component in the minus y direction. From Eq. (2) it is evident that ∇E^2 is normal to $\vec{E}$. Thus the confinement conditions for the first case appear to exist.

For the second case, let the waves intersect symmetrically about the y axis as shown in Fig. 1, and let the electric vectors be given in the mks system as

$$\vec{E}_1 = (-x_0\hat{i}+y_0\hat{j})E_0 \exp[i(\vec{k}_1\cdot\vec{r} - \omega t)], \qquad (6)$$

$$\vec{E}_2 = (x_0\hat{i}+y_0\hat{j})E_0 \exp[i(\vec{k}_2\cdot\vec{r} - \omega t - \alpha)], \qquad (7)$$

where $x_0 = \cos\theta$ and $y_0 = \sin\theta$.

As before, we find

$$\vec{E} = E_0 \exp[i(\beta_y - \omega t)]\{(y_0\hat{j} - x_0\hat{i}) \exp(i\beta_x) + (y_0\hat{j}+x_0\hat{i}) \exp[-i(\beta_x+\alpha)]\}, \qquad (8)$$

$$\vec{B} = \hat{k}(E_0/u) \exp[i(\beta_y - \omega t)]\{\exp(i\beta_x) + \exp[-i(\beta_x+\alpha)]\}. \qquad (9)$$

The electric vector is now elliptically polarized, while $\vec{B}$ remains plane polarized.

We can now form $\nabla(\vec{E}\cdot\vec{E})$ and examine the result to see if it is parallel to $\vec{E}$. Because of the complexity of ∇E^2, we present here the result for the special case when $\theta = \frac{1}{4}\pi$. In this case

$$\nabla E^2 = \sqrt{2}\, E_0^{\,2}(\omega/u)i \exp[2i(\beta_y - \omega t)] \times\{\hat{i}[\exp(2i\beta_x) - \exp(-2i(\beta_x+\alpha))] + \hat{j}[\exp(2i\beta_x) + \exp(-2i(\beta_x+\alpha))]\}. \qquad (10)$$

Taking the vector cross product of ∇E^2 with $\vec{E}$ and setting the result equal to zero, we find that the condition for ∇E^2 to remain parallel to $\vec{E}$ is

$$2\beta_x + \alpha = \tfrac{2}{3}n\pi, \qquad (11)$$

where n is an integer with values 0, 1, 2...

Hence one would expect regions to exist within the intersecting volume where strong focusing occurs.

Detailed calculations of individual particle trajectories must still be made. It may be necessary to use multiple plane waves to block open ends which may exist in the volume wherein only two waves intersect.

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[1] F. B. Knox, Australian J. Phys. 10, 221 (1957).
[2] F. B. Knox, Australian J. Phys. 10, 565 (1957).
[3] H. A. H. Boot, S. A. Self, and R. B. R-Shersby-Harvie, J. Electronics and Control 4, 434 (1958).
[4] H. A. H. Boot and R. B. R-S. -Harvie, Nature 180, 1187 (1957).
[5] E. S. Weibel, J. Electronics and Control 5, 435 (1958).
[6] E. S. Weibel, in Plasma in a Magnetic Field, edited by R. K. M. Landshoff (Stanford University Press, Stanford, 1958), p. 60.
[7] R. F. Wuerker, H. Shelton, and R. V. Langmuir, J. Appl. Phys. 30, 342 (1959).
[8] E. S. Weibel, Phys. Rev. 114, 18 (1959).
[9] V. Cushing and M. S. Sodha, Phys. Fluids 2, 494 (1959).

DURATION OF NUCLEOSYNTHESIS*

G. J. Wasserburg
Charles Arms Laboratory of the Geological Sciences, California Institute of Technology, Pasadena, California

and

William A. Fowler and F. Hoyle
W. K. Kellogg Radiation Laboratory, California Institute of Technology,
and Mount Wilson and Palomar Observatories, Carnegie Institution of Washington,
California Institute of Technology, Pasadena, California
(Received January 11, 1960)

In a recent Letter on a determination of the age of the elements, Reynolds[1] reported the important discovery of isotopically anomalous xenon in the stony meteorite Richardton. The isotopes which appear to occur in significant excess over atmospheric xenon are Xe^{128}, Xe^{129}, Xe^{130}, and Xe^{131}, with the Xe^{129} dominant by an order of magnitude. At present it does not appear possible to explain all of these data by any single mechanism. Because of the existence of these four anomalies, it is difficult to conclude that the Xe^{129} excess is simply the product of I^{129} decay.

