The Tunneling Time of an Electron

K. K. Thornber, Thomas C. McGill, and C. A. Mead

Citation: Journal of Applied Physics **38**, 2384 (1967); doi: 10.1063/1.1709888
View online: http://dx.doi.org/10.1063/1.1709888
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/38/5?ver=pdfcov
Published by the AIP Publishing

Communications

The Tunneling Time of an Electron

K. K. THORNBER,* THOMAS C. MCGILL,† AND C. A. MEAD
California Institute of Technology, Pasadena, California
(Received 21 July 1966; in final form 11 January 1967)

THERE is a widely spread misconception regarding the physical significance of the various tunneling times currently used to describe metal–insulator–metal tunneling phenomena. Using quantum mechanics, the *transition* time of an electron tunneling from a state on one side of the barrier to a state on the other side can be determined. This time is the period of interaction between the electron and the barrier, since before and after the transition, the electron is in a quantum state of one of the metals. Furthermore, the RC time constant of the sandwich-like device and the electron transition or interaction time are equivalent representations of the same physical parameter.[1–3] But none of these times is the quasiclassical "transmission time" analyzed by Hartman,[4] which has become widely accepted as the electron–barrier interaction time, although this was clearly not his intention. In this communication we wish to point out that it is the (quantum mechanical) transition time which is the characteristic time of tunneling phenomena.

The transition time, and hence the interaction time, can itself be quite different depending on the particular tunneling phenomena, as we point out below.[5] The actual duration of this interaction is important, for besides being the RC time constant, it can tell us under what circumstances the free-space, the optical, and the low-frequency (static) dielectric constant should be used in representing the potential of the barrier region.

Let us first recall that tunneling is a phenomenon of quantum mechanical origin and has no classical counterpart. Consequently, we cannot expect to describe all aspects of tunneling completely, unambiguously by a classical model. However, this has not prevented the formulation and adoption of such models. These originated partially from misinterpreting comments such as those of Bohm.[6] He has pointed out that the mathematical form of the transition time for tunneling out of a potential well may be factored into the product of the classical period of the electron in the well multiplied by the reciprocal of the quantum mechanical transmission coefficient through the barrier.[7] (It must be stressed at this point that the transmission coefficient, the probability of transmission per encounter, is in no way related to Hartman's[4] transmission time. The transmission coefficient is derived from the solution of a time-*in*dependent Schrödinger equation.) Hence it appeared that a ballistic model of tunneling could be conceived, and indeed one was. But as will be evident from what follows, such a model is only satisfactory in certain particular cases. In this model the electron is pictured as bouncing back and forth in the well until upon one encounter it penetrates the metal–insulator interface and traverses the insulating barrier during what is referred to as the transmission time. In this picture the interaction between the electron and the dielectric appears to occur only during this transmission time.

But quantum mechanically something quite different is going on. The electron is in fact interacting with the barrier during the entire transition. Pictured in a quantum mechanical manner we note that the wave function of the electron has a finite tail in the barrier region throughout the transition. If it did not, the matrix element of the transition would vanish and there would be no transition. Also $|\psi(t)|^2$, where $\psi(t)$ is the electronic wave function, is exponentially small in the barrier throughout the transition. Most of this electronic (probability) cloud is divided between the two metals, decreasing on one side of the barrier and increasing on the other side during the transition. The transition time or interaction time of the electron with the with the barrier is significantly different from the quasiclassical transmission time: the former is strongly dependent on barrier thickness, as is seen experimentally[3a] in the RC time constant and theoretically, as developed below; the latter is theoretically essentially independent of thickness[4] and, experimentally, it corresponds to no measurable phenomenon. Viewed in this manner, the conclusion that the electron transition time is the physically meaningful and characteristic time of metal–insulator–metal devices is apparent. This is not to say that the classical model should be completely abandoned, for it may be used to determine the tunneling current,[8] because the result of such a calculation is identical to the quantum mechanical solution.[9] This is also not to say that transmission time is never physically meaningful for all barrier problems. An examination of three different tunneling situations will make this evident.

We begin with a tunneling phenomenon with a particularly well-defined transition time—the quantum mechanical resonance[10] between the ground state (or any bound state) of well A and that of well B in Fig. 1. Restricting ourselves to these two levels, knowing initially that the electron is in well A (it is injected say at time $t=0$), it is straightforward to determine that in a time $\tau=\pi/M_{12}$ the electron will have traversed the barrier and appear well-defined in well B. M_{12} is just the transition matrix element of Bardeen[11] which in our simple case is proportional to $\exp[-\bar{k}(0)x]$ (see Ref. 2), as computed by Harrison.[9] Clearly the electron must interact with the barrier the entire time τ, for any attempt to follow the motion in between will start the experiment all over again.[12] As a specific example, using a similar model, Feynman[13] has calculated the Josephson[14] tunneling current between two superconducting metals. The problem may be treated as a two-level system because in the superconducting state nearly all the electrons are in the ground state.

