Caustics Produced by Waves Through the Earth's Core*

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(Received 1958 June 23)

Summary

Caustics produced by *SKP*, *PKS*, *PKP* and other phases through the Earth's core are investigated. No impulses from diffracted waves with periods of two seconds or more have been found beyond five degrees from caustics. Short waves preceding the main *PKIKP*-phase at distances between 125° and 140° probably have their deepest point in or near a transition zone between the liquid outer and the probably solid inner core and it is unlikely that they are related to the caustic of *PKP* near 145° although their travel-time curves end near this caustic. The observed range of long-period waves diffracted beyond the caustics of several phases is smaller than the maximum range calculated by Jeffreys.

1. The problems

Caustics and cusps of travel-time curves of waves travelling through the Earth's core are produced by two different phenomena. (1) The sudden decrease in the velocity of longitudinal waves at the core boundary results in travel-time curves consisting of two separate branches (Figure 1 a). In the following discussions

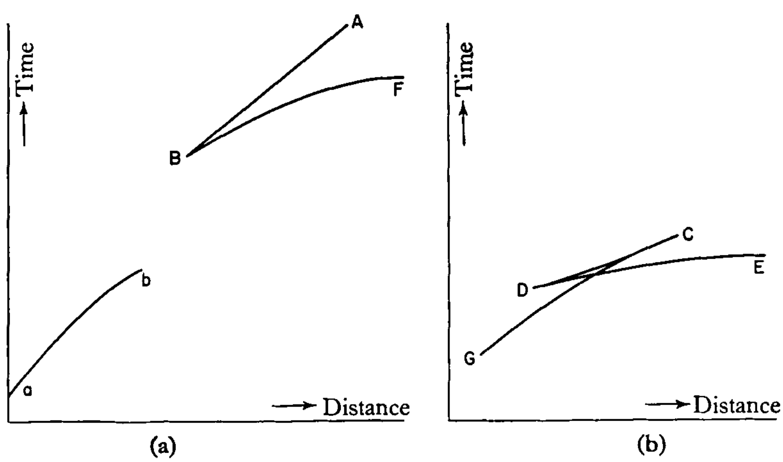
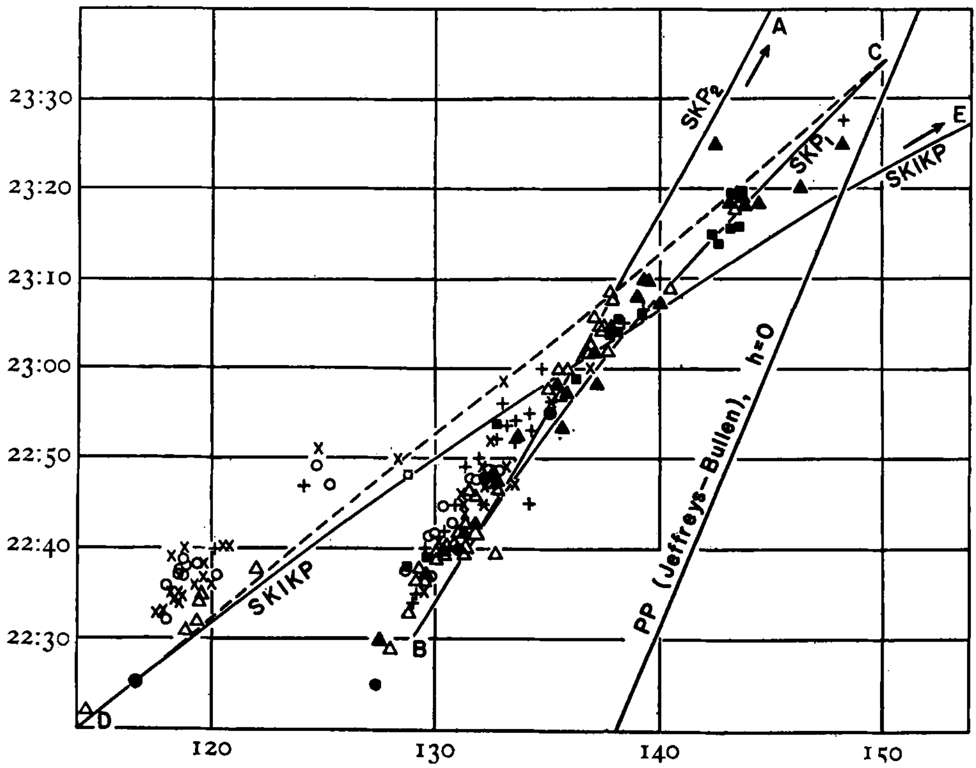


