THE USES OF COHERENT STRUCTURE

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Abstract

The concept of coherent structure in turbulent flow is a revolutionary idea which is being developed by evolutionary means. The main objective of this review is to list some solid achievements, showing what can be done by using the concept of coherent structure that cannot be done without it. The nature of structure is described in terms of some related concepts, including celerity, topology, and the phenomenon of coalescence and splitting of structure. The main emphasis is on the mixing layer, as the one flow whose structure is well enough understood so that technical applications are now being made in problems of mixing and chemistry. An attempt is made to identify some conceptual and experimental obstacles that stand in the way of progress in other technically important flows, particularly the turbulent boundary layer. A few comments are included about the role of structure in numerical simulations and in current work on manipulation and control of turbulent flow. Some recent developments are cited which suggest that the time is nearly right for corresponding advances to occur in turbulence modeling.

1. Introduction

In this paper I propose to point out some strengths and weaknesses of the concept of coherent structure in turbulent flow, list some achievements, and suggest what I believe are some useful lines of attack for the future. My first task, however, is to justify yet another review of the subject. In 1973, at a small Industrial Associates conference at Caltech, I noted that the new and revolutionary concept of coherent structure was encompassed by perhaps 30 to 40 papers, mostly published in the Journal of Fluid Mechanics. Today the number of papers is probably close to a thousand, and review articles have become the only practical avenue of approach for newcomers to the subject. In the nine years since Roshko's Dryden lecture[1] in 1976, various topics in coherent structure have been reviewed by Kovasznay[2], Falco[3], Lumley[4], Saffman[5,6], Cantwell[7], Coles[8], Hussain[9,10], Antonia[11], Laufer[12], Ho and Huerre[13], Rogallo and Moin[14], and others. There have also been a number of specialists' meetings. Prominent examples since 1976 are the workshops at Lehigh[15], Michigan State[16], and Madrid[17]. The frequency of such meetings is decreasing, perhaps because of surfeit (in some cases, the same material has been published several times), and perhaps because lines of polarization in the research community have by now become so well defined that they no longer need to be explored.

Among the reviews just mentioned, the paper by Cantwell stands out as a comprehensive and even-handed account of the experimental evidence. The other papers tend to emphasize some special problem or some special point of view. My present review is no exception. It is certainly neither comprehensive nor even-handed. It is best read as a Caltech white paper, presenting mostly my own point of view and using mostly my own experience or that of my colleagues. I believe that my subject is best painted with a broad brush; there is no room in this short paper for details that do not help to define the nature of coherent structure, however valuable these details may otherwise be. The flow which receives by far the most attention is therefore the mixing layer, because this is the flow whose mechanisms are best understood. For several other flows of great technical interest, particularly the boundary layer, many details have been documented, but the main mechanisms are still obscure, and I do not have very much to say. Lack of time and space also accounts for my failure to do justice to two important topics, numerical simulation of turbulent flow, and manipulation and control of turbulent flow. Both are active research areas, and each deserves a survey of its own.

Revolutions past and present. At the 1973 Caltech conference I also drew a parallel with a previous revolution in turbulence which took place during the period 1925-1933, when the mixing-length hypothesis was introduced. Before this event, the art of turbulence had seldom risen above the primitive level of log-log graph paper and power laws. It is not a great oversimplification to say that there were only two principal contributors to the revolution, Prandtl and Karman. There was one crucial experiment, the smooth-pipe profile measurements by Nikuradse, and there was one crucial result, the logarithmic law for the mean-velocity profile near a wall. For a full appreciation of these developments from the point of view of one of the participants, I recommend reading pages 134-140 of Karman's biography[18] along with the technical literature[19]. After 1935, the situation remained stable for several decades, during the period of exploitation that typically follows the introduction and acceptance of a new concept in science. In fact, at the first Stanford contest in 1968 I complained[20] that most of the computation methods presented there would not have been out of place, given adequate computing resources, at a similar conference in 1935. The main exception was the contribution by Bradshaw, which itself had the seminal effect of making fashionable the use of rate equations, or transport equations, for various important turbulence quantities.

Practically speaking, I believe that the element that made the first revolution viable was the evolution of the wind tunnel, or more generally the evolution of flow-management techniques which enlarged the class of classical shear flows by adding the boundary layer, the mixing layer, and the wake to the pipe flow, the channel flow, the jet, and the plume. A foundation was thus laid for the development of the similarity laws that today provide a workable synthesis of experience with the classical flows, and with their generalizations to include effects of surface roughness, heat transfer, mass transfer, and compressibility.

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The element that is making the second revolution possible is the digital computer, or more generally the whole of digital instrumentation. In my opinion, access to information about velocity fluctuations through the hot-wire anemometer, or later the laser-Doppler velocimeter, has not by itself led to substantial advances in understanding of turbulence. Neither has the analytical machinery of Wiener's generalized harmonic analysis, or the associated experimental machinery of analog correlators and spectral analyzers, or the application of all this machinery to the arch-problem of homogeneous isotropic turbulence. During the period 1940-1970, in particular, all of the classical turbulent shear flows were re-examined and re-documented with the aid of hot-wire instrumentation. The results, in the form of power spectra, intermittency profiles, space and space-time correlations, detailed energy balances, and so on, are a monument to the dedication of the experimental community. With few exceptions, however, these efforts were not productive of deep insights into the nature of turbulence. One of Liepmann's occasional aphorisms, not entirely facetious, is that the invention of Reynolds averaging may have delayed the development of concepts in turbulence in much the same way that the invention of the vacuum tube delayed the development of the transistor. The revolutionary contribution of digital instrumentation is that it preserves phase information, which is the key to any study of structure, and thereby transforms the operation of averaging. Another promising application, quantitative use of flow visualization through computer-aided image analysis, is still in an early stage of development.

Coherent structure. It is past time to define what I mean by coherent structure. This particular term seems to have been first used publicly in the title of a conference organized by Davies and Yule in 1974 at Southampton[21]. In the present context the word "coherent" means recognizable, as in "coherent speech". Roshko prefers the term "organized structure", which avoids the ambiguity associated with another meaning of the word "coherent" in describing wave phenomena. In any event, the term "coherent structure" is now generally accepted as meaning a recognizable concentration or accumulation of vorticity at the largest lateral scale of a turbulent shear flow. Operationally, I define coherent structure as the pattern that emerges when an ensemble average is constructed in coordinates moving in such a way as to best preserve the phase of the organized motion with respect to some definable origin in time and space. The velocity with which the coordinates must move is often called convection velocity and occasionally phase velocity. Because the latter term involves the ambiguity just mentioned, I prefer to use the term "celerity" which was originally introduced by Favre and co-workers in their work on space-time correlations in boundary layers[22].

The concept of coherent structure is illustrated by Figure 1, which is a famous photograph taken in the sublayer of a boundary layer by Kline and co-workers at Stanford[23]. The stated purpose of the research was to use flow visualization to learn more about the various mechanisms that operate in the three main regions of the boundary layer--the intermittent wake region, the fully turbulent logarithmic region, and especially the viscous sublayer, where much of the turbulence production is known to be concentrated. The ubiquitous presence of streamwise vortices in the sublayer was not suspected before this research was undertaken. These vortices evidently satisfy my operational definition of coherent structure, although the scale is the scale of the viscous sublayer rather than of the full boundary layer.

