Interference Effects of the Retardation Term in Pion Photoproduction*

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It is shown that the difference in behavior of the high-energy (>450 Mev c.m.) total photoproduction cross sections for π^+ and π^0 can be explained by the presence of the retardation term in the case of the π^+ production. The analogy with the behavior at the $(\frac{3}{2}, \frac{3}{2})$ resonance is noted. The discussion naturally provides an explanation for the difference in center-of-mass energies of the lower high-energy peak found in π^--p scattering and the corresponding peak in the π^+ photoproduction. It is felt that the discussion contributes some evidence for the resonance nature of the peaks.

I. INTRODUCTION

RECENT experiments on the photoproduction of single charged and neutral pions from hydrogen[1] have indicated the presence of a resonant-like increase in the total cross sections in the photon energy region of 700–800 Mev. This increase in cross section has been ascribed to the existence of a new isobaric state of the proton[2] characterized by isotopic spin $\frac{1}{2}$, angular momentum $\frac{3}{2}$, and a center-of-mass energy of about 600 Mev. Since this suggestion was advanced, the spin and parity of the state have been under discussion[3] and methods for determining these quantum numbers have been proposed.[4] Experiments on the polarization of the recoil protons from π^0 photoproduction[5] have lately indicated that the state may, possibly be specified as $D_{\frac{3}{2}}$.

Negative pion-proton scattering experiments[6] have exhibited two high-energy peaks in the total scattering cross section. The lower of these is to be associated with those found in the photoproduction cross sections, although it has been noted, by Burrowes *et al.*,[6] that the c.m. energy at the scattering peak is about 75 Mev higher than the corresponding energy at which the π^+ photoproduction peak is observed by Dixon and Walker.[1]

A striking feature of the high-energy maxima in the π^+ and π^0 photo cross sections is their difference in shape. The π^+ peak, at about 700 Mev, is sharp and falls steeply on the high-energy side, while the π^0 cross section shows a broad and smooth increase with a maximum apparently somewhat above 700 Mev. This difference has been locally considered to be due to interference between the resonant $T=\frac{1}{2}$ state and a $T=\frac{3}{2}$ state with the same spin and parity. Such an interference between a resonant $T=\frac{1}{2}$ state and a resonant $T=\frac{3}{2}$ state, at a higher energy, has been invoked in a discussion of pion pair production.[7]

The purpose of this note is to present an alternative, and perhaps more satisfactory explanation for the difference in behavior of the π^+ and π^0 photoproduction cross sections and, also to account for the discrepancy in c.m. energies of the π^+ photopeak and the π^- proton scattering peak. The procedure used to achieve these aims is felt to provide some additional evidence for the resonance character of the $T=\frac{1}{2}$ state.

A natural explanation of the effects, without arbitrarily introducing extra states, may be given by the interference between the $T=\frac{1}{2}$ state and the retardation part of the photon-meson current interaction, which is present for π^+ photoproduction, but not, in the same order, for π^0 production. The importance of including the retardation term in the analysis of charged-pion photoprocesses has been emphasized by Moravcsik.[8] The Born approximation to the term has the same form in all the theories proposed and its effects have been observed recently in experiments at around 300 Mev.[9]

So far the qualitative analysis of the high-energy data[2,3] has ignored contributions from the retardation term. Inclusion of the term, which contains states of all angular momenta, leads to considerable complication in the analysis of the π^+ photoproduction differential cross sections. The complication is caused by the interference of the states contained in the retardation term with all others present and the result is that it is not yet possible to make a firm statement about the states involved from the π^+ differential cross sections alone. A discussion of the total cross sections is, however, simpler as inter-