The phenomenon of tunneling between two normal metals must be treated differently. Near a given energy there are a large number of states from which an electron can tunnel, but we can never determine the actual pair of different initial and final states. Such problems are quite common. They owe their solution to Fermi's golden rule, which gives the transition probability per unit time for electron transit, the reciprocal of which is the mean time per transition, or just the mean time that the electron interacts with the barrier. Using the WKB approximation and this independent particle model, Harrison[9] found a transition probability per unit time proportional to $\exp[-2\bar{k}(0)x]$. Reciprocating gives a transit or interaction time[15] proportional to $\exp[+2\bar{k}(0)x]$.

As a third example, Hartman[4] studies the tunneling of a wave packet through a potential barrier. Here a well-defined electron packet is incident on the barrier and the time it spends interacting with the barrier, i.e., until the reflected and transmitted packets are formed, is calculated. But it is clear that to determine whether or not the electron was actually transmitted in this single encounter requires actually determining the initial and final state of the electron. For a barrier in free space using ballistic electrons this is easily performed. In the two metals as a tunneling device, however, this cannot be done because we cannot specify *a priori* which electron will tunnel, or *a posteriori* which electron has tunneled, or that it tunnels on one encounter with the barrier. In tunneling devices the electron is injected in some arbitrary manner into the metal on one side of the barrier. The eigenstates of the metal by itself have become virtual (metastable) states when the work function barrier was replaced by a tunnel barrier (C in Fig. 1). Bohm[6] has discussed a similar case fully.

One consequence of interaction times longer than 10^{-12} sec is that the ions in the barrier can follow the electron through the transition. Therefore, the static dielectric constant of the barrier should be used to compute the strength of the image potential.[16] An elegant model[17] devised by Harrison of tunneling phenomena involving polarizable media also gives this result. This means for AlN and Al_2O_3, for example, that the correction to the potential of the oxide due to the image charge in the metal is reduced by

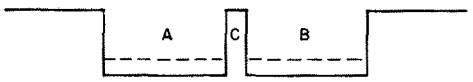

FIG. 1. A simple two-level quantum mechanical resonance system.

a factor of about two from that obtained with the optical dielectric constant, and by a factor of about eight from that using the dielectric constant of free space.[18,19] The latter corresponds to transit times shorter than 10^{-16} sec.

We hasten to point out that for tunneling near the top of the barrier, $\bar{k}(0)$ and x can be quite small, and one must check to see which dielectric constant is appropriate: $\tau > 10^{-12}$ sec, static; 10^{-13} sec $> \tau > 10^{-15}$ sec, optical; etc. For Schottky emission one uses the optical dielectric constant because the transit times clearly fall into this latter range.[20-22]

The rigorous treatment of tunneling between normal metals involves taking account of the many-body states in the metals, and the lattice-modified electron propagators in the insulator. But because of the accuracy obtainable with the free electron theory of metals and the effective-mass theories of bandgap materials, and the characterization of the transition time, such refinements do not yet appear to be necessary.

We wish to thank Walter A. Harrison, E. O. Kane, and R. Stratton for helpful and stimulating discussions; and William E. Spicer and John L. Moll for much help in clarifying the work. Support from the Air Force Office of Scientific Research, the National Academy of Sciences, National Research Council and the Office of Naval Research is gratefully acknowledged.

* National Science Foundation Cooperative Graduate Fellow. Present address: Stanford Electronics Laboratory, Stanford University, Stanford, California.

† National Science Foundation Graduate Fellow.

[1] Charge the diode to a voltage V_0; open the circuit to measure the voltage decay $V(t)$ across the diode: $V(t)=V_0\exp(-t/\tau)$, where $\tau \equiv RC$. Physically this decay results from the transition of the excess electrons N from one metal to the other. But $dN/dt=-Nr$, or $N(t)=N_0\exp(-rt)$, where r is the electron transition rate. Clearly $r=1/\tau$ follows.

[2] Recently Lewicki and Mead (Ref. 3a) have reported that the RC time constant for tunneling current in Al-AlN-Mg and Al-AlN-Al sandwiches varies as $A\exp[-2\bar{k}(0)x]$ from 10^{-5} sec to 1 sec with insulator thicknesses of 32Å–47Å. (Ref. 3b). Extrapolating to zero thickness, RC approaches 4.10^{-15} sec, and for a 10-Å layer, RC is about 10^{-12} sec.

[3] (a) G. Lewicki and C. A. Mead, Phys. Rev. Letters **16**, 939 (1966); (b) $\bar{k}(0)$ is the average value of k encountered in the tunneling path corresponding to an incident electron with zero transverse momentum and energy equal to the metal Fermi energy, and x is the thickness of the insulator.