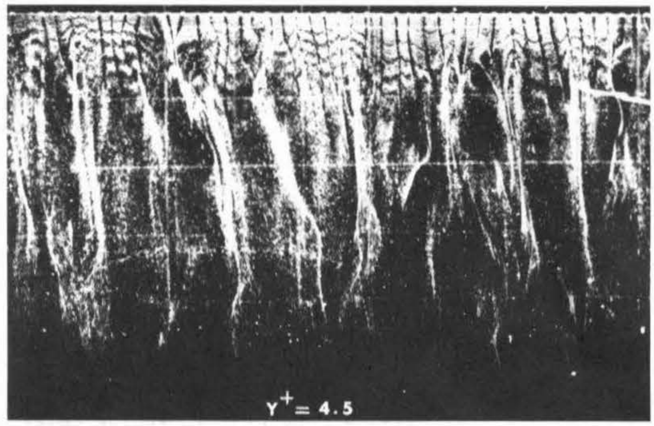

Figure 1. Coherent structure in the sublayer of a turbulent boundary layer. Photograph provided by S.J. Kline, Stanford University.

Figure 2 is an equally famous photograph taken in the Brown-Roshko shear-layer apparatus at Caltech[24]. The research in question was not designed or intended to produce this photograph. The original objective was to study the quite different behavior of mixing layers (a) when the speed is low and the two streams differ in density because the two gases are different, and (b) when the speed is high and the density difference is associated with compressibility. This objective was preempted (although Roshko has recently returned to it) when the existence and persistence of turbulent coherent structure was discovered. There was considerable skepticism at first about this discovery. However, the skepticism is now mostly quieted, and research on the problem of coherent structure in this and other flows is being pursued vigorously in many directions.

2. The Nature of Structure

I mentioned that there are perhaps a thousand papers concerned with one or another aspect of coherent structure. A superficial census shows that the flow represented most heavily is the boundary layer. Next is the mixing layer. Then come

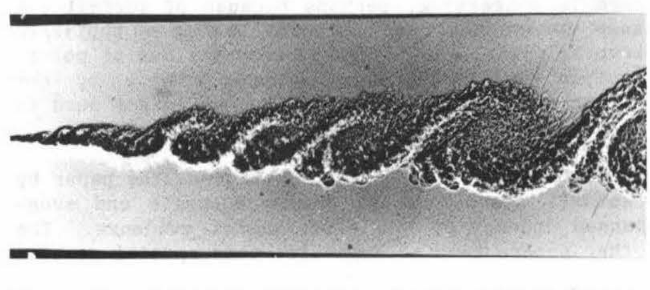

Figure 2. Coherent structure in the mixing layer. Photograph provided by A. Roshko, California Institute of Technology.

the pipe and channel, and finally the far jet and far wake. Plumes and wall jets have not received much attention.

Vorticity. By any reasonable measure of progress in achieving new understanding, the mixing layer leads all the rest, for reasons that are not hard to discover. At least for flow at high Reynolds numbers, coherent structure involves large-scale vorticity in circumstances where non-linearity is at least as important as diffusion. It should then be a good approximation to think of the bulk motion in terms of the traditional vortex laws for inviscid rotational flow. Thus vorticity is carried with the fluid. Vortex lines or tubes do not end in the fluid, but close on themselves, or go to infinity, or end at walls. In any morphology of coherent structure, therefore, an important distinction arises between flows containing large-scale mean vorticity of only one sense and flows containing large-scale mean vorticity of both senses. The mixing layer is unique among the classical plane flows (if it is supposed that the boundary layer includes its image in the wall) in that it is the only flow that is driven naturally toward a two-dimensional structure. It is primarily this property that makes the mixing layer experimentally transparent and accounts for the substantial progress that has been made with this flow.

Celerity and topology. Implicit in this characterization is the idea that a coherent structure moves as a unit, preserving its geometry, while the ambient flow accommodates itself to the kinematic and dynamic demands of the structure. Several isolated structures--the spiral turbulence, the puff in a pipe, the spot in a boundary layer, the vortex ring--are unequivocal examples of such behavior. It is also clear that a knowledge of celerity is a prerequisite to any study of structure, because particle paths and instantaneous streamlines are very sensitive to the velocity of the observer. My own experience has been that celerity is best determined by tracking peaks in ensemble-mean vorticity, or suitable contours of mean velocity or intermittency. Space-time correlations can be very deceptive. Such correlations uniformly indicate that the celerity is not constant across the lateral extent of a given flow, but is biased in the direction of the mean-velocity profile. It is known that the bias can be significantly reduced by removing the high-frequency part of the signals[22]. However, I know of no experiment in which the passbands for two probes have been systematically adjusted to maximize the envelope of the correlation. The problem seems made to order for computer-aided instrumentation based on an array processor. It is regrettable that data are not available to allow a direct test in a variety of flows of Roshko's conjecture[1] that the correlation envelope is a measure of life expectancy for coherent structure.

In any single realization of a turbulent flow, the large structure is often obscured by motions of smaller scale. The structure emerges only after averaging at constant phase over an ensemble, according to the operational definition given in the Introduction. The language which emerges is the language of topology. My own opinion, which I concede is not widely shared, is that topology is a crucially important element of the new turbulence. In fact, topology is the central theme in my survey paper of 1981[8], where I assembled some ideas developed in discussions with colleagues at Caltech over several years. The two cartoons shown in Figure 3 are taken from this paper. They are intended to suggest the topology of the mixing layer and the vortex street, both portrayed as growing in time rather than space. I propose to interpret the flow as steady and the patterns as mean particle paths, ignoring the fact that the mean flow cannot be both unsteady and steady at the same time. Despite these defects, cartoons like the ones in Figure 3 have a number of important uses.

Turbulence production. It is known experimentally that a dominant site for turbulence production (the ensemble-averaged scalar product $P = -\langle u_i'u_j'\rangle\, \partial\langle u_i\rangle/\partial x_j$) is at saddle points in the moving flow patterns of Figure 3. This property was established for the mixing layer by Hussain[9] and independently for the near vortex street behind a cylinder by Cantwell and Coles[25]. Data from the latter paper, expressed in coordinates aligned with the wind tunnel, give the peak in turbulence production at one particular saddle as

$$P = -\begin{pmatrix} 0.060 & -0.034 \\ -0.034 & 0.077 \end{pmatrix} \cdot \begin{pmatrix} 0.028 & 0.330 \\ 0.570 & -0.044 \end{pmatrix} = 0.032$$

where all quantities are made dimensionless with the cylinder diameter and the free-stream velocity. The mean continuity equation is not satisfied by the data; the discrepancy $\partial\langle u\rangle/\partial x + \partial\langle v\rangle/\partial y = -0.016$ probably provides a rough gauge of experimental accuracy. In these coordinates, the numerical result for P is dominated by the product $-\langle u'v'\rangle(\partial\langle u\rangle/\partial y + \partial\langle v\rangle/\partial x)$. However, it does not follow that the measured large turbulence production is associated with shearing deformation near the saddle. First, note that the measured mean vorticity $\partial\langle v\rangle/\partial x - \partial\langle u\rangle/\partial y$ at the saddle is 0.24. For heuristic purposes, I propose to neglect this vorticity compared to the measured peak mean vorticity of 1.99 at the nearest vortex. If it is stipulated that the vorticity at the saddle is zero, the separatrices intersect at right angles and coincide locally with the principal axes of the strain tensor. In these principal coordinates, P becomes

$$P = -\begin{pmatrix} 0.034 & 0.006 \\ 0.006 & 0.103 \end{pmatrix} \cdot \begin{pmatrix} 0.444 & -0.120 \\ 0.120 & -0.460 \end{pmatrix} = 0.032$$