FIG. 1.—Schematic travel-time curves, (a) if the velocity decreases, (b) if it increases suddenly or rapidly with depth.

* Contribution No. 890 from the Division of the Geological Sciences, California Institute of Technology.

only the second branches (*ABF*) are involved. With decreasing angle of incidence the epicentral distance at which the rays arrive at the Earth's surface decreases to a minimum (at *B*), which corresponds to a caustic, and then increases again. (2) The increase in velocity between the outer and the inner core results in cusps of the travel-time curves (Figure 1 b). Most travel-time curves of waves through the core begin with a section *ABC* (where *C* is on the curve *BF* of Figure 1 a), then continue with *CDE* of Figure 1 b. For some phases, e.g. *PKS* and *SKP* (Figure 2), the



Abscissa: distance in degrees.
Ordinate: Travel time in minutes and seconds.

Observed data, corrected for *h*
Data from SKP
Gutenberg (1958) *h* = shallow +
h = 50 to 150 km ×
h = 270 km, Sweden ✱
h = 600 km ■

Forester (1953) *h* = shallow,
short-period Z △
long-period Z ▲
h = 100 km, short-period Z o
Data from PKS intermediate *h* ●
Curves are calulated for *h* = 0

FIG. 2.—Travel times of *SKP* group, reduced to zero focal depth.

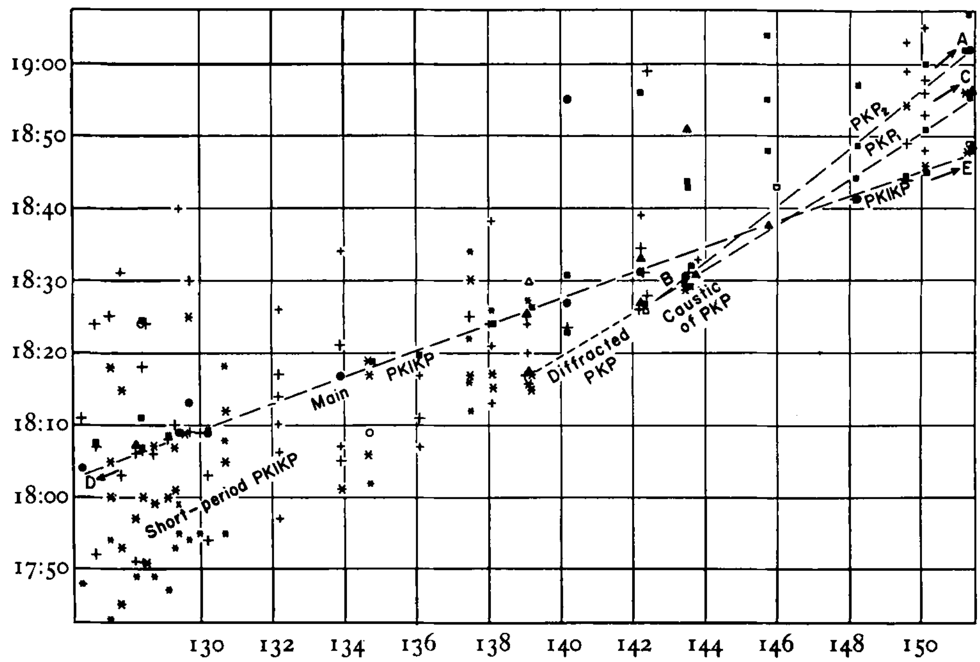
waves arrive noticeably earlier at the caustic *B* than waves to the branch *DE* at the same epicentral distance; for others, e.g. *PKP* (Figure 3), the caustic *B* is near the branch *DE*.