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[1] Heinberg, McClelland, Turkot, Woodward, Wilson, and Zipoy, Phys. Rev. **110**, 1211 (1958); F. P. Dixon and R. L. Walker, Phys. Rev. Letters **1**, 142 (1958); **1**, 458 (1958). J. I. Vette, Phys. Rev. **111**, 672 (1958); DeWire, Jackson, and Littauer, Phys. Rev. **110**, 1208 (1958); P. C. Stein and K. C. Rogers, Phys. Rev. **110**, 1209 (1958). R. M. Worlock, California Institute of Technology thesis (to be published).
[2] R. R. Wilson, Phys. Rev. **110**, 1212 (1958).
[3] Walker, Wilson, and Gell-Mann, discussion in CERN, 1958 (unpublished); R. F. Peierls, Phys. Rev. Letters **1**, 174 (1958); H. H. Bingham and A. B. Clegg, Phys. Rev. **112**, 2053 (1958).
[4] J. J. Sakurai, Phys. Rev. Letters **1**, 258 (1958); M. J. Moravcsik, Phys. Rev. Letters **2**, 171 (1959).
[5] P. C. Stein, Bull. Am. Phys. Soc. **4**, 24 (1959); P. L. Connolly and R. Weill, Bull. Am. Phys. Soc. **4**, 23 (1959); and private communication.
[6] Burrowes, Caldwell, Frisch, Hill, Ritson, Schluter, and Wahlig, Phys. Rev. Letters **2**, 119 (1958); Crittenden, Scandrett, Shephard, Walker, and Ballam, Phys. Rev. Letters **2**, 121 (1959).
[7] A. B. Clegg, see reference 3.
[8] M. J. Moravcsik, Phys. Rev. **104**, 1451 (1956).
[9] J. H. Malmberg and C. S. Robinson, Phys. Rev. **109**, 158 (1958); Uretsky, Kenney, Knapp, and Perez-Mendez, Phys. Rev. Letters **1**, 12 (1958); Lazarus, Panofsky, and Tangherlini, Phys. Rev. **113**, 1330 (1959).

ference terms between states of different spins and parities do not contribute.

The next section discusses an interference effect in the total pion photoproduction cross section at the $(\frac{3}{2}, \frac{3}{2})$ resonance. The point brought out has been remarked by Chew,[10] but it was thought useful to show the magnitude of the effect using a specific theory to calculate the magnitude and to exhibit the similarity of the behavior with that found at the second resonance.

Section (3) describes a simple resonance model determined by reference to pion proton scattering data. The model is used to compute the contribution of the retardation-resonance interference to the π^+ cross section at the second resonance.

The last section compares the result of the calculation with the experimental data and discusses the significance of the result.

II. THE $(\frac{3}{2}, \frac{3}{2})$ RESONANCE

A study of the π^+ and π^0 total photoproduction cross sections at the $(\frac{3}{2}, \frac{3}{2})$ resonance shows a small relative displacement between the maxima, of the order of 25 Mev. It will be shown that the difference is caused by interference between the resonance and the retardation term in the π^+ photoproduction.

In order to attempt to compare the total cross sections associated with the low-energy resonance it is necessary to subtract out from the π^+ cross section the contributions due to the nonresonant states present. This has been done by calculating the sum of the contributions due to the $S_{\frac{1}{2}}$ state, the retardation term and their mutual interference using the relevant terms contained in the complete photoproduction amplitude to order $1/M$ given by Chew *et al.*[11] This amplitude is obtained from an application of the dispersion theory and contains an attempt to include rescattering and recoil processes together with the dominant resonant amplitude. In the evaluation the coupling constant f^2 was set equal to 0.081, the S-wave phase shifts used were those suggested by Puppi,[12] and the small amplitude $F^{(0)}$ was multiplied by $(1+w/M)^{-1}$. Nonresonant states other than those mentioned above were neglected.

The result of the subtraction is shown in Fig. 1 in which the experimental data are compiled from several sources.[13] Also shown is the π^0 total photoproduction cross section scaled down by an isotopic spin weight equal to 2. It should be noted that the π^0 cross section contains small contributions from rescattering and recoil which have not been subtracted. In the form shown in Fig. 1 the shift of the π^+ maximum relative to the π^0 is quite marked.

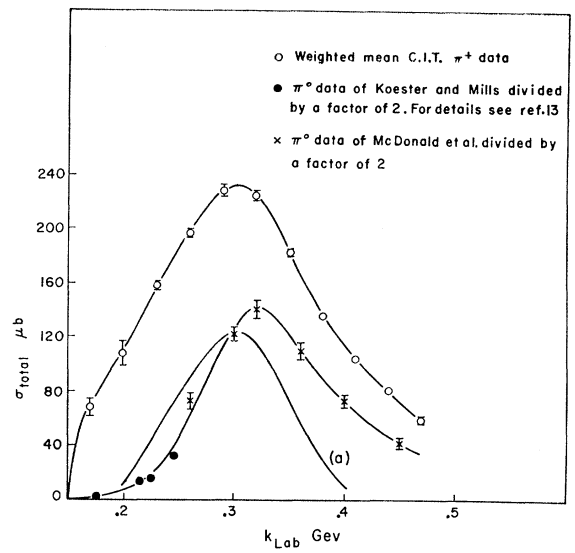

FIG. 1. Total π^+ photoproduction cross section and total π^0 photoproduction cross section, the latter scaled down by a factor of 2. Curve (a) is the result of subtracting the s wave and retardation contributions from the π^+ total cross section.