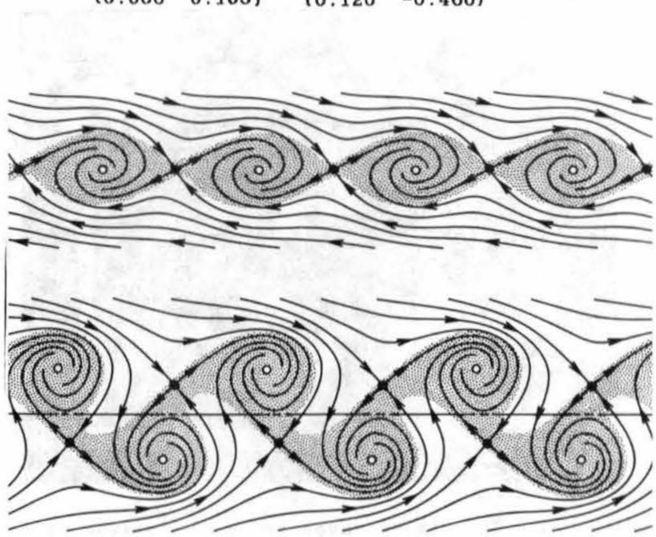

Figure 3. Topological cartoons of the vortex street and the mixing layer, from Coles[8].

The difference between these two calculations is important, because two quite different conclusions can be drawn from the same data, depending on the point of view. The clear message of the second calculation is that turbulence production in such flows can be associated with normal stresses, and specifically with vortex stretching along the diverging separatrix at the saddle. The new turbulence is continuously transported to the adjacent centers, where it accounts for the observed high turbulence level.

Several traditional rules of turbulence are threatened by the last two statements. They imply that turbulent energy is not necessarily produced in the streamwise component and transferred to the other components by the action of a pressure-strain mechanism. Turbulent energy is not necessarily produced at the largest lateral scale of a flow and delivered to smaller scales by a cascade mechanism. In short, real turbulent shear flow has little in common with the flow described by the Reynolds-averaged equations together with the boundary-layer approximation and the concept of gradient diffusion. There are, of course, new difficulties with the new view, and with the broad-brush technique that is being used throughout this paper. For example, an appreciable fraction of the turbulence production in both flows occurs in the vortices

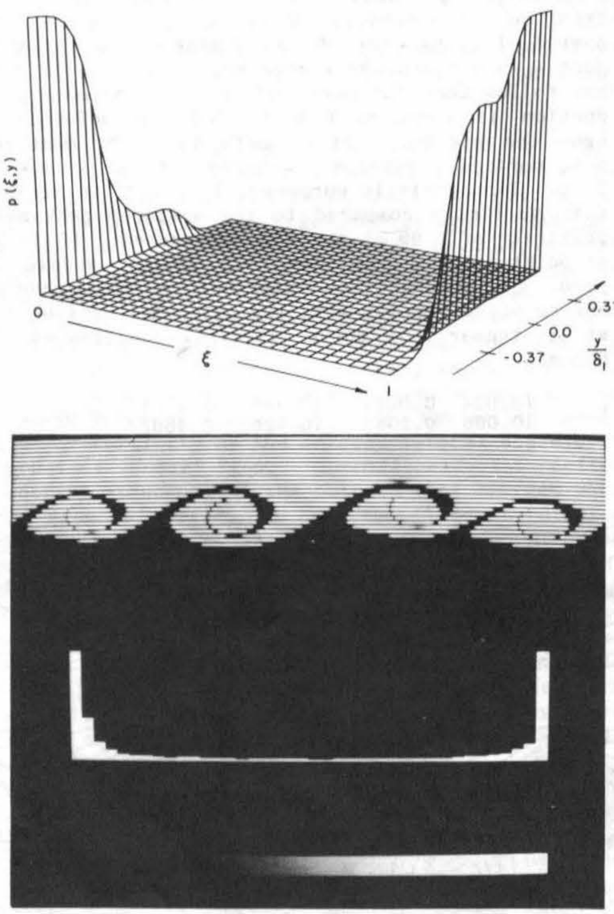

Figure 4. The mixing layer below the mixing transition, with the measured probability density for concentration, from Koochesfahani[30]. The concentration ξ is zero for pure low-speed fluid and unity for pure high-speed fluid.

themselves. Moreover, the vorticity being tilted and stretched in the bridges must be continuously replenished, to prevent the pattern from breaking up into isolated islands of turbulence. Essential elements of structure, at least in these two flows, therefore include a strain field that maintains a finite amount of spanwise vorticity in the bridges, and an instability mechanism resembling (say) the one proposed by Lin and Corcos[26].

Mixing. An important technical issue here is the mixing transition in the mixing layer. This transition was first described by Konrad[27] and later studied by Breidenthal[28], Bernal[29], Koochesfahani[30], and others. The mixing transition begins with the initial appearance of streamwise vorticity in the bridges. It is consummated by the appearance of three-dimensional small-scale motions throughout the large spanwise vortices, usually after one or more coalescence events. To illustrate the point, flows at Reynolds numbers below and above the mixing transition are shown in Figures 4 and 5. These images[30] are not photographs of the flow. They are photographs of a black-and-white monitor screen* on which is displayed a y-t diagram of light intensity along a line normal to the main flow. The high-speed fluid carries a fluorescent dye, a laser beam is used for excitation, and the light intensity along the beam is measured by a scanning photodiode array. The streamwise or time scale has not been adjusted to match the celerity, so that there is some distortion. Figures 4 and 5 also include the measured probability density for relative concentration, plotted as a function of position through the layer. The data leave no doubt that the flow in the large spanwise vortices is essentially unmixed when the flow is below the mixing transition, and quite well mixed above. In the latter case, the most probable composition for the mixed fluid is independent of position within the structures, and is biased toward a higher concentration of high-speed fluid. The latter property will be discussed again in the next section.

Measurements like these in the mixing layer are relatively straightforward (the graduate students involved might not agree). The various elements that need to be understood in order to formulate both the fluid mechanics and the chemistry of the mixing layer are gradually coming under control. Recent contributions include analytical work by Broadwell and Breidenthal[32] and by Effelsberg and Peters[33], as well as experimental studies of the $NO\text{-}O_3$ reaction by Masutani and Bowman[34] and the H-F reaction by Mungal and Dimotakis[35].