Wiechert (1922) has suggested that longitudinal waves through the core arriving at epicentral distances smaller than that of the caustic *B* which is at about 145° are diffracted, but Lehmann (1936) has pointed out that their amplitudes at distances as short as 110° are too large for diffracted waves. She suggested the existence of an inner core in which the velocity is greater than in the outer core. Gutenberg & Richter (1938) have found that assumption of an increase in velocity from about 10·2 to 11·4 km/s between about 1600 and 1200 km from the Earth's centre explains the various observed travel-time curves of waves through the core.

Jeffreys (1939, p. 553) applying Airy's theory of diffraction has concluded that waves with periods T of 1 s could be diffracted up to 4 deg, and those with T = 10 s up to 15 deg from the caustic. To check these results, Bullen & Burke-Gaffney (1958) studied records produced by hydrogen bombs exploded near Bikini and written at epicentral distances slightly smaller than that of the caustic of *PKP*. However, the phases believed by Bullen and Burke-Gaffney to be diffracted *PKP* waves may actually be early short-period *PKIKP* waves (compare Figure 3).

2. Materials used

The material used includes data for *PKP* and *PKIKP* by Gutenberg (1951, 1957a, 1958), for *PKP* by Denson (1950, 1952), for *PKS* by Forester (1953, 1956) and for *SKS* by Nelson (1952, 1954). Moreover, seismograms of an earthquake at a focal depth h = 600 km, have been kindly furnished by stations with



Abscissa: distance in degrees.
Ordinate: Travel time in minutes and seconds.

Travel times

T	e	ei,i
< 1s	* (small)	*
1 – 1¾	+ (small)	+
2 – 2¾	o	●
3 – 4¾	□	■
⩾ 5	△	▲

FIG. 3.—Travel times of the *PKP*-group, observed for the earthquake at 4 h: 04 min: 04 s G.C.T. on 1957 April 16, focal depth about 600 km near 4½° S 107½° E.

epicentral distances of over 119° in North and Central America (details in Table 1); for most of them the range of azimuths at the source is within 50°, so that the error in location (probably less than 1°) does not affect noticeably the resulting relative travel times.

Travel times t for *PKP*, *SKP* etc. to distances θ in degrees have been calculated by combining the travel times for K through the core found by Gutenberg (1958, Table 4) with the travel times for *PcP*, *PcS* and *ScS* for zero focal depth listed by Jeffreys & Bullen (1940). If $dt/d\theta$ for K and for *PcP* etc. is measured in seconds per degree, corresponding values of θ and t for *PKP* etc. are found by adding values for θ and t for K to those for *PcP* etc. which correspond to the same value

