can be interpreted as equivalent breadths of two different "apparent masses" based, respectively, on horizontal momentum and vertical momentum, and the essence of Karman's approach is the assumption that the two breadths are equal.

7. CONCLUSIONS. It is seen from the foregoing analysis that the momentum-balance method is just a way of implicitly assuming that $b_x = b_y$, that is, for any y:

$$\int_0^\infty \rho a_x dx = \int_0^\infty \rho a_y dx$$

This is equivalent to assuming that during the time Δt the resultant momentum in each (dy) strip, shown in Fig. 8, has a vector direction that makes an angle of 45° with the floor of the reservoir. Intuitively this seems plausible for a vertical dam face, but not so plausible for a sloping dam face. Even for the vertical dam face it is clear that the assumption is incorrect at $y = 0$ and $y = h$. At $y = 0$ the vertical velocity is zero and so the angle is zero, and at $y = h$ the angle must be close to 90° for the integral of ρa_x must be close to zero. In view of this, it is surprising that the momentum balance method gives such good approximations. It is another example of Karman's well-known flashes of insight. He did not explain how he happened to hit upon this method, but it would seem that he was motivated by the spirit of a remark by Lamé: "When one obtains a simple result by means of complicated calculations, there must exist a more direct method of obtaining the result; the simplifications which occur and the terms which disappear during the course of the calculations are certain indications that a method exists for which these simplications have already been made and in which these terms do not appear." - M. Lamé, Théorie de L'Elasticité (1866).

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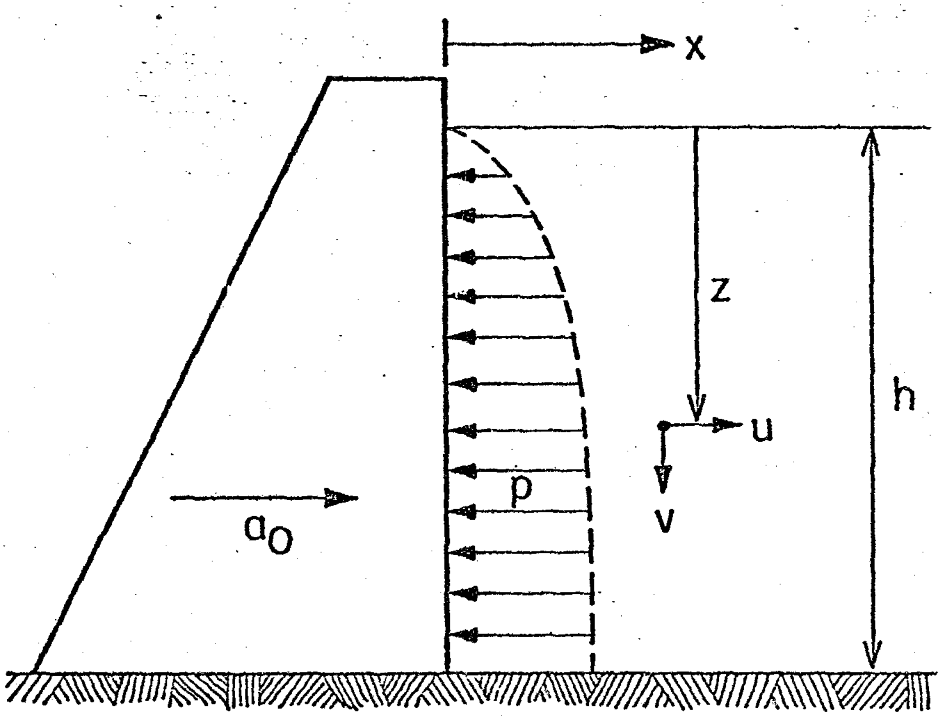

Fig. 1. Westergaard's problem of a rigid dam accelerating into the water in a reservoir and experiencing hydrodynamic pressures.