Instabilities. One substantial obstacle to progress in research on coherent structure, especially in the boundary layer, has been that an inviscid conceptual model of large-scale structure is difficult to reconcile with results of experiments carried out at low Reynolds numbers. Fortunately, the obstacle becomes less substantial if the structure appears in both a laminar and a tur-

* These and other images in spectacular false color can be found in (some copies of) the thesis by Koochesfahani. Similar false-color images of vortex streets can be found in (some copies of) the thesis by Roberts[31]. Such images are part of the experimental basis for my topological cartoons in Figure 3.

bulent version. Obvious examples are again the mixing layer and the vortex street. The laminar and turbulent versions of these two flows have in common that the primary source of structure is a two-dimensional instability of inflection-point type. Such an instability is essentially inviscid, with viscosity playing a stabilizing role by regulating the thickness of the basic velocity profile and thus limiting the range of unstable wave numbers. In the non-linear stages of the instability, a nearly stationary climax state can result from an approximate balance between transport and diffusion (the corresponding balance is exact in laminar circular Couette flow and Benard convection). The climax state is itself unstable, but in a different sense, in which excessive structural stability leads to slow growth, crowding, coalescence, and reappearance of structure at a larger scale. Within this description, the mixing transition that connects the laminar and turbulent states becomes almost incidental.

One further point should be made about the case of the vortex street at low Reynolds number. This flow was once thought to be subject to a pairing instability, and some authors still take this view[36]. However, some remarkable photographs by Cimbala[37], reproduced in Figure 6, tell a different story. A smoke-wire is used for flow visualization. Moving the location of the smoke wire progressively downstream reveals that the original vortices decay rapidly. However, the smoke concentrations are very persistent, and it is therefore easy to reach the wrong conclusion that the original street is still present (say) at x/d = 100. Actually, the street that is visible near x/d = 150 is an unrelated pattern, arising from the classical instability of the smooth, laminar wake profile after the original street has decayed. In the present context of coherent structure, however, the real message of these photographs is in the plan view, which shows that the second vortex street immediately develops a three-dimensional pattern. There is a strong impression that only one side of the wake is marked by smoke. Each vortex on that side of the street becomes wavy and inclined, with alternate vortices 180° out of

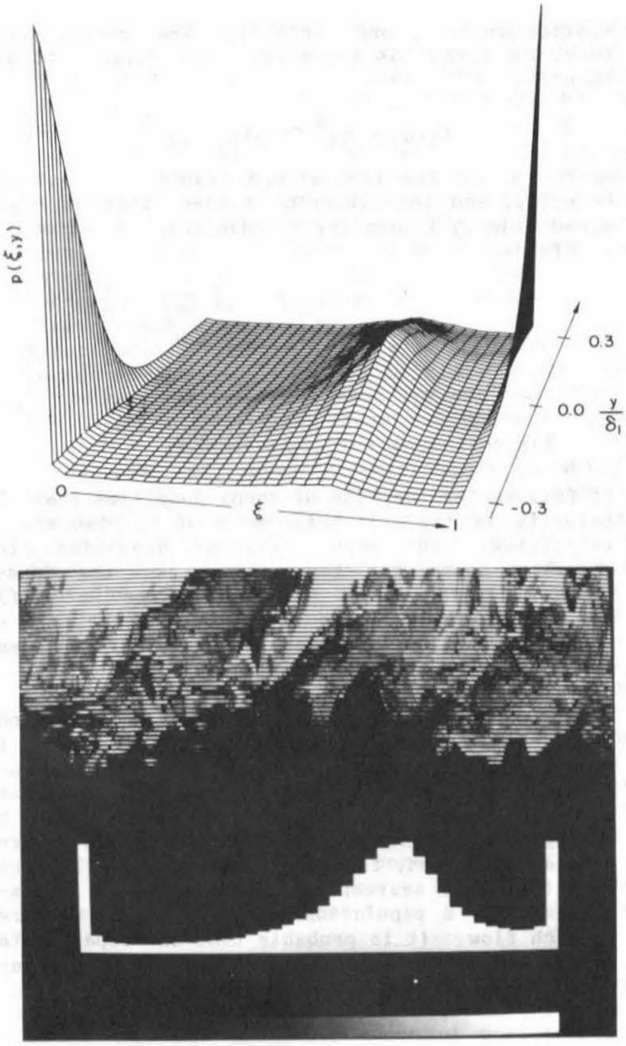

Figure 5. The mixing layer above the mixing transition, with the measured probability density for concentration, from Koochesfahani[30]. The concentration ξ is zero for pure low-speed fluid and unity for pure high-speed fluid.

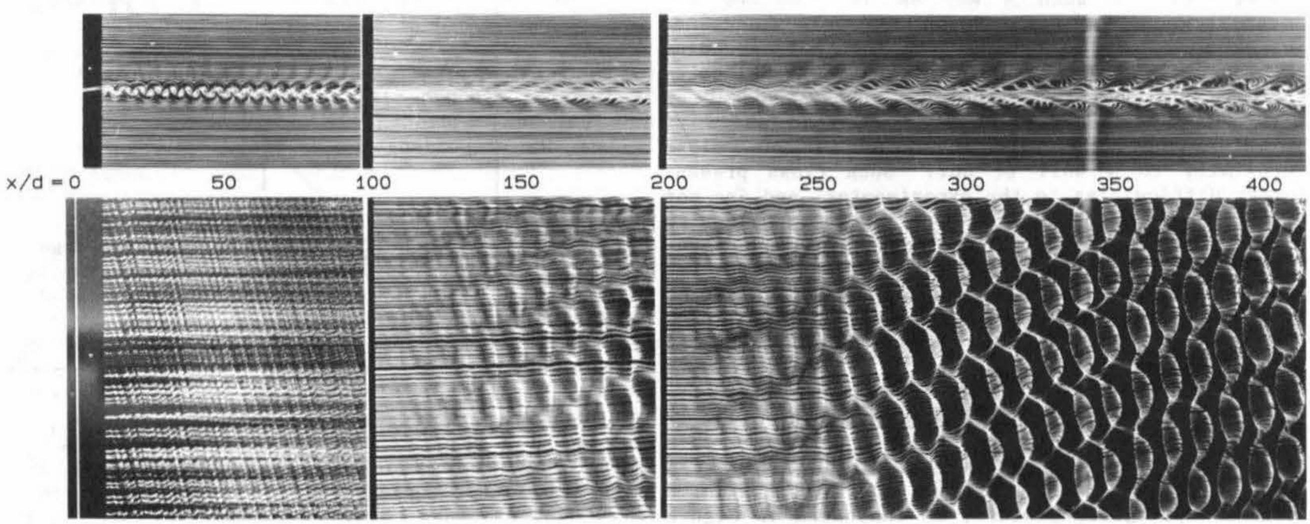

Figure 6. The first and second vortex streets at Re = 140. Photograph provided by J. Cimbala, Pennsylvania State University.

phase, and with some overlap in the plan view. One more downstream position for the smoke wire would have been useful, to guard against the possibility that the second street is also decaying. As matters stand, the picture is unclear, and further study of this flow should be classified as urgent unfinished business. One reason is that a roughly similar wavy structure (not including the phase shift) has recently been detected experimentally in the turbulent wake by Hussain (private communication). Another is that Savas[38] has independently proposed a topologically similar pattern for the turbulent boundary layer. Because I have a vested interest in any structural similarity which can be found between the wake and the boundary layer, I follow these developments closely.