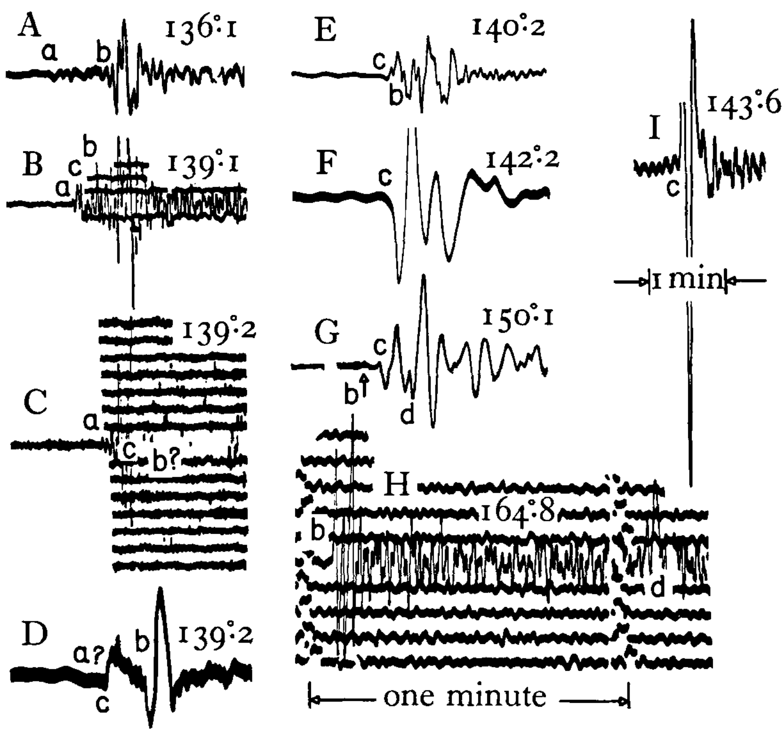


FIG. 4.—Beginning of vertical records of the shock of 1957 April 16 (Table 1). A. at Kirkland Lake; B. at Montreal; C. and D. at Ottawa, C from short-period, D from long-period instrument; E. at Lubbock; F. at Florissant; G. at Tacubaya; H. at San Juan; I. from ultra-long instrument at Palisades.

a. indicates short-period *PKIKP*; b. long-period *PKIKP*; c. *PKP*1; d. *PKP*2.

of $dt/d\theta$. This procedure permits rapid calculation and is sufficiently accurate in most instances. For *PKIKP*, travel times through the core for short as well as for long waves have been used, for all other phases only the main travel-time curve for K ($T \geqslant 2$ s).

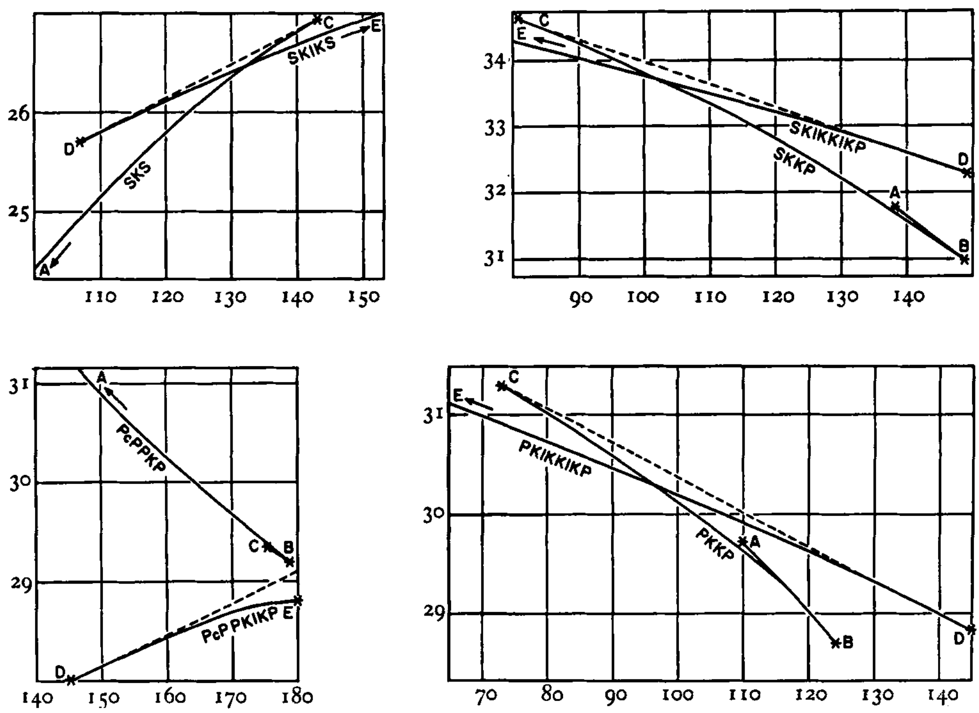
Where observed travel times have been reduced to values for a surface focus, the necessary corrections have been taken from Jeffreys & Bullen (1940) or from

