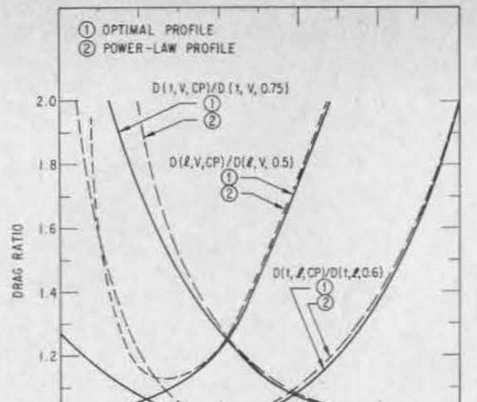

Fig. 3 Drag penalty for adding a constraint on center-of-pressure location.

power law. This is the known solution[2] when length and diameter (but not CP) are given. With a three-quarter-power body as an initial design, the center of pressure can be moved aft more than 2% before the optimal profile becomes bell-shaped.

Direct integration of the profiles (3a) and (3b) in (1) provides $C_D(CP)$ and a measure of the penalty in added drag paid by adding a constraint on CP. When CP is not specified, the optimal profile is characterized by a center of pressure at 0.6, where $C_D(l/t)^2 = \frac{27}{64}$. The penalty in drag D for specifying a CP other than 0.6 is the ratio

$$\frac{D(CP)}{D(0.6)} = \frac{4}{27}\left(\frac{1-z}{z}\right)^2\left[B_z\left(\frac{2}{3}, -\frac{1}{3}\right)\right]^3 \times \left[1 + \frac{3}{2}\,\frac{z^{2/3}(1-z)^{-4/3}}{B_z(\frac{2}{3}, -\frac{1}{3})}\right] \qquad 0.6 \leq CP < 0.75 \quad (5a)$$

or

$$\frac{D(CP)}{D(0.6)} = \frac{4}{27}\left(\frac{1}{z}\right)^2\left[B_z\left(\frac{2}{3}, \frac{2}{3}\right)\right]^3 \times \left[1 + \frac{3}{2}\,\frac{z^{2/3}(1-z)^{2/3}}{B_z(\frac{2}{3}, \frac{2}{3})}\right] \qquad 0.6 > CP \geq 0.5 \quad (5b)$$

The penalties are plotted in Fig. 3. The penalty for using a power-law profile for each CP

$$\frac{D(CP)}{D(0.6)} = \frac{4}{27}\,\frac{CP^3}{(1-CP)^2(2CP-1)}$$

is also plotted. For any CP, this is a larger penalty than (5). The added penalty is relatively small in the range $0.622 < CP < 0.75$, where the optimal profiles are inflected, but is unbounded as $CP \rightarrow 0.5^+$. This is contrary to what might be expected, since power-law profiles are not themselves inflected.

The same profiles given previously for the case of given CP, t, and l also apply when we are either given CP, t, and volume V or given CP, l, and V. The reason is that CP, t, l, and V are related for any slender body through

$$V[4/(\pi t^2 l)] = 1 - CP \qquad (6)$$

If any three are given, the fourth follows immediately. In the limit $CP \rightarrow 0.75^-$, the upper curve of Fig. 2 yields an optimal profile $\eta = \xi^{3/2}$ which is the optimal solution when t, and V (but not CP) are given. When $CP = 0.5$, the lower curve of Fig. 2 yields $\xi = B_{\eta^2}(\frac{2}{3}, \frac{2}{3})/B_1(\frac{2}{3}, \frac{2}{3})$, which can be expressed in terms of incomplete elliptic integrals, and which is the optimal solution when l and V (but not CP) are given. In these two latter cases, it can be shown by application of the transversality condition[1] that (3) minimizes the drag as well as $C_D(l/t)^2$. Figure 3 gives the drag penalty for adding a constraint on CP to existing constraints on any two of t, l, and V. The three curves shown for the optimal profiles have different shapes because the drag varies as (5) times t^4/l^2. When l and V are given, t^4 depends on CP through (6), as does l^2 when t and V are given. For reference, $C_D(l/t)^2$ is $\frac{27}{32}$ or $[\Gamma(\frac{2}{3})]^2/(16[\Gamma(\frac{4}{3})]^3)$ when only t and V or l and V are given, respectively. $\Gamma(x)$ is the gamma function.