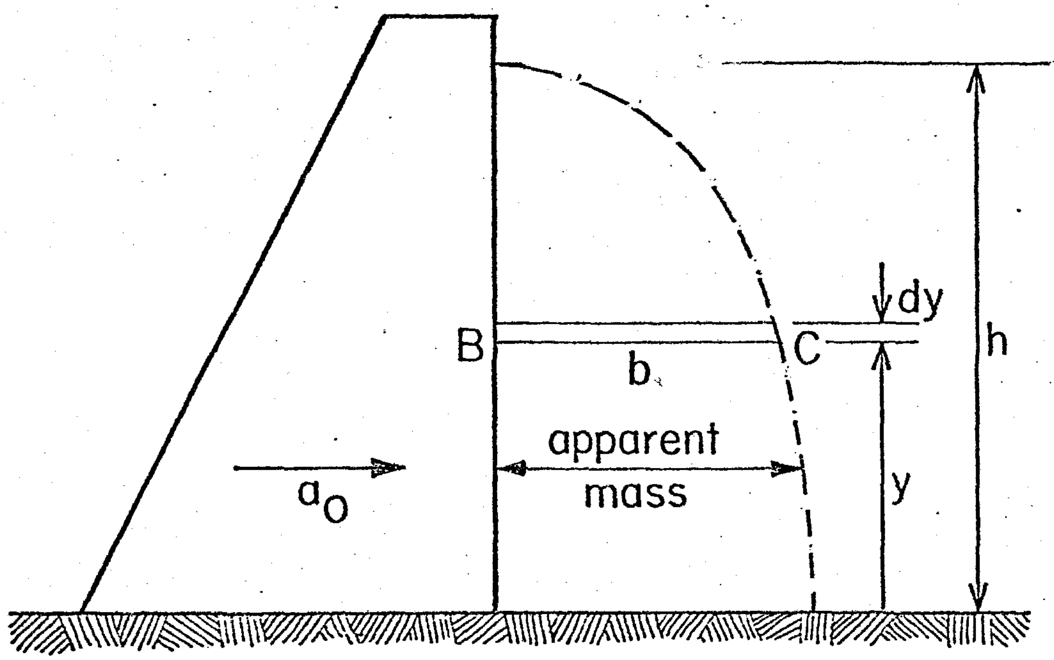

Fig. 2. Karman's problem of a rigid dam accelerating an apparent mass of fluid.