The contribution due to the interference between the retardation and $(\frac{3}{2}, \frac{3}{2})$ terms was computed from the dispersion theory amplitudes taking the resonance energy $w_r = 2.0$ (pion mass units) and using $(\frac{3}{2}, \frac{3}{2})$ phase shifts calculated from the effective-range formula of Chew.[11]

The major effect of the interference term, as noted by Chew,[10] arises from the resonant part of the amplitude and involves $\cos\delta_{33} \sin\delta_{33}$ which changes sign on going through resonance. Apart from this there are smaller effects due to rescattering and small magnetic moment Born terms contained in the complete amplitude. The rescattering terms arise as corrections to the renormalized Born approximation to the meson current term. The effect of these contributions is to shift the point at which the interference changes sign to an energy slightly lower than the resonance energy.

The shape of the interference contribution is shown in Fig. 2 together with points taken from the difference between the curves indicated in Fig. 1. The general agreement is good and clearly indicates the origin of the relative shift in the π^+, π^0 peaks as being due to the interference term.

III. THE SECOND RESONANCE

(a) Scattering Peak

The maxima in the π^- proton scattering cross section were given by Burrowes *et al.*[6] as occurring at 615 ± 40 and 775 ± 40 Mev c.m. The maximum in the π^+ total photoproduction cross section, according to Dixon and

[10] G. F. Chew, Phys. Rev. **95**, 1669 (1954); G. F. Chew and F. E. Low, Phys. Rev. **101**, 1579 (1956).

[11] Chew, Low, Goldberger, and Nambu, Phys. Rev. **106**, 1345 (1957); **106**, 1337 (1957).

[12] G. Puppi, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

[13] Walker, Teasdale, Peterson, and Vette, Phys. Rev. **99**, 210 (1955); Tollestrup, Keck, and Worlock, Phys. Rev. **99**, 220 (1955); L. J. Koester and F. E. Mills, Phys. Rev. **105**, 1900 (1957); McDonald, Peterson, and Corson, Phys. Rev. **107**, 577 (1957).

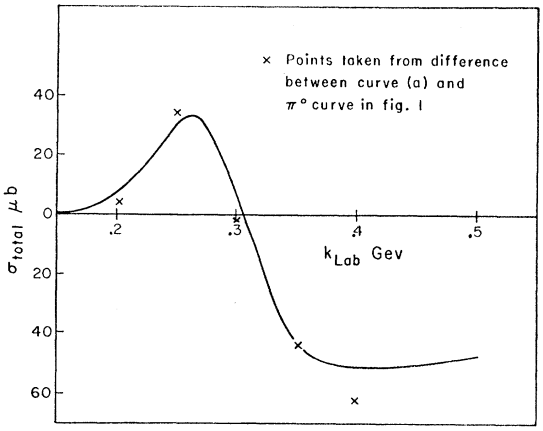

FIG. 2. Contribution to total π^+ photoproduction cross section due to interference between resonance and retardation term.

Walker,[1] occurs at about 540 Mev c.m. It is rather difficult to place the peak in the π^0 photoproduction cross section, but the subtraction of a resonant-like tail due to the low-energy resonance, as performed by Vette,[1] seems to indicate that the maximum due to contributions other than the $(\frac{3}{2}, \frac{3}{2})$ state lies at about 590 Mev. This agrees, within the uncertainties involved, with the position of the lower peak in the scattering experiments, but the π^+ position is low. Photon energies high enough to explore the region of the 775 Mev c.m. maximum have not yet been attained, but the 1000-Mev photon energy (720 Mev c.m.) π^+ angular distribution of Dixon and Walker[1] indicates the presence of a strong high-angular-momentum wave which is probably associated with the large scattering cross section observed at 775 Mev c.m. The change in character of the π^0 angular distributions between 785 and 940 Mev, as observed by Vette,[1] may also be evidence for the presence of another state.