Cases where surface area is given are not considered in this note, because these cases do not lead to universal curves such as those in Fig. 2. For the present constraint cases, when a CP less than 0.5 is specified, the optimal profiles have noses that are too blunt to be treated within the slender-body approximation. When $0.5 \leq CP < 0.622$, the optimal profiles are blunt and not flared. For $0.622 < CP < 0.75$ the optimal profiles are inflected, and for $CP > 0.75$ the optimal profiles are believed to have needle noses of zero thickness in order to achieve the specified length.

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Effect of a Shoulder Modification on Turbulent Supersonic Base Flow

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It has been observed experimentally by Hama[1] and discussed theoretically by Weinbaum[2] that effects of the fast expansion and consequent lip shock at the shoulder of a supersonic base or downstream-facing step can be quite appreciable at high Mach number. Hama found that the lip shock can be much stronger than has been assumed. He also drew attention to characteristic humps or peaks in the pressure distribution on the reattachment surface; these, he showed, could be attributed to secondary waves directed toward the surface from the point of interaction of the lip shock with the main recompression shock. Scherberg and Smith[3] have also drawn attention to the possible strong effects connected with a lip shock. In this note, we report some further observations of the occurrence of this phenomenon, and its elimination by a small modification at the shoulder to alleviate the fast expansion there.

The experiments were carried out on the same axisymmetric body and for the same conditions described in Ref. 4, where dimensions, nomenclature, and measurements on the basic (square shoulder) body may be found. The modification consisted of boat-tailing the shoulder, as shown in Fig. 1. This was accomplished by fitting a ring, contoured to the

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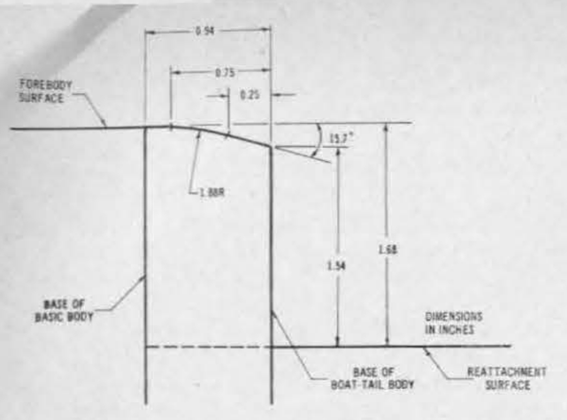

Fig. 1 Model modification.

boat-tail section, to the basic body. The slight change in flow conditions resulting from the 0.94-in. downstream displacement of the base is not considered to be significant. In all cases, the boundary layer approaching the shoulder was turbulent, transition having occurred well ahead of the shoulder.

The boat-tail angle ($\beta = 15.7°$) was chosen to be approximately equal to the Prandtl-Meyer deflection angle α through which the flow was expected to expand (differing by only a few degrees for the various Mach numbers). The length of the boat-tailed portion, 0.75 in., designed to allow the flow to expand more gradually than at a square shoulder, was equal to 4–6 boundary-layer thicknesses.

The surface pressure distributions downstream of the base with and without boat-tail are compared in Fig. 2 at four values of Mach number M_s. For both cases, h is the step height measured in the base plane and x is the axial coordinate measured from the base plane.

In Table 1 are listed the following values: p_b is the base pressure for the boat-tailed body, p_s is the pressure, and M_s the Mach number at the beginning of the boat-tail; α is the flow direction of the free shear layer toward the body (calculated from p_b); $\alpha - \beta$ is the difference between the base flow direction and the boat-tail angle (that is, the residual expansion angle at the shoulder); p_{b_0} is the base pressure on the basic body without boat-tail[4]; δ is the inclination of the lip shock toward the surface for the case without boat-tail, determined from schlieren pictures.