Gutenberg & Richter (1936; 1939, p. 113). For *PKIKP* characteristic corrections are as follows:

Focal depth	25	100	300	600 km
correction $\theta = 145°$	+4	+14	+37	+69 s
correction $\theta = 180°$	+4	+14	+38	+70 s

For *PKP* and *PKS* they are about 2 s smaller than for *PKIKP* if $h = 600$ km. For *SKS* they are 125 s for $h = 600$ km at $\theta = 130°$ and 129 s near 180°; for *SKP*, most corrections are up to 2 s smaller than those for *SKS*. Errors in the corrections probably do not exceed a very few seconds.

The epicentral distance of the caustic of *PKP* is about 2° shorter for shocks at a depth of 600 km than for shallow shocks. However, while depth corrections for travel times depend mainly on the phase (*P* or *S*) with which the wave starts, the path in the core plays an additional role for the distance of the caustic.



Abscissa: for all diagrams. Distance in degrees.
Ordinate: for all diagrams. Travel time in minutes.

FIG. 5.—Calculated travel times of (a) *SKS*-, (b) *SKKP*-, (c) *PcPPKP*- and (d) *PKKP*-groups for zero focal depth.

3. SKP and PKS

In Figures 2, 3 and 5, *ABC* corresponds to waves which have not entered the inner core, the caustic is at *B*, and *CD* results from the increase in wave velocity between outer and inner core. Its proximity to *BC* and *CE* prevents observation, and its details are always uncertain. *DE* corresponds to waves refracted through the inner core. At *E*, which for *SKP* is at $\theta = 180°$, the amplitudes of *SKP* or

PKS are zero, since for vertical incidence at the core boundary no energy can be refracted from *S* to *K* or vice versa.

Most observed points in Figure 2 are either close to *SKP* near the caustic, or to *SKIKP* at shorter distances. Most observations of *SKIKP* are from records of earthquakes at intermediate depth (compare Gutenberg 1958, Figure 5). Observed times are frequently a few seconds greater than the calculated, probably mainly as a result of too late readings of phases emerging in the preceding motion.

For $h = 0$, the calculated epicentral distance of the caustic of *SKP* is 129°. Forester (1953, 1956) has found the largest amplitudes of *SKP* at distances near $131\frac{1}{2}°$ for waves with $T < 5$ s and between 130° and 131° if $T > 5$ s. The smallest distances at which he has identified *SKP* are 129° for short-period and 125° for long-period waves. The shortest distance at which *SKP* has been measured in the present research without searching for it is 129°; one clear *PKS*-wave ($T = 7$ s) has been found by Gutenberg (1958) from a 270 km deep shock at $\theta = 127°$. Thus, the observed distances of diffracted *SKP*- and *PKS*-waves are well within the theoretical limits found by Jeffreys (1939).

4. The caustic of PKP

Gutenberg (1914) had found that on records of the long-period instruments used before 1910 the first *PKP*-waves emerge at about 141° and that their largest amplitudes (periods 4 to 12 s) occur near 145°. Denson (1952) observed that *PKP*-waves with periods *T* of 5 to 8 s show the caustic near 143° while for those with *T* about 2 s it is near 147°. According to Gutenberg & Richter (1935, p. 325) the largest amplitudes of *PKP* are between 143° and 147°. In the present investigation, the largest amplitudes of *PKP* are observed at distances of between 142° and 144°, (compare Figure 4, F, I) where at several stations the first impulse is so large that its turning point is not visible. This corresponds to about 145° in a shallow shock. Gutenberg (1951, p. 382) has observed the amplitude maximum for *PKPPKP* at a distance of 68°, corresponding to a distance of 146° for *PKP*. An epicentral distance of 145° for the caustic of *PKP* in shallow shocks is in good agreement with all these data.