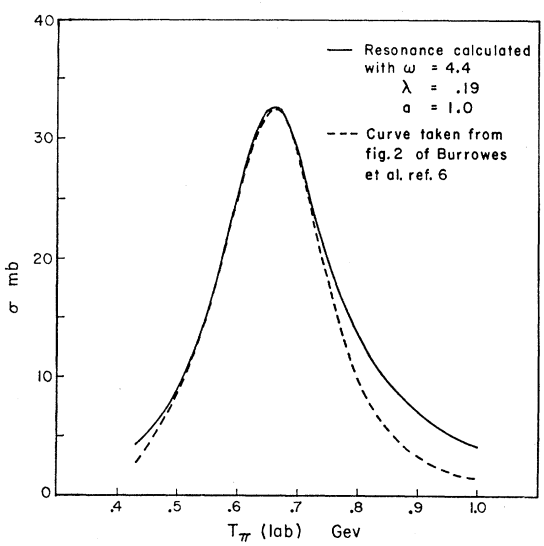

FIG. 3. Form of resonance used in discussion; it is normalized to the peak of the curve given by Burrowes *et al.*

In order to discuss phenomena connected with the 615-Mev c.m. peak in the scattering cross section, a simple single-level resonance type of description[14,2] will be used and the state concerned specified as $D_{\frac{3}{2}}$. The procedure for constructing the resonance is set out in the following formulas.

For a state with $j=\frac{3}{2}$, the total scattering cross section is given by

$$\sigma_s = 8\pi \left| \frac{e^{i\delta} \sin\delta}{q} \right|^2, \tag{1}$$

where δ is the phase shift and q the c.m. momentum. A resonant phase shift may be characterized by[15]

$$\tan\delta = \frac{\frac{1}{2}\Gamma}{W_r - W}, \tag{2}$$

where W is the total c.m. energy and W_r is the resonance energy. Γ is the total resonance width and is given by

$$\Gamma = 2(qa)v\lambda; \tag{3}$$

here a is the channel radius, λ is the reduced width, and v is the penetration factor. For a state with $l=2$, v is given by[15]

$$v = (qa)^4/[9+3(qa)^2+(qa)^4]. \tag{4}$$

Setting $W_r = 615$ Mev and using (1) through (4), the parameters a and λ were chosen to give a total width at half height of about 200 Mev, as required by the $T=\frac{1}{2}$ cross section shown in Fig. 2 of the Letter of Burrowes *et al.*[6] The values for a and λ used were, $a=1$ pion Compton wavelength, $\lambda=0.19$ (pion mass). The maximum elastic scattering cross section observed by Crittenden *et al.*[6] was about 30 mb compared with the absolute maximum of 44 mb for a $j=\frac{3}{2}$ state at a c.m. energy of 615 Mev. This indicates absorption of the $j=\frac{3}{2}$ wave, and the resonance calculated has been fitted to a maximum absolute value of about 30 mb. The result is shown in Fig. 3 with the curve drawn in Fig. 2 of Burrowes *et al.*[6]

(b) Interference With the Retardation Term in Photoproduction

The considerations of the last subsection are considered to have determined the $D_{\frac{3}{2}}$ elastic scattering amplitude in terms of the phase shift, given by Eq. (2), and the adjusted parameters a and λ.

A resonant $E1$, $D_{\frac{3}{2}}$ photo matrix element, involving the same scattering amplitude, may be written down

[14] K. A. Brueckner, Phys. Rev. **86**, 106 (1952); M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1954), Vol. 4.

[15] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

in the form

$$\mathfrak{M}=-iefC\frac{e^{i\delta}\sin\delta}{q^5}(3\boldsymbol{\sigma}\cdot\mathbf{q}\mathbf{q}\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}q^2). \tag{5}$$

The amplitude term has been formed with reference to Watson's remarks[16] on final-state interactions and the angular part has been taken from the general form given by Chew *et al.*[11] C is considered a constant which is adjusted to give a fit to the contribution of the $E1$, $D_{\frac{3}{2}}$ state to the total π^0 photoproduction cross section and e and f have been included explicitly for convenience.

The extraction of the $E1$, $D_{\frac{3}{2}}$ contribution from the experimental data has been done in a rather arbitrary way. The tail of the $(\frac{3}{2},\frac{3}{2})$ cross section was calculated from the dispersion amplitude, using experimental δ_{33} phase shifts given by Blevins *et al.*[17] The result was subtracted from the experimental cross section and the remainder considered to be due mainly to the $D_{\frac{3}{2}}$ resonant state. Specifically, the maximum $D_{\frac{3}{2}}$ cross section was taken to be about 70% of the maximum remaining after subtraction, the other 30% being presumably due to small contributions from rescattering processes induced by the nonresonant S wave and retardation interactions, and higher partial waves. The low-energy side of the high angular momentum state indicated by the data of Dixon and Walker[1] and Vette[1] probably contributes substantially at the upper end of the range of measurements. In the absence of further information the above procedure was followed and C obtained by comparing

$$I=\left(\frac{q}{k}\right)\left(1+\frac{W}{M}\right)^{-2}\int|\mathfrak{M}|^2d\Omega, \tag{6}$$

with the estimated $D_{\frac{3}{2}}$ cross sections.