From the point of view of morphology, appropriate weight should be given to the fact that flows near walls seem not to pass through a laminar climax state, but rather proceed directly to the turbulent one. For example, there is no stable laminar analog of the turbulent spot or the spiral turbulence. The laminar instability in such flows is also more subtle than the inflection-point instability discussed above, inasmuch as the role of viscosity is destabilizing. In any event, it is not obvious that coherent structure in such flows can be thought of as basically inviscid. Moreover, the basic turbulence-production mechanism may be associated with shearing strain, because slow lateral growth seems to imply that any coherent structure may be much flatter than the ones sketched in Figure 3. This point will be taken up again in the next section.

Mechanisms. At least for the two free turbulent flows discussed so far, there is no doubt that turbulence production is strongly coupled to the strain field associated with large-scale vorticity concentrations. The concentrated vortices, once formed, generate a strain field which guarantees that there is energetic turbulence production near saddles in the flow pattern. The newly turbulent fluid, while it is being transported to and incorporated into the centers, controls the viscous part of the entrainment; i.e., the local propagation velocity of various turbulent-nonturbulent interfaces, in such a way as to preserve the overall geometry of the turbulent regions and thus of the coherent structure. The corresponding mechanisms in other flows are greatly complicated by the fact that the structures probably occur as three-dimensional patterns and in some cases as flat or elongated structures, whether in close proximity to a wall or not. Such flows present great difficulties to the experimenter, and are not yet close to being reduced to a few relatively simple elements like those characterizing the shear layer. One reason may be that most experimenters, myself included, have a bad habit of distorting their structural diagrams by blowing up the lateral coordinate, thus losing contact with the real geometry and the real topology.

3. Entrainment and Coalescence

Celerity. Granted that the strain field of a train of structures produces saddle points in a steady topological pattern, as indicated in Figure 3, a practical conclusion can be drawn about celerity in the mixing layer at low Mach number. The two streams must meet at the saddle with the same stagnation pressure. They must have the same static pressure, and hence the same dynamic pressure, at large distances from the layer. Consequently,

$$\rho_1(u_1 - c)^2 = \rho_2(c - u_2)^2$$

where u is the free-stream velocity, c is the celerity, and the subscript 1 identifies the high-speed side in laboratory coordinates. Solution for c yields

$$\frac{2c}{u_1 + u_2} = 1 + \frac{\left(\frac{u_1}{u_2} - 1\right)\left(\sqrt{\frac{\rho_1}{\rho_2}} - 1\right)}{\left(\frac{u_1}{u_2} + 1\right)\left(\sqrt{\frac{\rho_1}{\rho_2}} + 1\right)}$$

It follows for the case of equal densities that the celerity is the arithmetic mean of the two stream velocities. For very different densities, the celerity approaches the velocity of the denser stream. Incidentally, there should be no difficulty in extending this derivation to the case of compressible flow, for which shock waves may appear (and be detectable experimentally).

Figure 7 shows the formula just derived, together with some values of celerity measured by Wang[39] for different values of velocity ratio and density ratio. The mixing layers in question happen to be curved, but the celerity seems not to be strongly dependent on the sense of the curvature, provided that body forces are not large. The error bars represent scatter in slopes measured from x-t diagrams for a population of individual structures in each flow. It is probable that the experimental celerities, being measured for the trailing interface only, are slightly understated.

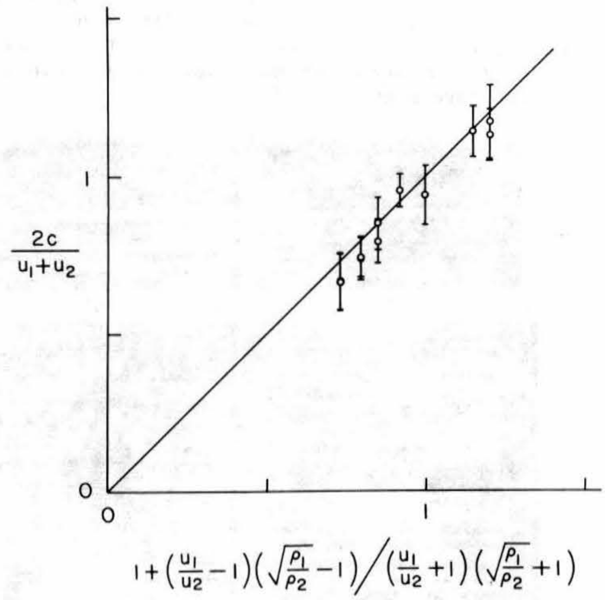

Figure 7. Celerity c of structure in mixing layers having different velocity ratios and density ratios, adapted from Wang[39]. The subscript 1 refers to the high-speed side.

Entrainment. Dimotakis[40] has independently derived the same formula for celerity as part of an effort to account for the unexpected experimental finding, first reported without fanfare by Konrad[27], that the volume entrainment into a shear layer with equal densities is about 30 percent higher on the high-speed side. The argument requires an estimate of two entrainment velocities normal to the layer, together with an associated area for each structure. Dimotakis takes each entrainment velocity to be proportional to the corresponding free-stream velocity in moving coordinates; i.e., to $(u_1 - c)$ or $(c - u_2)$. It is clear that the entrainment area for each vortex cannot be the saddle-to-saddle distance in Figure 3, because this is the same for the two sides of the layer. Dimotakis therefore appeals to the streamwise asymmetry of the real flow (cf. Figure 2) by using the center-to-center distance, which he measures downstream on the high-speed side and upstream on the low-speed side. The result is a quite plausible formula for layer growth in the general case. The argument recognizes implicitly that the entrainment is not uniform from saddle to saddle, but is small along the bridges and large in the downstream lee formed by the flow relative to each structure. This conspicuous non-uniformity in local entrainment has been noted by other authors, and has given rise to a gastronomical description of entrainment of free-stream fluid as "gulping". Dimotakis uses the term "induction" to refer to de facto entrainment of fluid that is irreversibly committed to entering the turbulent region, but has not yet done so. However, it is not clear how or where a line can be drawn between this fluid and fluid whose turn will come a little later. The issue of entrainment seems to be at least partly a subjective issue.

Coalescence. Coalescence is a conspicuous phenomenon in the free shear layer. There is no doubt that the proximate cause of coalescence is entrainment, which continuously increases the volume of fluid involved in the motion of a given structure. The scale in the streamwise direction is constrained by conservation of number, but the scale in the direction normal to the shear flow is not. Eventually the two scales must diverge sufficiently so that distortion in the strain field triggers a fundamental change in the geometry. The change provided by nature is coalescence. According to x-t diagrams of vortex trajectories reported by various investigators, this process is rapid at large Reynolds numbers. I therefore tend to think of coalescence events as punctuation in the text of turbulence.

To test this description of the mixing layer quantitatively, Hernan and Jimenez have carried out a computer-aided image analysis[41] of a high-speed shadowgraph movie taken by Bernal of a constant-density flow in the Brown-Roshko apparatus. Although the numerical values show considerable dispersion, the results suggest that an average vortex increases its area between pairings by about 77 percent, and that the combined area increases by a further 16 percent during pairing. The area balance is therefore satisfactory. Moreover, the x-location for successive pairings is found to double, on the average, when measured from the apparent origin defined by the growth of the mean flow. Allowable configurations for inviscid rotational models of the climax state have also been studied by Pierrehumbert and Widnall[42] and by Saffman and Szeto[43]. Although the numbers obtained for a hypothetical coalescence event are somewhat different from those quoted above, the analyses tend to reinforce the view that coalescence is essentially an inviscid phenomenon, requiring only growing vortices.