5. Travel times of PKP and PKIKP

Seismograms of the shock of 1957 April 16, 4 h: 04 min: 04 s, in the East Indies show clear phases of waves through the core (compare Gutenberg, 1957 a, Figure 1; 1958, Figure 4), and have been selected for detailed study. Twenty reports of *pP–P* ($50° < \theta < 104°$) give focal depths between 570 and 640 km (average 590 ± 5 km). For calculations $h = 600$ km, given by the U.S. Coast and Geodetic Survey and by the U.S.S.R., has been used.

Figure 3 shows travel times of the *PKP*-group; arrival times of the main phases are listed in Table 1. Short- and long-period records give the same time for the first impulse of *PKP*, which is not preceded by any detectable emergent waves (Figure 4, F, I) and near its caustic arrives a few seconds earlier than *PKIKP* (Figure 3).

At Ottawa ($\theta = 139{\cdot}2°$) a long-period wave which precedes *PKIKP* by about 9 s (Figures 3, 4 D) is probably a diffracted *PKP* wave. There is no indication of diffracted long-period *PKP* waves for $\theta < 138°$, that is, over 5° from the caustic. Consequently, we cannot expect short-period diffracted *PKP* waves for $\theta < 140°$, and it is unlikely that the short-period waves preceding the main *PKIKP* phase at $\theta < 140°$ are connected with *PKP*.

Table 1

Arrival times t (*after* 4 *a.m.*) *and corresponding periods* T *of first PKIKP-waves* (a), *main PKIKP-waves* (b), *PKP1* (c), *and PKP2* (d) *of earthquake on* 1957 *April* 16; *assumed epicentre at* 4½° *S* 107½° *E, focal depth near* 600 *km, magnitude m* = 7·2. *θ, epicentral distance. Parentheses indicate that the interpretation is doubtful, including effect of microseisms.*

Station	θ	a		b		c		d	
		t	T	t	T	t	T	t	T
	deg	min s	s	min s	s	min s	s	min s	s
Seattle	119·5	no		21 55!	1½				
Ukiah; Shasta	122·7	no		21 59*	?				
Hungry Horse	123·8	no		22 01*	?				
Berkeley	123·9	no		22 02!	1;5				
Santa Clara	124·3	no		22 02	5				
Lick	124·5	21 49	¾	22 03!	1½				
King Ranch	126·7	(21 52)	¾	22 08!	2				
Tinemaha	127·1	21 56	1	22 11!	1;3				
Isabella	127·5	21 58	¾	22 09!	¾				
Eureka, Nevada	127·5	21 47*	?	?	?				
Woody	127·8	21 49	¾	22 07!	1				
China Lake	128·1	21 53	¾	22 10!	1½				
Pasadena	128·4	21 55	¾	22 11!	1;4;8				
Dalton	128·7	21 53	¾	22 11!	¾				
Riverside	129·1	21 51	¾	22 12!	1½;3				
Big Bear	129·3	21 57	¾	22 14!	1				
Salt Lake City	129·4	(22 03)	1	22 13!	2				
Schefferville	129·6	21 59	½	22 13!	½				
Palomar	129·7	21 58	¾	22 13!	1				
Boulder City	130·0	21 59*	?	22 13*	?				
Barrett	130·2	21 58	1	22 13!	2;4				
Hayfield	130·7	21 59	¾	22 16!	¾				
Rapid City	132·2	22 01	1	22 18!	1½				
Boulder, Colo.	133·9	22 05	¾	22 21!	2				
Denver	134·2	22 06	¾	22 21	2½				
Tucson	134·7	22 06	¾	22 23!	4				
Kirkland Lake	136·1	22 11	1	22 24!	3				
Seven Falls	137·5	22 16	¾	22 29	1				
Shawinigan Falls	138·1	22 17	1	22 28!	1;3				
Montreal	139·1	22 20	½	22 29!	1½;3	22 24	1		
Ottawa	139·2	22 19	¾	22 30!	4½	22 21!	½;3;5		
Halifax	139·3	22 21*	?						
Lubbock	140·2			(22 31)	2?	(22 27)	3		
Florissant	142·2					22 31	5		
Harvard	142·2					22 30!	1		
Weston	142·3			22 35	1;2	22 31!	1;4		
Cleveland	142·3					22 31!	1?		
St. Louis	142·4					22 32!	1		
Mazatlan	142·4					22 30	3		
Fayetteville	143·5					22 33!	1;3		
Pennsylvania State	143·5					22 35	3;10		
Palisades	143·6					22 35!	1;10		
Pittsburgh	143·6					22 36!	?		