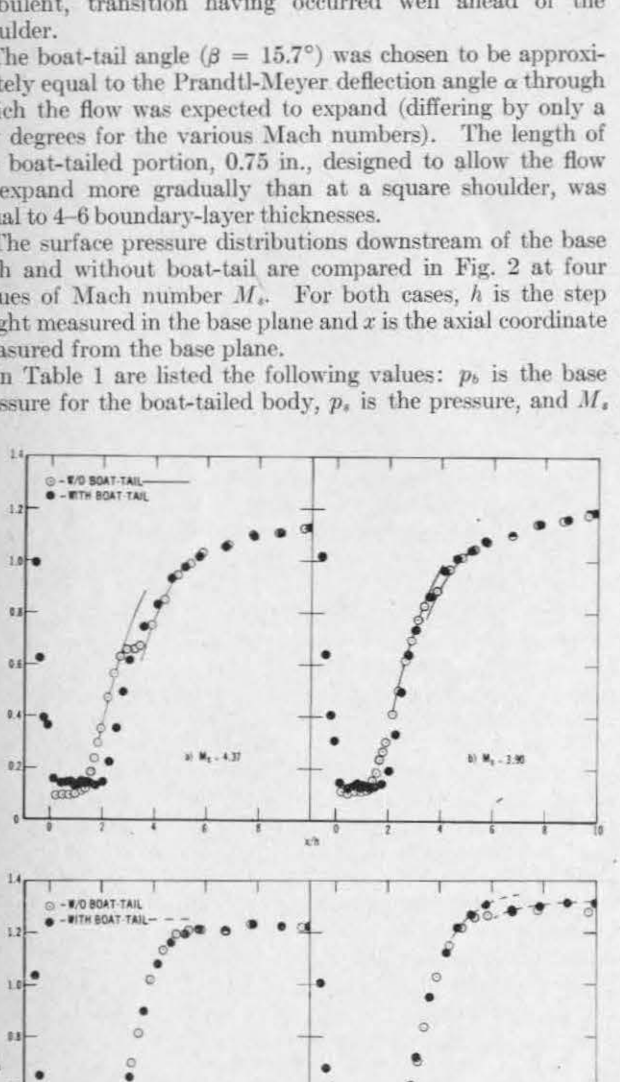

Fig. 2 Effect of shoulder modification on shapes of measured pressure distributions.

The largest effect occurs at the highest Mach number, $M_s = 4.37$, at which there is a local hump at about $x/h = 3$ in the pressure distribution for the square shoulder (Fig. 2a). With the boat-tail, this hump is completely eliminated and the pressure rises smoothly; furthermore, the base pressure is increased by 39%. At the next lower Mach number, $M_s = 3.90$, there is a smaller hump in the base pressure distribution at about $x/h = 4$ (Fig. 2b); this is eliminated by the boat-tail and the base pressure is increased by 12%. (Lines have been faired to the measured pressure distributions to better display the hump or "discontinuity" that occurs.)

We believe that these humps are associated with the lip shock-recompression shock (LSRS) interaction phenomena described by Hama. At these two higher Mach numbers (4.37 and 3.90), the lip shock is inclined toward the body (see Table 1) and is probably embedded in the free shear layer. Thus, the pressure perturbation from the LSRS interaction appears in the region of reattachment pressure rise downstream of the reattachment point.

At the two lower Mach numbers (3.02 and 2.09), the lip shock is inclined so slightly toward the body, or even away from it (at $M_s = 2.09$), that any waves from the LSRS interaction would reach the body well downstream of the reattachment region. Furthermore, as shown by Hama,[5] the magnitude of the perturbation decreases with increasing base pressure ratio. Therefore, the absence of pressure disturbances in the data for $M_s = 3.02$ (Fig. 2c) and the corresponding lack of boat-tail effect are not surprising.

On the other hand, at $M_s = 2.09$, boat-tailing does change the base pressure slightly and appears to produce rather than remove a hump in the pressure distribution (at $x/h \doteq 6$, Fig. 2d). We are at a loss to explain these effects, but feel certain that they are not LSRS phenomena, which, at this Mach number, would occur much further downstream. One possibility for producing a perturbation at this location is a secondary wave associated with the formation of the compression shock from the compression wavelets in the reattachment region.

Another noteworthy feature of Fig. 2d is the overexpansion of the flow at the end of the boat-tailed surface (c. f., the point corresponding to the lowest pressure). There must be a recompression (lip) shock at the shoulder to bring the pressure back to the base value.

We summarize our observations with the following general remarks about lip shock and related phenomena.

Fast expansion of supersonic flow over the shoulder of a base can produce the following effects:

1) Distortion of the shear layer profile.[6]

2) A lip shock may be formed in the free shear layer some distance downstream of the shoulder, from the interaction of the expansion wave with the vortical shear layer, along the lines described by Weiss and Weinbaum.[7]

3) A lip separation shock may result from the overexpansion of the flow over the shoulder and subsequent

Table 1 Experimental values of base flow quantities

M_s	p_b/p_s	α, deg	$\alpha - \beta$, deg	δ, deg	p_b/p_{b_0}
2.09	0.440	12.9	−2.8	−9	1.06
3.02	0.259	15.2	−0.5	2	1.03
3.90	0.136	17.2	1.5	12	1.12
4.37	0.154	14.6	−1.1	...	1.39

separation off the base, as observed by Hama.[1] The interrelation and contribution of each of these phenomena to the single lip shock usually observed is not clear.