Control of coalescence. Two types of experiments have shown that control can be exerted over growth and coalescence of vortices in a shear layer. Kibens[44] applied a small perturbation at the natural frequency of the initial region and obtained the results for vortex spacing shown in Figure 8. The plotted data were inferred from the known celerity and from the position of extrema in the auto-correlation signal in time for the u-component of velocity. The boxed regions are regions of coalescence, and the spacing has a unique value only in the short intervening intervals. The upper part of the figure presents a proposed skeleton of the same flow. The staircase shows the streamwise vortex scale increasing discontinuously during coalescence events, and the straight line shows the lateral vortex scale (normal to the layer) increasing continuously. This approximation of continuous change in the lateral scale is consistent with numerical calculations, admittedly for laminar flow at low Reynolds numbers, by Corcos and Sherman[45]. As several authors have noted, Figure 8 suggests that an adequate model of the flow might require only one cycle of the evolution, this cycle then being repeated indefinitely with the appropriate increases in scale.

The second type of control involves a perturbation at a frequency small compared to the natural frequency of the initial region. The effect is to consolidate all of the structures which form within each cycle of the perturbation, after which coalescence is suppressed for a considerable distance

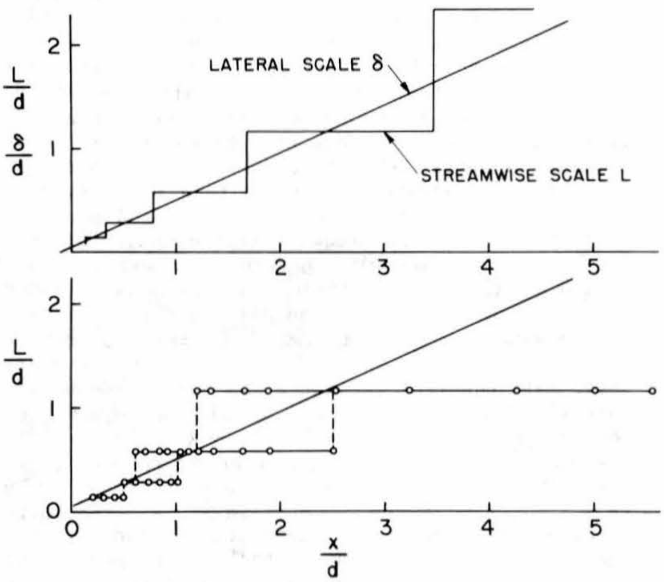

Figure 8. Change of scale in a controlled mixing layer during successive coalescence events, adapted from Kibens[44], with a proposed skeletal model. The parameter d is the nozzle diameter.

before it resumes. Experiments of this kind have been carried out by Ho and Huang[46], Oster and Wygnanski[47], and others. The flow in the plateau region is, to say the least, abnormal. The Reynolds shearing stress changes sign, so that the energy budget in Reynolds-averaged form indicates energy transfer from the turbulence to the mean flow. Entrainment is also greatly inhibited[31] The topology apparently shifts from that shown in Figure 3 to the corresponding closed pattern of a train of Kelvin cat's-eyes. Whatever else can be said about this flow, it is not an auspicious place to study the entrainment properties of a normal shear-layer structure. On the other hand, the fact that attempted control of structure can lead to a very unnatural flow is itself a remarkable and valuable result.

Morphology; linear growth. There is some prospect that the skeletal model of shear-layer development in the upper part of Figure 8 can be adapted to certain other flows. As a primitive universal model, suppose that coalescence when it occurs is instantaneous and volume preserving, that the streamwise scale doubles during coalescence, and that the lateral scale goes from being too large by a factor $\sqrt{2}$ (with respect to a mean ratio of scales) to being too small by a factor $1/\sqrt{2}$. Entrainment then resumes and the cycle repeats itself. The rate-controlling reaction is evidently the process of entrainment between pairings. Unfortunately, this process is experimentally almost inaccessible in an unforced general flow because of experimental limitations, uncertainty about the location of pairing events, three-dimensionality of the structure, variations in age and size, and the need to use unfamiliar Lagrangian techniques of data analysis.

Nevertheless, other flows than the mixing layer can be contemplated in these same terms, using in each case the standard similarity law for mean lateral growth in the Reynolds-averaged sense[7]. As a basic premise, suppose that the standard structure in each flow has three (or two) natural scales which tend to remain in a constant ratio to each other; i.e., the average streamwise and spanwise scales are constant multiples of the average lateral thickness of the layer. In a side view, the streamwise scale of coherent structure is assumed to double at each coalescence. The interval between coalescences should then be determined by growth of the mean lateral scale, and particularly by the distance required for doubling of this scale. For linearly growing flows ($\delta \sim x$), including the mixing layer, the plane and round jet, the radial jet, and the plane and round plume, the nominal ratio x_2/x_1 for two successive coalescence events should be 2. For the boundary layer at constant pressure ($\delta \sim x^{4/5}$, approximately), the ratio x_2/x_1 should be about 2.3. For the plane or round wake ($\delta \sim x^{1/2}$, $\delta \sim x^{1/3}$), the ratio x_2/x_1 should be 4 or 8, respectively. For the plane or round momentumless wake ($\delta \sim x^{1/4}$, $\delta \sim x^{1/5}$), the ratio x_2/x_1 should be 16 or 32, respectively. These observations suggest that more attention might be paid to the problem of coherent structure in the latter two flows, despite an awkward non-uniqueness in the similarity laws which arises because the initial momentum flux vanishes as a flow parameter. All of the plane flows listed here except the mixing layer contain mean vorticity of both senses, so that three-dimensional structures should be expected. These structures then have a spanwise scale which must also double, to replicate the geometry in a plan view. Hence coalescence might be expected to involve four structures rather than two.

Unfortunately, this reasoning cannot be correct in general, because it leads in certain cases to a fatal contradiction. The argument requires all of the flows which grow less than linearly, beginning with the boundary layer, to develop very flat or elongated structure. Thus the basic premise, that the natural scales remain in a constant ratio to each other, is violated. I am not sure where the fault lies, but I am reluctant to give up my premise concerning scales. I think it more likely that my assumptions about skeletal models, coalescence, and doubling of scale are only correct for flows that grow linearly. If so, it may be necessary to think in terms of splitting (fission) as well as coalescence (fusion) of structure. There is at least one precedent here; splitting of structure is known to occur for the puff in pipe flow at low transition Reynolds numbers. The topic is a difficult one, and I have made no progress with it. In any case, a fundamental morphological distinction is dictated by the presence or absence of a global scale for the mean flow. If the flow boundary defined by the appropriate similarity law is curved rather than straight, such a scale is inherent in the geometry. Finally, I will mention for the record one flow that is especially difficult to fit into the morphology just outlined. This is the sink flow; i.e., the boundary layer in a wedge-shaped converging channel. The streamwise/spanwise/lateral scales for a fixed volume of fluid within the boundary layer are increasing/constant/decreasing, so that splitting of large structure seems much more likely than coalescence. On the other hand, it may not be an accident that the mean-velocity profile in this flow is logarithmic everywhere outside the sublayer, and that there is no net entrainment. The question of of large-scale coherent structure may be moot.