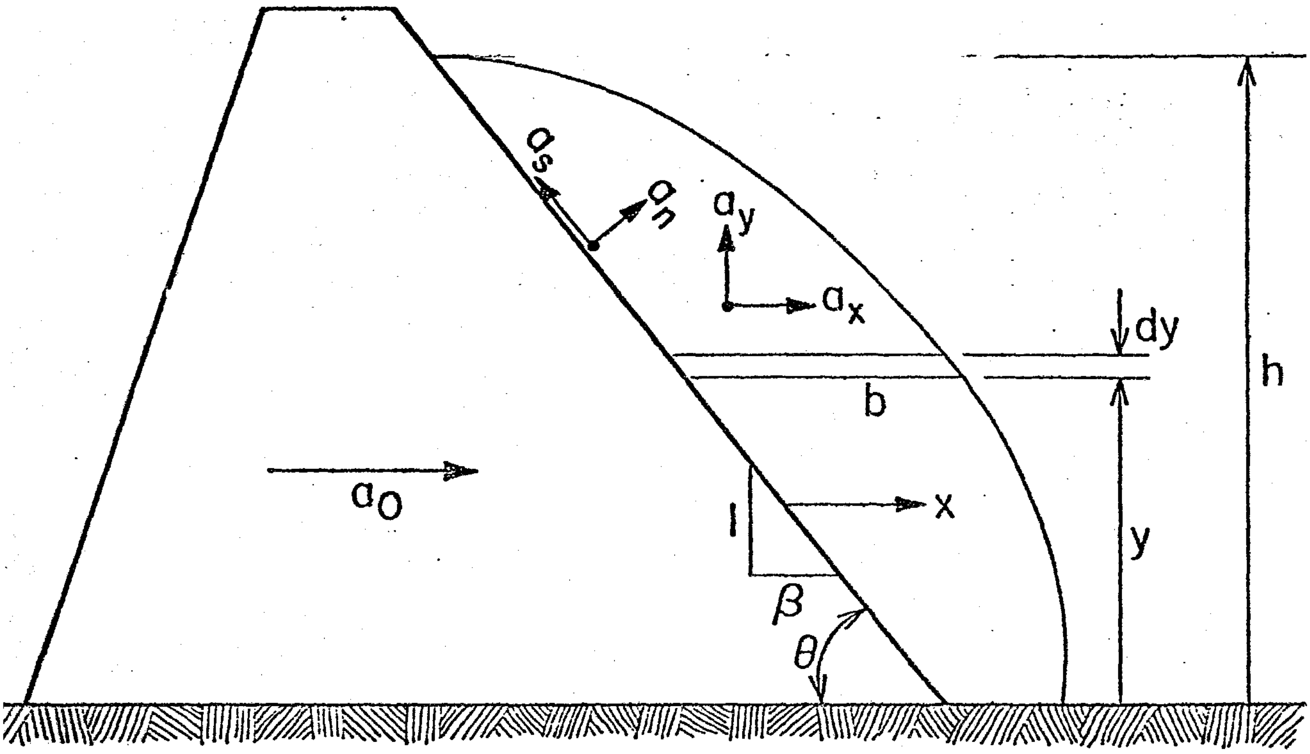

Fig. 3. Dam with sloping face accelerating an apparent mass of breadth b.