Table 1 (*continued*)

Station	θ	a		b		c		d	
		t	T	t	T	t	T	t	T
	deg	min s	s	min s	s	min s	s	min s	s
Fordham	143·8					22 36*	?		
Dallas	143·8					22 37!	1;2		
Washington, D.C.	145·7					22 42!	5		
Chapel Hill	148·2			22 45	2;4	22 48!	4	22 52!	3
Columbia	149·6			22 48!	1;3	22 53!	1	22 58	½
Tacubaya	150·1			22 49!	¼;3	22 56!	1;3	23 02	1;5
Bermuda	151·3			22 52	¾	23 00!	½	23 06	2
Mérida	156·6			23 02	3			(23 38)!	3
Comitan	157·5			(22 59)	3				
Oaxaca	157·8			23 04	3				
Antigua	163·6			23 05!	¾			24 00	¾
San Juan	164·8			23 06	1			24 07	1½
St. Lucia	165·2			23 06!	1			24 07!	1
St. Vincent	165·9			23 06!	¾			24 10!	¾
Trinidad	167·5			23 10!	¾			24 19!	¾
Galerazamba	173·2			23 15*	?				
Chinchina	176·9			23 12!	3			25:02!	3
Bogota	178·4			23 12!	4				

* From reports.

At distances between about 148° and at least 152°, *PKIKP*, *PKP1* and *PKP2* can be identified (Figures 3, 4 G). At greater distances, up to nearly 180°, *PKIKP* and *PKP2* are clearly recorded (Figure 4 H).

At epicentral distances of $124° < \theta < 140°$, many seismograms of short-period instruments begin earlier than those of long-period instruments (Figures 3, 4 A). Travel times of these short-period phases, and especially the earliest that can be measured on a record beyond doubt, depend on properties of the instrument, the shortest periods existing in the waves, effects of the ground under the station, properties of the microseisms, and possibly the wave path, including effects of the ellipticity of surfaces in the earth. The time interval between these short-period waves and the main branch of *PKIKP* decreases with increasing θ. Ottawa ($\theta = 139{\cdot}2°$) seems to be the most distant station where these short-period waves are visibly recorded (Figure 4 C).

These short-period waves preceding the main *PKIKP*-phase have been reported repeatedly (e.g. by Gutenberg & Richter 1934; Denson 1952; Båth in bulletins of Uppsala). They have been considered by Gutenberg (1957, 1958) to be *PKIKP*-waves which have higher velocities in the transition zone from the outer to the inner core than the longer waves. Kuhn & Vielhauer (1953) have concluded from theoretical considerations and from laboratory experiments that in a material near its melting point the velocity of longitudinal waves depends on the wave length, and that for a given material the "boundary" between its solid and its liquid phase, found from observed increase in wave velocities, should extend farther towards the liquid phase for short than for relatively long waves. If we assume (probably not correctly) that all the earliest short "*PKIKP*"-waves belong to the same travel-time curve, an increase in velocity of longitudinal waves from about 10 to 11 km/s would start at a distance r of about 1700 km from the Earth's centre; exact values of r cannot be found, since neither the epicentral distance of the end of *PKP* (point C in Figure 3) nor the branch CD of the travel-time curve is observed. For waves with travel-times about half way between the first and the main branch of *PKIKP* in Figure 3, Gutenberg (1958) found on similar assumptions that the longitudinal wave velocity would increase from about 10·1 km/s at r about 1500 km to 11·2 km/s at r = 1 300 km; for the main branch (T usually 2–3 s) a similar increase in velocity would occur between 1 300 and 1 200 km from the Earth's centre.