4. Comments on Modeling

If research on the mixing layer can be described as a developed industry, then research on the boundary layer should be described as a flea market. The boundary layer has been studied so extensively that it is exasperating that so little is known about it. I can conceive of several reasons. The most important is that coherent structure in the boundary layer is three-dimensional. Use of single probes, or of multiple probes with conditioning, or of lines or sheets of light with smoke or fluorescent dye, has not so far illuminated the large-scale three-dimensional motion. Almost nothing is known about topology. Finally, the progression from laminar structure to turbulent structure is missing, and information about structure during transition, while extensive, is not of much pragmatic value.

The viscous sublayer. Nevertheless, recognition of small-scale coherent structure in the sublayer of a boundary layer has cleared up one old mystery associated with the behavior of fluctuations near a smooth wall. Power series in y for various mean quantities in Reynolds-averaged form can be derived by straightforward operations of repeated differentiation of the equations of motion, followed by evaluation at the wall (the details are left as a tedious exercise for the reader). If the mean flow is two-dimensional, the leading term in the equation for the turbulent shearing stress is

$$-\overline{u'v'} = \left[\frac{3}{2\mu^2}\frac{d}{dx}\overline{(\tau_w')^2} + \frac{3}{\mu^2}\overline{\tau_w'\frac{\partial}{\partial z}\sigma_w'}\right]\frac{y^3}{6} + O(y^4)$$

where $\tau_w' = (\mu\,\partial u'/\partial y)_w$ and $\sigma_w' = (\mu\,\partial w'/\partial y)_w$. The overbar can be interpreted as a time average, an ensemble average, or a spatial average, as appropriate. Given the wide-spread belief that $\overline{u}$ eventually behaves like log y outside the sublayer, the radius of convergence y_* of such series is certainly finite and probably small (say $y_* < 15\nu/u_\tau$, where $u_\tau = \sqrt{\tau_w/\rho}$).

The first term in the coefficient of y^3 vanishes for a channel flow, and is easily shown to be too small by several orders of magnitude to represent the observed behavior of $-\overline{u'v'}$ in a standard boundary layer. For a structureless sublayer, it is at least plausible that the second term should also vanish, leaving the term $O(y^4)$ as the leading term (see, for example, the discussion in Monin and Yaglom[48], Section 5.3, or Hinze[49], Section 7-5). A reasonable argument might be that, given a particular value for $\partial\sigma'_w/\partial z$, positive or negative values for τ_w' are equally probable. However, the sublayer is not structureless. It contains counter-rotating streamwise vortices, and the phase relationship in z for τ_w' and σ_w' is manifestly such that τ_w' and $\partial\sigma_w'/\partial z$ are in phase. Hence the leading term in $-\overline{u'v'}$ is $O(y^3)$. I have been able to estimate the coefficient with the aid of a deterministic model which I proposed some time ago[50] for the part of the sublayer flow associated with the streamwise vortices. The model yields

$$-\frac{\overline{u'v'}}{u_\tau^2} = 0.00103\left[\frac{yu_\tau}{\nu}\right]^3 + O(y^4)$$

A preliminary value extracted by Spalart from his recent full numerical simulation of the boundary layer is 0.001 (private communication). The agreement is encouraging. However, I consider the quantitative content of this or any other deterministic sublayer model to be less important than my qualitative conclusion in the cited paper that three transport mechanisms, rather than two, must be at work in flow past a smooth wall. Just at the wall, the mechanism is molecular. Outside the sublayer, the mechanism is bulk transport by large-scale eddying motions, coherent or not. For reasons which are at present obscure, the latter mechanism is apparently incompatible with a no-slip boundary condition at the wall for the large-scale fluctuations. Nature has therefore invented the sublayer vortices to make up the deficit in transport, particularly in the region $0 < yu_\tau/\nu < 50$. The instability which creates these sublayer vortices, and keeps them energized, is still a mystery.

One reasonable inference from this discussion is that the main effect of surface roughness may be to compensate for the incompatibility in the no-slip boundary condition by other means. Another is that a deterministic description of the sublayer structure can be expected to have important applications in treating heat or mass transfer at a wall, especially at large Prandtl or Schmidt numbers. Finally, any discussion of the drag-reducing effects of high-molecular-weight polymers in liquids, especially the existence of a critical velocity, should benefit from estimates of the maximum local rate of normal strain in such a sublayer.

<u>Hierarchy of structure</u>. Analytical work on structure has failed to keep pace with experimental work. I know very little about turbulence modeling, but I am sure that turbulence models have an important role to play in the problem of structure when the curtain finally goes up, at some time in the future. Although deterministic models will have to give way at some scale to statistical ones, this scale may not be the largest scale in the flow. Evidence is accumulating that at least a modest hierarchy of structure can be identified in some flows, although not the hierarchy proposed by Theodorson, which requires an orthogonal horseshoe coordinate system. The motion which first renders a mixing layer three-dimensional has already been mentioned in Section 2. Other less substantial evidence refers to the spot in a boundary layer[51] and to the slug in a pipe[52], although the issue here may really be the issue of splitting. My own position on the question of hierarchy has softened, after some recent experience with the turbulent vortex ring. Figure 9 (Glezer, private communication) shows sections in two orthogonal planes through such a ring in water. Fluorescent dye and a sheet of laser light were used for flow visualization. The side view presents a contradiction. Any reasonable topological cartoon of the vortex ring would place the region of main entrainment at the rear. However, the side-view photograph, like numerous similar photographs by other investigators, shows that dyed fluid is left behind in the wake; there is substantial de-entrainment at the rear. Clarification was obtained when the light sheet was turned normal to the path of the ring, yielding the sequence of pictures shown as satellites in Figure 9. At certain azimuthal positions, the region outside the main core is either visible or not visible through the whole sequence. The pictures suggest the presence of vortex tubes wrapped around the main core in azimuthal planes. At some azimuthal positions the induced flow corresponds to entrainment; at others, to de-entrainment. It is the latter regions which move dyed fluid into the wake. There is a small net

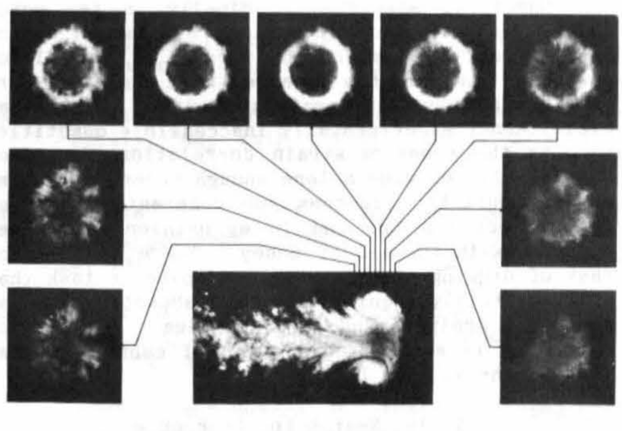

Figure 9. Secondary vortex structure in a turbulent vortex ring. Photograph provided by A. Glezer, University of Arizona.

entrainment into the ring, which grows conically but exceedingly slowly, with a total cone angle of about 1 or 2 degrees.

