results are

$$C_F = (2\sigma/Re_1)(1 - Q)\exp(\sigma\tau/Re_1)\,\mathrm{erfc}\,(\sigma\tau/Re_1)^{1/2} \quad (5)$$

$$U_s/u_1 = Q + (1 - Q)\exp(\sigma\tau/Re_1)\,\mathrm{erfc}\,(\sigma\tau/Re_1)^{1/2} \quad (6)$$

when $Q = 0$—i.e., the wall is isothermal—Eqs. (5) and (6) revert to the solution of reference 3.

The "analogy" between this nonstationary problem and the semi-infinite flat plate consists in setting the time from the start of the motion proportional to the distance from the leading edge of the stationary plate. In the present problem it is necessary to simultaneously identify the leading edge of the stationary plate with a point—for instance, the origin of the nonstationary plate. If the first assumption is true, which implies that the boundary layer has no "memory" of its past development, then the second one appears to be also acceptable because the Rayleigh flow along the moving plate is identical over all sections at any time. The solution is physically meaningful only in the region where the temperature is finite and positive. However, the "analogy" between the stationary and nonstationary problem under the transformation $t = Ux$ is weak. As the inspection of the complete set of the governing equations reveals, the conditions for equivalence (such as $v = 0$) are not rigorously valid even for the isothermal-wall compressible case.[3] (They are valid for the incompressible flow.[2]) The addition of a streamwise variation of wall temperature—consequently a streamwise variation in density (the pressure was forced to be constant)—and, by continuity, the existence of a cross-flow velocity make the analogy even more approximate.

In spite of the shortcoming of this analysis, a qualitative estimate may be attempted. There is no reason to foresee that the trend indicated by this model should be incorrect. The ratio of skin friction with and without wall temperature gradient is

$$C_F/C_{F_0} = (1 - Q) = 1 - 4.24 \times 10^{-3}\,(1/u_1 P_S) \times [\partial(T_s^{3/2})/\partial x]$$

where $C_{F_0}$ is simply given by Eq. (5) with $Q$ zero.

The numerical constant was calculated using the ideal-gas equation of state and the values of gas constants characteristic of diatomic air. The units are in degrees Kelvin, centimeters, and atmospheres. This ratio exceeds unity when the temperature falls downstream of the leading edge. At typical altitudes (for instance: $P = 10^{-5}$ atm.) and for high temperature gradients (for instance: 20°K. per centimeter at 1,000°K.) the "thermal strain" increases the skin friction by about 5 per cent at 20,000 ft./sec. flight speed.

The result is quite sensitive to the assumption of temperature gradient and ambient pressure. The thermal stress term can be as high as 50 per cent of the slip-flow friction, if a higher temperature gradient or a lower pressure were assumed. However, if one considers an aerodynamic surface, the influence of these factors tends to balance out; higher pressures would lead to higher aerodynamic heating and therefore to higher gradients.

### References


1 Patterson, G. N., *Molecular Flow of Gases*, pp. 179–187; John Wiley & Sons, Inc., New York, 1956.

2 Schaaf, S. A., *A Note on the Flat-Plate Drag Coefficient*, University of California Inst. Eng. Res. Report HE-150-66, 1950.

3 Mirels, H., *Estimate of Slip Effect on Compressible Laminar Boundary Layer Skin Friction*, NACA Tech Note 2609, 1952.


## On the Effect of Air Pressure on Strouhal Number


Anatol Roshko
*Guggenheim Aeronautical Laboratory, California Institute of Technology, Pasadena, Calif.*
August 25, 1958


The experimental measurements of reference 1 show an effect of free-stream pressure on the Strouhal Number-Reynolds Number relation for a vortex-shedding cylinder. Since it is unlikely that Buckingham's $\pi$-Theorem needs re-examination, and since other possible parameters (Mach Number, etc.) would not appear to be relevant under the conditions of the experiments, one is at a loss to explain the results. It is interesting that if the Reynolds Numbers of reference 1 are divided by the pressures, normalizing on standard pressure, one obtains a single "standard" curve. Such a correction to Reynolds Number would be needed if account had not been taken of the density changes corresponding to the pressure changes.

### Reference


1 Rimoldi, R. F., Clifford, W. D., and Bacigalupi, R. J., *Effect of Air Pressure on Vortex-Shedding Frequency of Cylinders*, Journal of the Aero/Space Sciences, Vol. 25, No. 8, p. 532, August, 1958.


## *Author's Reply*


William D. Clifford
*National Aeronautics and Space Administration, Lewis Research Center, Cleveland, Ohio*
October 16, 1958


In the experimental program presented on the effect of air pressure on Strouhal Number, the effect of pressure variation on density was taken into account in computing Reynolds Number. Temperature was held constant at 80°F. Equipment was calibrated and extraneous effects including cylinder blockage and free-stream velocity profile were calculated. The data were not extensive but did indicate some interesting and temporarily unexplainable trends. These data, along with other data showing similar temperature effects,[1] indicate that the Strouhal Number is not solely a function of Reynolds Number, as previously speculated.

### Reference


1 Clifford, William D., *Effect of Air Temperature on Vortex Shedding Frequency of Cylinders*, Journal of the Aeronautical Sciences, Vol. 24, No. 11, p. 852, November, 1957.


## A Generalized Porous-Wall "Couette-Type" Flow


Kenneth R. Cramer
*Aeronautical Research Engineer, Aeronautical Research Laboratory, WADC, ARDC, Wright-Patterson Air Force Base, Ohio*
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Recently, it was observed that the two existing boundary-layer texts (references 1 and 2) did not contain a solution for the case of Couette flow with a constant, uniformly distributed suction or blowing. Thus, the following analysis considers a "Couette-type" flow between a stationary flat surface and a slightly inclined flat plate moving at a constant velocity. In addition, the flow is subjected to a constant, uniformly distributed suction or blowing at the fixed surface. For this case, the boundary-layer Eqs. (1) and (2) are easily reduced to a simple ordinary differential equation in the following manner:

$$u(\partial u/\partial x) + v(\partial u/\partial y) = U(dU/dx) + \nu(\partial^2 u/\partial y^2) \quad (1)$$

$$(\partial u/\partial x) + (\partial v/\partial y) = 0 \quad (2)$$

Boundary conditions:

$$y = a: \quad u = U$$
$$y = 0: \quad u = 0; \quad v = \text{const.} = b$$

where $b$ is negative for suction and positive for blowing. Letting $\bar{u} = u/U$; $\bar{v}$ = the constant wall value $= b/U$