The greatest difference in travel times for short- and long-period *PKIKP*-waves should occur at the shortest distances. With increasing θ the length of the path of short-period waves in the transition zone between outer and inner core decreases, and the various travel-time curves should approach each other as it is observed (Figure 3). Unfortunately, periods T of less than 1 s cannot be measured accurately on usual seismograms; theoretically, the effect of the transition zone should differ appreciably for waves having different periods of less than 1 s (length $10T$ km). Irregularities in the transition zone should produce greater scattering and higher absorption of short than of longer waves. The curves for the melting point and the temperature in the transition zone probably intersect at a very small angle, so that a thick transition zone with irregularities may be expected.

The rather sudden end of the short-period waves at about the same distance and time where the long-period diffracted *PKP* begins looks suspicious. However, it is unlikely that short-period diffracted *PKP*-waves extend many times as far from the caustic as long-period diffracted *PKP*-waves. Another problem concerns the short-period impulses observed frequently simultaneously with the main long-period *PKIKP*-impulse. However, the shortest periods observed in this impulse are usually noticeably longer than those observed in the earlier short-period waves (compare Table 1). The finding of several distinct impulses between the first and the main *PKIKP*-waves and the lack of a continuous spectrum corresponds to the observation (Gutenberg, 1957 b) that in many seismograms from distances of less than 100° *P* as well as *S* show the prevalence of a few distinct periods and not a continuous spectrum.

6. Travel-time curves of other phases through the core

Observed travel times of *SKS* follow the curve for *SKS* (Figure 5 a) at short epicentral distances as far as its intersection with the curve for *SKIKS* and are near the *SKIKS*-curve for greater distances (Nelson 1954).

Travel times for *PKPPKP* can be found from Figure 3 by doubling distances and times. Observations by Gutenberg (1951, Figure 3) are close to all major branches. There is no indication that *PKPPKP* is observed more than 8° beyond the caustic; this corresponds to 4° for *PKP*.

Figure 5 d shows travel-time curves for *PKKP*, which is a short-period phase. There are many observations by Gutenberg (1951, Figure 12), most between 2 s earlier and 12 s later than calculated. *PKIKKIKP* has not been definitely observed. There is no indication of *PKKP*-waves diffracted noticeably beyond the caustic.

Figure 5 b shows travel-time curves for *SKKP*, which usually consists of a train of short-period waves. Impulses of *SKKP* have been recorded at eleven stations ($129° < \theta < 144°$) on 1957 April 16. They are between 3 and 13 s later than calculated. *SKIKKIKP* has not been observed.

The travel-time curve of *PcPPKP* (or *PKPPcP*) should have a caustic near

180° (Figure 5 c). The curve for *PcPPKIKP* has a strong curvature near 175°, so that relatively large amplitudes can be expected there. Strong impulses have been observed by Gutenberg & Richter (1934, p. 123) at distances between 175° and 176° with travel times between 29·0 and 29·3 min.

Other phases through the core and their caustics have been discussed by Gutenberg & Richter (1934, 1939) and by Jeffreys & Bullen (1940).

7. Conclusions

In all instances where seismograms were available showing phases at distances near their caustic, or where corresponding data have been reported, no waves having periods $T \geqslant 2$ s have been found in the "shadow zone" at distances exceeding about 5 deg beyond the caustic. The amplitudes of these diffracted waves decrease rapidly with increasing distance from the caustic. The range of distances showing waves diffracted at the caustic surfaces in the Earth is even shorter than that expected theoretically by Jeffreys. It is unlikely that diffracted waves with periods of two seconds or less exist at epicentral distances exceeding five degrees from these caustics. Waves with periods of less than 2 s, observed at distances of between 19° and 4° from the caustic of *PKP* are probably *PKIKP* waves with their deepest point in or near the transition zone between the liquid outer and the solid inner core; in this zone, short waves travel faster than longer waves.

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1958 *June.*

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