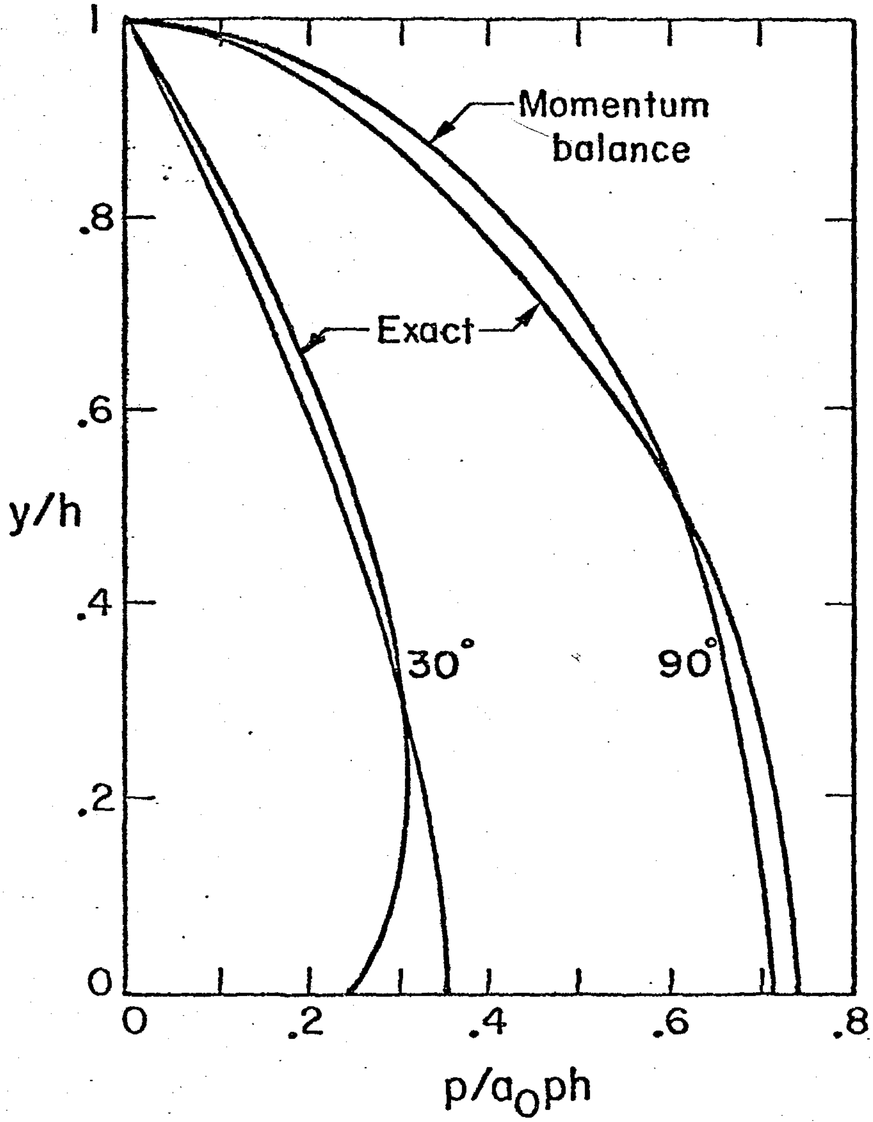

Fig. 4. Comparison of exact pressure distribution with that given by the momentum-balance method.