In (6) k is the c.m. photon momentum and W the total c.m. energy.

Figure 4 shows the shape of the π^0 resonance calculated from Eq. (6) with the subtracted cross section. The position of the calculated π^0 maximum is consistent with the experimental points.

The Born approximation to the retardation part of the π^+ meson current matrix element is

$$R=\sqrt{2}ief\frac{\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})\mathbf{q}\cdot\boldsymbol{\varepsilon}}{q_0k(1-\beta\cos\theta)}. \tag{7}$$

The contribution to the total π^+ photoproduction cross section due to the interference of the resonance and retardation matrix elements is calculated from the cross term between $\sqrt{2}\mathfrak{M}$ and R.

Explicitly it is given by the expression

$$\mathcal{I}=4e^2f^2C\frac{(\cos\delta\sin\delta)}{q^5}\left(1+\frac{W}{M}\right)^{-2}\left(\frac{q}{k}\right)A. \tag{8}$$

[16] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[17] Blevins, Block, and Leitner, Phys. Rev. **112**, 1287 (1958).

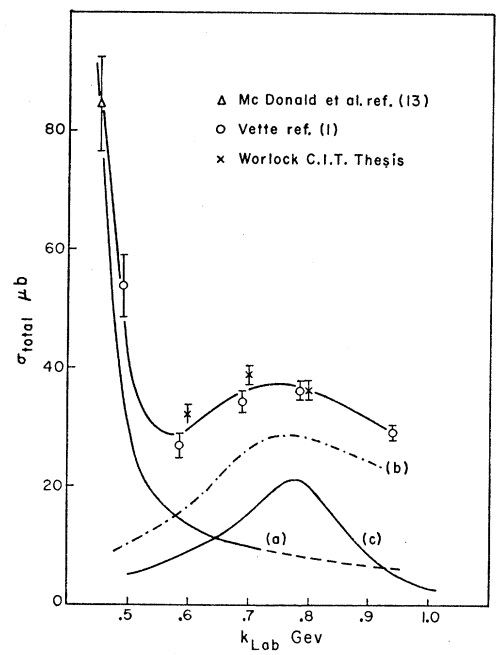

FIG. 4. High-energy total π^0 photoproduction cross section. Curve (a)—calculated tail of $(\frac{3}{2},\frac{3}{2})$ resonance; (b)—difference between experimental curve and curve (a); (c)—$D_{\frac{3}{2}}$ resonance calculated from scattering amplitude, maximum normalized to 70% of maximum of (b).

The result of the angular integration, A, is given by

$$A=2\pi\gamma^2k^2[2\beta^2(1+\gamma)-3]-\pi\frac{\gamma^2k^2}{\beta}(1-\beta^2)\times(3-2\beta^2\gamma)\ln\left(\frac{1-\beta}{1+\beta}\right), \tag{9}$$

where $\beta=q/q_0$, $\gamma=q_0/k$ and q_0 is the c.m. pion total energy. As $\mathcal{I}$ contains $\cos\delta\sin\delta$, the term changes sign at resonance and reaches a maximum around $\delta=45°$. The curve calculated from (8) and (9) is shown in Fig. 5. Also shown is the sum of $\mathcal{I}$ and $2\times$ the π^0

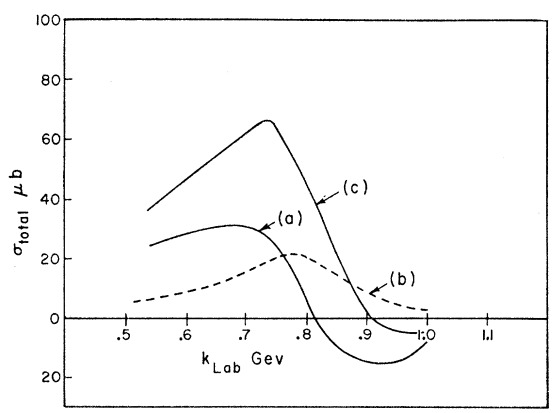

FIG. 5. Estimated contributions to π^+ total photoproduction cross section. Curve (a)—calculated interference contribution; (b)—calculated π^0 resonance; (c)—sum of (a) and $2\times$(b).