If, however, it is assumed that most of the Xe^{129} can be attributed to the decay of I^{129} remaining after some element-building processes, some deductions may be made regarding these processes. In these considerations we will assume that after the termination of nucleosynthesis, a time interval Δt passed during which I^{129} underwent unsupported decay and the resulting daughter product Xe^{129} was not accumulated in solid objects such as meteorites. After the end of this time interval, the crystals in the meteorites are supposed to form closed systems until the present time. In the calculation of Δt given by Reynolds, he assumes that the nucleosynthetic processes during which iodine was made were instantaneous, in accord with such short-time scale models as proposed by Alpher, Bethe, and Gamow.[2] On the other hand, Burbidge, Burbidge, Fowler, and Hoyle[3] have proposed that nucleosynthesis has taken place in stars at a more or less uniform rate over the lifetime of the Galaxy. The period of nucleosynthesis is taken by these authors to be of the order to 10^{10} years. There is considerable uncertainty as to the exact time dependence of stellar evolution and nucleosynthesis in the Galaxy. However, in the following discussion we assume that nucleosynthesis occurred at a uniform rate over a time large compared to the mean life ($\tau = 2.5 \times 10^7$ yr) of I^{129}.

If the production of iodine occurs uniformly over a period of time T ($T \gg \tau$) and is then halted, and if a time interval Δt passed before the Richardton meteorite existed as a closed system, the following relationship is obeyed: $\ln(I^{127}/Xe^{129}) = \Delta t/\tau + \ln(T/\tau) + \ln(K_{127}/K_{129})$. Here, K_{127} and K_{129} are the production rates of I^{127} and I^{129}, respectively. Reynolds' experiment yields $\ln(I^{127}/Xe^{129}) = 14.1$. In interpreting this result Reynolds took $T < \tau$, in which case $\ln(T/\tau)$ does not occur in the equation. Then, assuming $K_{127}/K_{129} \sim 1$, he found $\Delta t = 3.5 \times 10^8$ years.

Because of the fact that the term involving T occurs in a logarithm, the calculation of Δt does not depend critically on T. For any mechanism of element production where $K_{127}/K_{129} \lesssim 10^4$, the result obtained by Reynolds is hardly restrictive as to the value of T purely from such considerations. In addition, the introduction of T permits considerable latitude in Δt. For example, if $\Delta t = 0$ and $K_{127}/K_{129} \sim 1$, then $T \sim 3 \times 10^{13}$ years. A consideration which might yield a more strict limit on T is the evaluation of the time necessary to form stable solar objects. If a large fraction of the time calculated by Reynolds is required for this purpose, then the possible values of T will be small. On the other hand, for $\Delta t = 2.0 \times 10^8$ yr, then $T \sim 10^{10}$ yr. Decreasing the free decay period by less than a factor of two permits a reasonable period of duration for nucleosynthesis.

We conclude, therefore, that Reynolds' number is the maximum time interval which could exist between the termination of nucleosynthetic processes and the formation of the Richardton meteorite. It does not directly date the time of formation of the elements.

In addition to this matter, there is a question regarding the assumption that the Richardton meteorite formed a closed system with respect to xenon and iodine. An A^{40} - K^{40} age of $(4.15 \pm 0.10) \times 10^9$ yr has been determined on Richardton by Geiss and Hess.[4] This value is somewhat younger than the A^{40} - K^{40} ages on some other meteorites and is about 0.45×10^9 yr less than the Pb^{207} - Pb^{206} age obtained on the stony meteorite Nuevo Laredo by Patterson.[5] If the argon age represents the time that Richardton became a closed system, then, ignoring experimental errors, the maximum time since the termination of nucleosynthesis is $4.15 + 0.35 = 4.50 \times 10^9$ yr. If the true time of formation of Richardton is 4.6×10^9 yr and the xenon has been retained since then, then the maximum time is 4.95×10^9 yr, as reported by Reynolds. If the true time of formation of this meteorite is 4.6×10^9 yr and if the last major outgassing is given by the argon age, then it is possible that the relatively short-lived ($\sim 10^6$ yr) radioactivities could contribute significantly to the heat production in the early history of the solar system. From the point of view of long-term nucleosynthesis the abundances of radioactive isotopes with mean lives (τ') shorter than I^{129} will be proportional to τ'/T. This will still permit a significant contribution to the heat production if $\Delta t = 0$. The most critical parameter is the interval Δt. If the time interval between the formation of meteorites and the termination of nucleosynthesis is within an order of magnitude of the maximum of 3.5×10^8 yr as reported by Reynolds, then there will be no such effect.