Concerning effects on base pressure and reattachment pressure distribution, the following observations may be made:

1) For a weak but fast expansion (i.e., for expansion over a shoulder at low supersonic Mach number), the lip shock is weak and so is its effect on base pressure. The surface pressure distribution downstream of the reattachment region may have perturbations from the LSRS phenomena. The most important effect of the fast expansion may be shear layer distortion and consequent effects at reattachment.

2) With increasing Mach number the lip shock becomes stronger; its effect on base pressure becomes particularly important when it is inclined toward the surface so far as to be embedded in the shear layer. The LSRS phenomena then have a direct effect in the reattachment region, in addition to other possible effects such as shear layer distortion. For these cases, significant changes in the near-wake flow and base pressure can be produced by modifications of the shoulder.

3) If the near-wake flow is in a transitional range of Reynolds number, distortion of the initial shear layer profile can affect the transition process and so the base pressure.

If the surface ahead of the base is boat-tailed smoothly to alleviate the fast expansion and its distortive effects on the boundary layer, the following situations may arise:

1) If the boat-tail angle β is smaller than the Prandtl-Meyer expansion angle needed to reach base pressure, then an expansion will still be required at the shoulder, and the previously described phenomena will occur, but in weakened form.

2) If β is just equal to the expansion angle needed to reach base pressure, i.e., if the base pressure is equal to the boundary-layer pressure ahead of the shoulder, then the flow will separate smoothly at the shoulder, without a shock, and the boundary-layer profile will suffer the least distortion.

3) If β is larger than required for matching, the flow will overexpand on the boat-tail surface to pressures below base pressure. It then will separate, either at the shoulder through a lip separation shock, or, if β is too large for that, from the surface ahead of the shoulder through a surface separation shock.[8]

Our remarks are for a step-base, but the same general phenomena should occur for free base flows.

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On the Mechanism of the Decomposition of Ammonium Perchlorate

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DESPITE technological interest in ammonium perchlorate because of its extensive use in solid-propellant rockets, the mechanism of the decomposition is not well understood at the present time. The intention of this note is brief comment, in the light of recent new experimental data, on the various theories of decomposition that have appeared in the literature, and the presentation of some new proposals regarding the mechanism which are more in accord with experimental fact.

It was first reported by Bircumshaw and Newman[1,2] that below about 300°C ammonium perchlorate (AP) decomposes only to a limited extent (~30%). The residue from this low-temperature decomposition is AP chemically identical with the starting material. If the temperature is raised, sublimation of the residue occurs at an increasing rate, but if the ambient pressure of inert gas is increased, further chemical decomposition occurs instead of sublimation. The principal nitrogen-containing products are reported to be N_2O[1] and HNO_3[3] at low-temperatures and NO[1] at high temperatures.

Bircumshaw and Newman[1,2] discussed three mechanisms for the decomposition: 1) electron transfer, 2) proton transfer, and 3) breakdown of the anion, ascribing the low-temperature reaction to 1) the high-temperature reaction to 3) and sublimation to 2).

Apart from differences in the temperature range and the products the kinetics of the low- and high-temperature reactions[4,5] are very different, the activation energies for the low-temperature[4] and high-temperature[5] processes being ~30 and ~39 kcal/mole, respectively (for compressed pellets of AP). Because the activation energy for the sublimation process had been reported to be ~21 kcal/mole,[6] Galwey and Jacobs considered that three different mechanisms must be operative. Because the low-temperature reaction is catalyzed[1,7] by ions such as Mn^{4+}, which can change their valency rather readily, they followed Bircumshaw and Newman in ascribing the low-temperature process to electron transfer. They differed, however, in considering that the high-temperature reaction involved proton transfer followed by rapid decomposition of $HClO_4$ in the gas phase and the oxidation of NH_3 by radicals (mainly O atoms) produced by this decomposition. At this time there was no quantitative information on the stability of $HClO_4$ in the gas phase. Galwey and Jacobs assumed that it would be very unstable and thus made a tentative suggestion that sublimation might involve an NH_4ClO_4 "molecule" or ion pair. It is now known that this suggestion was not soundly based. Levy's work[8] on the gas-phase thermal decomposition of $HClO_4$ has provided quantitative information on the stability of this molecule, showing that it is perfectly possible for AP to sublime at low ambient pressures as free $HClO_4$ and NH_3. Furthermore, infrared[9] and mass spectrometric[10] investigations have failed to reveal any evidence for a molecular NH_4ClO_4 species.

Recent kinetic data[11] (details of which will be published in a more extended form) have shown that when the kinetics of decomposition and sublimation are all measured by weight

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