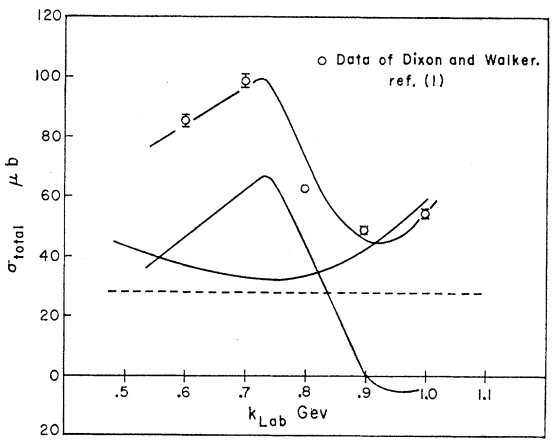

FIG. 6. Estimated π^+ total photoproduction cross section given by curve (c) of Fig. 5 added to a "background" cross section discussed in the text.

photoproduction cross section calculated from (6). The maximum in the sum is seen to be about 50 Mev below the maximum in the π^0 cross section and also the peak falls off sharply on the high-energy side due to the change in sign and rapid variation of the interference term in the cross section.

In order to compare the result of this discussion directly with the experimental data, a guess must be made as to the cross section due to other processes. Figure 6 shows the result of adding on a "background" cross section composed of an essentially energy-independent contribution, a resonance tail from the $(\frac{3}{2}, \frac{3}{2})$ state and a tail from the 775-Mev c.m. resonance.

IV. DISCUSSION AND CONCLUSIONS

The shape of the estimated π^+ photoproduction cross section, shown in Fig. 6 is in good agreement with the experimental data. There is a hint that the estimated peak is a little too high in energy and this could be remedied by choosing the resonance energy a little less, say 600 Mev, than the 615 Mev taken from the scattering data. On the other hand, other effects could cause a further small shift in the peak and such effects due, for example, to rescattering processes were remarked in the case of the $(\frac{3}{2}, \frac{3}{2})$ resonance. Another way of saying this is that the Born approximation to the retardation term is inadequate. It is certainly clear, however, that the difference in c.m. energy between the maximum in the π^+ photoproduction cross section and that in the π^--proton scattering cross section can be essentially explained by the present discussion. Of course, the absolute value of the resonance and interference cross sections have been determined in a rather arbitrary way, but the general shape of their contribution is not too sensitive to variation of the normalizing constant C. Another arbitrary procedure was the addition of the background cross section shown in Fig. 6. Apart from its absolute magnitude, the main assumption was that it did not vary rapidly with energy and, therefore, did not distort the general shape of the peak. This assumption is probably not unreasonable.

It is easy to verify[18] that the present analysis leads to consistency with the sign of the polarization found in the recent π^0 experiment.[5] According to Sakurai,[4] the polarization is supposed to be due to interference between the $D_{\frac{3}{2}}$ state and the tail of the $(\frac{3}{2}, \frac{3}{2})$ resonance. Taking the phase of the $P_{\frac{3}{2}}$ amplitude to be moving from 90° to 180° and using the $D_{\frac{3}{2}}$ phase assignment of the present discussion, one finds that the polarization of the recoil nucleon should be positive in the sense of the direction $(\hat{q}\times\hat{k})$. The measurement of Stein[5] shows that this is the case. The correlation between the sign of the shift of the π^+ photopeak and the polarization of the recoil proton in π^0 production provides useful information for determining the possible states present and should also be applicable to the case of the "third" resonance when experimental data is available.

Finally, the present discussion brings out the important point that the difference in c.m. energies of π^--proton scattering peak and the π^+ photoproduction peak, together with the singular shape of the latter, can be explained in terms of a state characterized by a single resonant phase shift and the presence of the charge dependent meson current. Insofar as this is true, the argument can be said to contribute some evidence that the rise in the pion photoproduction and scattering cross sections at around 600 Mev c.m. is caused by a state with a phase shift which passes through 90°.

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I am grateful to the Commonwealth Fund of New York for a grant.

[18] I am indebted to Professor M. Gell-Mann for this remark and to Professor R. L. Walker for an independent check on signs.