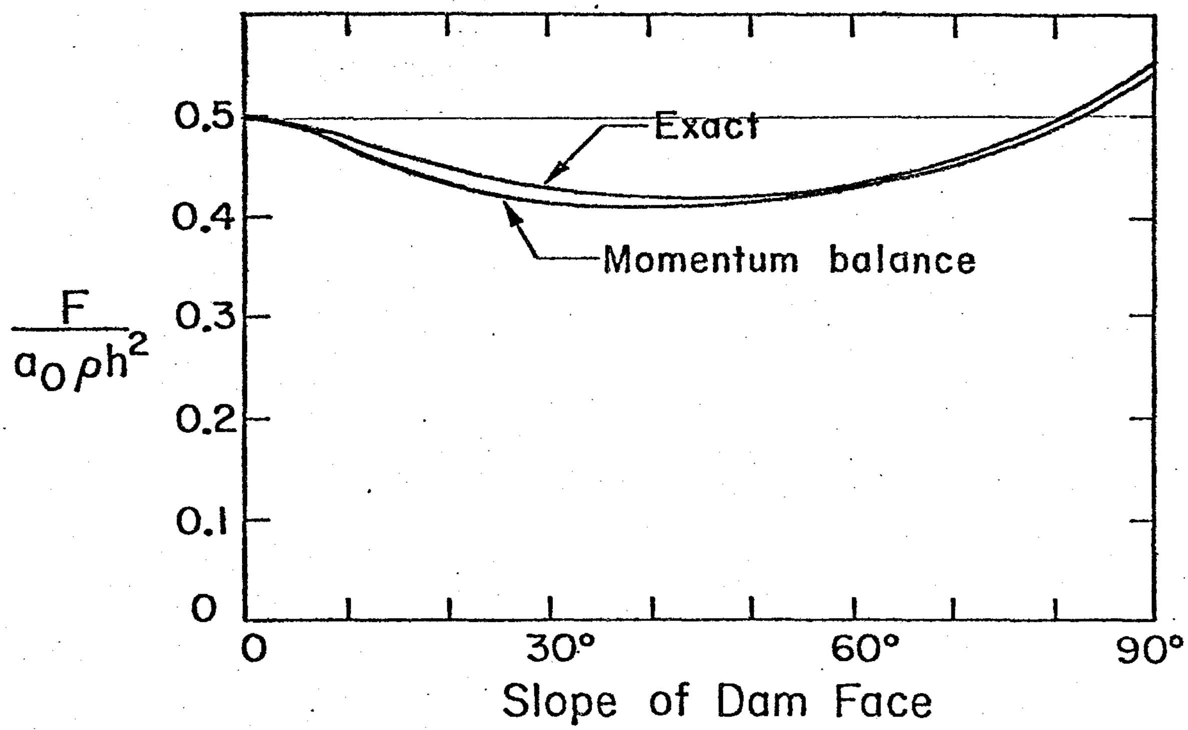

Fig. 5. Total hydrodynamic force exerted on the face

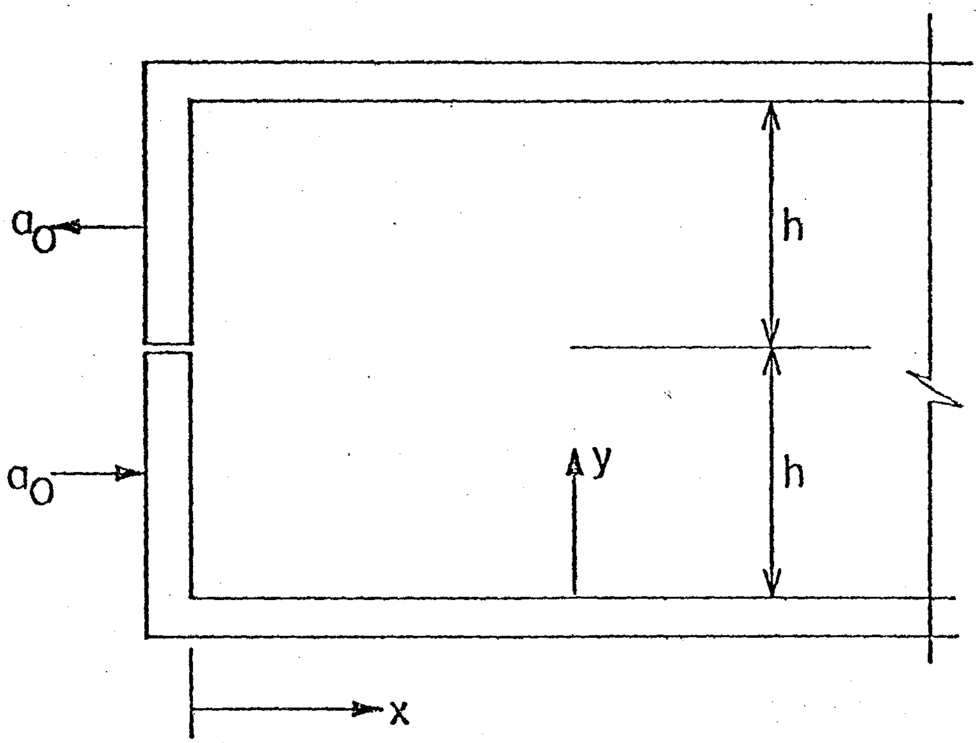

Fig. 6. Hydrodynamic problem exactly representing Westergaard's solution.

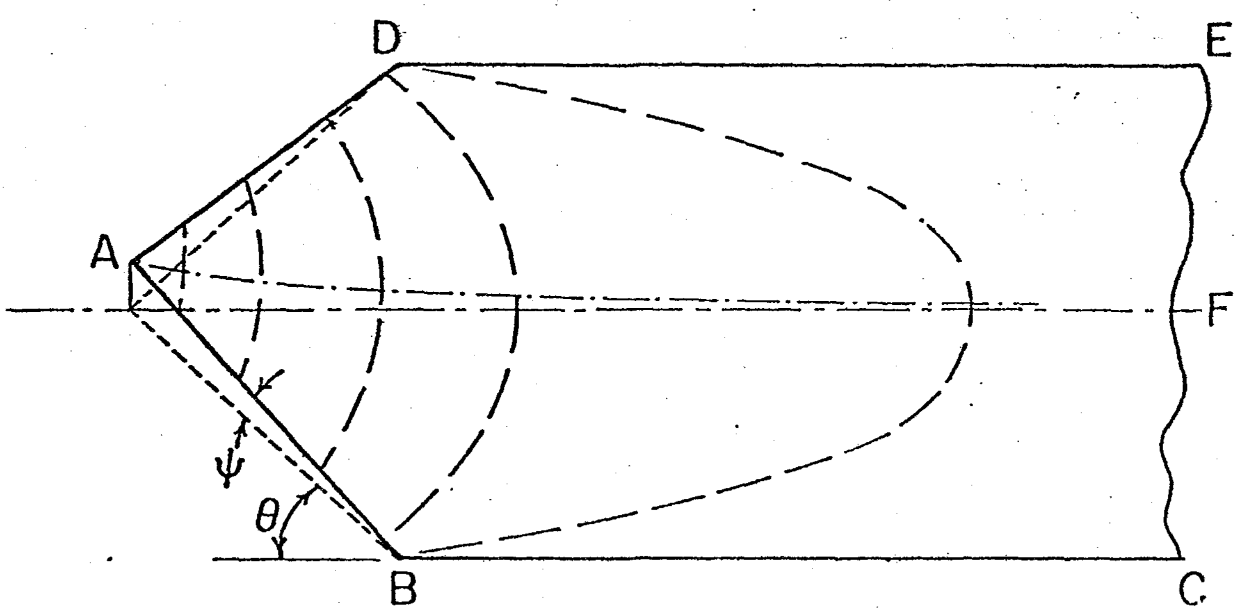

Fig. 7. Stretched membrane with tilted apex. Dotted lines indicate contours.