The positive result obtained by Reynolds of excess Xe^{129} on Richardton is a factor of ten greater than the upper limit reported on the Beardsley meteorite by Wasserburg and Hayden.[6] Since Beardsley has an A^{40} - K^{40} age of (4.30±0.10)

$\times 10^9$ yr, this implies that either the analytical data on the xenon in Beardsley are incorrect, or that these two meteorites were formed as closed systems separated by a time interval of $\sim 10^8$ yr. This would just be possible within the limits of analytical error.

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[1]J. H. Reynolds, Phys. Rev. Letters 4, 8 (1960).
[2]R. A. Alpher, H. A. Bethe, and G. Gamow, Phys. Rev. 73, 803 (1948).
[3]E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. R. S. Hoyle, Revs. Modern Phys. 29, 547 (1957).
[4]J. Geiss and D. C. Hess, Astrophys. J. 127, 224 (1958).
[5]C. C. Patterson, Geochim. et Cosmochim. Acta. 10, 230 (1956).
[6]G. J. Wasserburg and R. J. Hayden, Nature 176, 130 (1955).

DIRECT OBSERVATION OF OPEN MAGNETIC ORBITS

E. I. Blount
Westinghouse Research Laboratories, Pittsburgh, Pennsylvania
(Received December 23, 1959)

It is well known that in the band structure of the more complicated metals (polyvalent metals other than semimetals), the Fermi surface need not be singly-connected and that as a consequence, some of the orbits followed by electrons under the influence of a magnetic field may not be closed, but will extend indefinitely in some direction in $\vec{k}$ space and in the perpendicular direction in the real space. It has been widely supposed that these orbits are nonquantized and that no resonance can be expected due to them.[1,2] It is the purpose of this Letter to contradict both of these suppositions.

It has recently been shown by Kohn[3] that the practice of describing the motion of an electron in a crystal in a magnetic field by the Hamiltonian $\mathcal{H} = \delta_{mm'}E_m(\hbar\vec{k} + (e/c)\vec{A})$ is justified, though it would be more proper to write $\mathcal{H} = \delta_{mm'} \times E_m(\vec{H}; \hbar\vec{k} + (e/c)\vec{A})$ as E should be modified at higher values of the field H. This Hamiltonian is periodic in $\vec{k}$ space as are the basic functions in the representation in which it holds. Thus any wave function describing an electron motion must be periodic in $\vec{k}$ space, which thus effectively becomes a three-dimensional torus. These wave functions obey the equation

$$\{E - E_m(\hbar\vec{k} + (e/c)\vec{A})\}\varphi(k) = 0$$

($\vec{x}$ is represented by $i\partial/\partial\vec{k}$). In the semiclassical language which we shall employ from here on, Bohr-Sommerfeld quantum conditions can then be applied to the action integral for any orbit, the integration extending from any point in $\vec{k}$ space over the path followed by the electron until it returns either to the same point in $\vec{k}$ space or to an equivalent point.

Before applying this procedure it is necessary to consider the actual form of the orbit (in $\vec{k}$ space) followed by an electron. This orbit is the intersection of a constant-energy surface and a surface of constant $\vec{k}\cdot\vec{H}$. The latter, however, requires some elucidation. Let us choose a unit cell C and corresponding Brillouin zone B, which are convenient for the structure in question. If H is parallel to one of the basis vectors thus chosen, say the z axis, k is a constant of the motion, and if the orbit is open, it will leave the zone B at a point different from but equivalent to the point at which it entered, and will then retrace its previous course in the same k_z plane. The orbit is, therefore, a single connected curve, and the Bohr-Sommerfeld condition can be applied to one traversal of B.

If H is not parallel to a basis vector, but its components have rational ratios, it is parallel to some lattice vector, and a unit cell C_H and Brillouin zone B_H can be constructed such that H is in the z direction. In this zone the orbit may be closed or open as described above, a natural frequency exists, and the levels are quantized. This Brillouin zone, however, may be very thin and flat if the lattice vector R_H parallel to H is long. The frequency in this case may be smaller than that in the previous paragraph by roughly the ratio of the length of R_H to that of the basis vector. If we try to describe the motion in the zone B, the surface $\vec{k}\cdot\vec{H}$ = constant becomes a set of planes. The orbit may be closed in the part of one of three zones which