Interfaces. Given a turbulent shear flow, suppose that the largest scale, whose topology is certainly more than a descriptive device because it determines important energy transformations, can be adequately treated by deterministic modeling. Suppose also that the smallest scales can be adequately treated by an eddy-viscosity or mixing-length scheme. In conventional modeling, as far as I am aware, little or no use is made of the concept of intermittency; i.e., of interface. I am not quite ready to give up the hypothesis that statistically flat interfaces can exist, especially at large Reynolds numbers, and can propagate by the process gastronomically described as "nibbling." I would like to see more attention paid to the implications of turbulent diffusivities which are proportional to, or otherwise dependent on, the quantity being diffused. For example, consider the simple one-dimensional model equation

$$\frac{\partial c}{\partial t} = \tau \frac{\partial}{\partial y} e \frac{\partial e}{\partial y}$$

where e stands (say) for turbulent energy, and τ is a constant with the dimensions of time. This equation admits solutions of the form

$$e = -\frac{s}{\tau}(y - st)$$

where

$$s = -\tau \frac{\partial e}{\partial y}$$

Consequently, the quantity e vanishes at a front that propagates in the direction of increasing y with a velocity s which is proportional to the gradient of the turbulent energy near the interface. I have no instinctive feeling for the nature of the characteristic time τ. Moreover, in the absence of a viscous term, there seems no way to accommodate negative values for s, although these are needed, for example, to describe the puff in a pipe. Turbulence models with the desired property have been proposed by Kovasznay[53] and by Saffman[54], both of whom have discussed the problem of laminar-turbulent interfaces. The frequently used k-ϵ model also falls in this class, but I am not aware of much discussion of the point at issue.

Numerical simulation. Finally, a few words should be said about numerical simulation, which has become an important force in turbulence research. Leonard has remarked that numerical work provides abundant information about a flow, especially about experimentally inaccessible quantities such as the pressure-strain correlation, but does not normally provide a long enough record to accommodate heuristic searches for coherent structure. This defect can be, and in my opinion should be, overcome with the aid of money. There remains the task of digging structure out of noise, a task that will eventually require close collaboration between numerical analysts and experimenters. Research in this area is moving rapidly, and I cannot do justice to the subject here.

5. The Search for Coherence

Some applications. I have occasionally been asked by managers in industry if the concept of coherent structure is something that they should be applying in design and development. My answer until recently was negative: the concept was not ready, and the best course was probably to have somebody paying careful attention so that application could begin at the earliest possible moment. I am less negative now. I have in mind that much of the strength in turbulence modeling is concentrated in industry, and that modeling of coherent structure is a most conspicous lack at present.

Another new element of value to industry is progress in manipulation and control of turbulent flows. For example, Walsh and Lindemann[55] have consistently achieved reductions of several percent in surface friction in boundary layers by using longitudinal surface grooves called riblets, despite the fact that these devices substantially increase the surface area. The change in geometry affects the sublayer vortices in such a way that drag is reduced but heat transfer is increased. Plesniack and Nagib[56] have reported achieving net drag reductions of about 20 percent in boundary layers by a different method, using small, flat flow guides called large-eddy-break-up devices (LEBU's). The mode of interaction of these devices with the large-scale structure is not understood. So far the devices, and perhaps the thinking, are two-dimensional. In both methods of drag reduction, the control schemes are literally cut-and-try, and are highly intuitive. There are precedents to prove the value of intuition in research of this kind, going back many years to Roshko's use of a splitter plate to inhibit vortex shedding from a cylinder and thereby reduce the drag. I emphasize the value of intuition to suggest that there is room in the game for more players.

Some issues. In the Introduction, I mentioned some lines of polarization in the research community. I should say what a few of these are, according to my perception of the situation. One issue concerns the permanence of structure, and its dependence on initial conditions. A few authors hold that any detectable structure, particularly in boundary layers, can always be traced upstream to a transition region, and that structure should therefore become undetectable in a sufficiently mature flow. Counter-examples are abundant, beginning with the atmosphere. Certainly, there is no reason to suppose that the instabilities that lead to coherent structure operate only in the transition region. A more relevant issue involves a difference of opinion about the role of viscosity and the value of inviscid models. In research on the turbulent spot, for example, the laminar boundary layer is idealized as a vortex sheet in the work by Coles and co-workers[57], but is treated as an essential part of the ambient flow in the work by Wygnanski and co-workers[58]. Both positions are tenable; they are simply different. In research on boundary layers, a fundamental issue is the relative role of the sublayer flow and the large-scale outer flow. One school argues that the sublayer drives the outer flow; another school argues precisely the opposite. No mechanism has so far been proposed to accommodate both views, and thus eliminate the occasional myopia and presbyopia that have caused arguments in the research community from time to time. Finally, there is, or eventually will be, a difference of opinion on the question of hierarchy. There is no concensus about the smallest scale at which structure can be defined, or is worth defining, before conventional statistical ideas are reinstated.

There is even a serious problem with publication. For example, some of the earliest descriptions of the mixing-layer mechanism are in the subliterature. The importance of topology and the presence of saddles in the strain field were noted in an otherwise conventional thesis by another Brown[59]. The importance of vortex stretching along bridges and the association with the mean strain field were emphasized in a laboratory report by Corcos[60] which is full of insights. Unfortunately, the impact of these insights was weakened when they were dispersed into three journal papers which appeared five years later. The importance of coalescence as part of structure became public information only in 1974, in nearly simultaneous journal papers by Winant and Browand[61] and by Brown and Roshko[62]. The experimental evidence for large turbulence production near saddles appeared in journal papers after a lapse of three years and seven years, respectively, from the time that the theses by Zaman and by Cantwell were completed. It seems that senior members of the research community in turbulence, myself included, are accustomed to a leisurely pace, and are not yet persuaded that it is advisable during a revolution to publish quickly, carefully, and succinctly--and not only in the subliterature.

Some expectations. In this paper I have concentrated on showing what can be done by using the concept of coherent structure that cannot be done without it. After fifteen years of research, there are some definite gains. It has always been so that the most valuable products of research on turbulence are formulas capable of generating reliable numbers over a substantial range of conditions. Although the results so far are not very profound, the second revolution, like the first one, is visibly raising the art of turbulence to a higher level. When this level is reached, I hope that the variety of empirical constants which describe growth rates and energy levels in different flows will be consolidated into a much smaller number. I hope that a solid argument will emerge for the existence of a logarithmic mean-velocity profile in flow near a wall. I hope that a knowledge of flow patterns and processes associated with coherent structure will allow closer estimates of relaxation times (and will account for overshooting) in flows where rapid changes occur in the boundary conditions. All in all, the uses that have already been discovered for coherent structure promise well for the future.

I am well aware that an act of faith is required to believe, on the evidence presented here, that one key to turbulence lies in an understanding of the dynamics of a single deterministic coherent structure in a crowd of other structures, with the interaction adequately described in terms of the joint strain field. This belief has its origin in studies of the mixing layer, where experience with both laminar and turbulent flow and with numerical simulations all points in the same direction. For other flows, the evidence is much less conclusive. It does not follow that the emperor has no clothes, that experience with the mixing layer cannot be generalized, and that there is no such thing as a typical coherent structure in most or all of the other classical flows. There is no such thing as a hard spherical molecule, either; yet several useful properties of gases, such as the dependence of viscosity on temperature, and the lack of dependence on pressure, can be inferred from such a model, and primitive estimates can be improved by finite improvements in the model. I believe that this analogy is real.

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