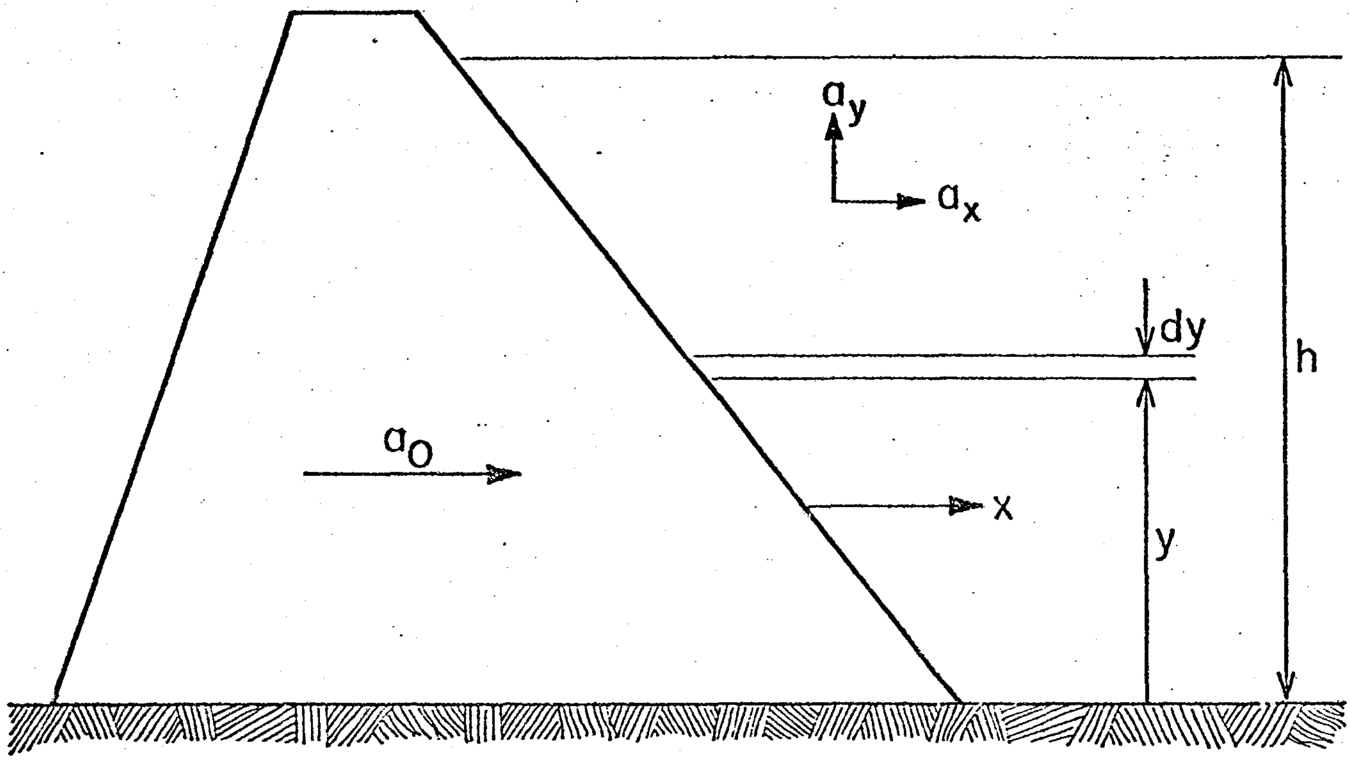

Fig. 8. Rigid sloping dam accelerating into fluid in a reservoir.