[4] Thomas E. Hartman, J. Appl. Phys. **33**, 3427 (1962). Transmission times of 10^{-16} sec are found, essentially independent of thickness.

[5] For example, the interference of electron "waves" is lost when we are able to specify an intermediate state. Great care must also be taken to determine what the initial and final states of the system truly represent.

[6] David Bohm, *Quantum Theory* (Prentice-Hall Co., Englewood Cliffs, New Jersey, 1951), pp. 290–291, especially Eq. (74a).

[7] The probability per unit time of transition through the barrier equals the number of encounters per unit time multiplied by the transmission coefficient (probability of transmission per encounter), when expressed using reciprocals.

[8] R. Stratton, J. Phys. Chem. Solids **23**, 1177 (1962).

[9] Walter A. Harrison, Phys. Rev. **123**, 85 (1961).

[10] Ref. 6, p. 467 ff.

[11] John Bardeen, Phys. Rev. Letters **6**, 85 (1961).

[12] An extreme case is α decay in which the lifetime (or the reciprocal of the transition probability) is very long. And if one attempts to determine whether an atom has decayed or not (by weighing it for example) the counting of the time for decay must begin anew. Thus the particle may be viewed as interacting with the barrier for a long time indeed.

[13] R. P. Feynman, *Lectures on Physics*, (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1965), Vol. III, Chap. 21, p. 9.

[14] B. D. Josephson, Phys. Letters **1**, 251 (1962).

[15] With some liberty of expression, the difference between transition rates proportional to $\exp[-k(0)x]$ and $\exp[-2k(0)x]$ (which are just the transition matrix element M and its square) is much like the difference between coherent and incoherent probabilities—$(1+M)^2 \approx 1+2M$ and $1+M^2$, respectively.

[16] The image-charge approximation to the contribution to the potential in the dielectric barrier region due to the induced charges in the neighboring metals is only valid if the electronic wave function in the dielectric ψ_d changes slowly with respect to the dielectric relaxation time of the metal (about 10^{-18} sec). (We are indebted to Richard P. Feynman for pointing this out.) This permits an adiabatic adjustment of the metallic charge during transition. The amount by which this image potential is screened by the dielectric depends further on how rapidly ψ_d changes with respect to the motion of the electronic and ionic charges. If ψ_d changes during a period longer than 10^{-12} sec, it is clear that the ions as well as the electrons of the dielectric can respond. And if this is the case, the static dielectric constant must be used.

[17] Walter A. Harrison (private communication).

[18] In recent tunneling work (Refs. 1, 19) it has been possible to neglect the image force in analyzing the results. However, had the full image force due to a free-space dielectric constant entered, this would have been detected. In fact in some cases there were indications that the contribution might be less than that predicted by using the optical value. With more detailed work in this direction, one should be able to see the effect of the image force and determine its magnitude.

[19] G. Lewicki, Ph.D. thesis (unpublished).

[20] G. M. Sze, C. R. Crowell, and D. Kahng, J. Appl. Phys. **35**, 2534 (1964).

[21] C. A. Mead, B. E. Deal, and E. H. Snow, Appl. Phys. Letters **9**, 53 (1966).

[22] An 0.01-eV electron will cover 100 Å in about 10^{-13} sec.

InSb and InAs Lattice Change during Zinc Diffusion

V. L. LAMBERT
Avco Electronics Division, Cincinnati, Ohio
(Received 7 November 1966; in final form 6 January 1967)

THIS note reports a change in lattice parameters of InSb when subjected to zinc closed-tube diffusions. The quartz diffusion ampoules used were 18-cm long, 1-cm diameter and evacuated to the 10^{-5} Torr range. The x-ray measurements were made by using Cu $K\alpha$ lines on a flat back reflection Polaroid camera. {553} reflections were used for InAs and {644} reflections for InSb. The precision is believed to be ± 0.002 Å on InSb and ± 0.003 Å on InAs for lattice changes. The values given here are to three figures only, and are believed to be significant. The reproducibility of results for different diffusion runs is good. Typical results for InSb are given in Table I.

TABLE I. Typical results for InSb diffusions.

Condition	A_0 (Å)	Remarks
1	6.48	Undiffused crystal from Cominco, n-type, tellurium-doped (10^{15} cm^{-3})
2	6.48	Cadmium diffusion, 64 h, 400°C, 8–10 mg, 90% In–10% Cd spheres
3	6.48	Zinc diffusion, 3 h, 400°C, 6 mg Zn
4	6.47	Zinc diffusion, 4 h, 410°C, 250 mg Zn
5	6.47	Zinc diffusion, 16 h, 410°C, 2 mg Zn

It is to be noted that a definite lattice change occurs in the case of zinc diffusion. This same trend was noticed in the case of InAs, but the results are not as conclusive. In conditions 3, 4, and 5, the temperature difference is not believed to be important. The lattice change appears to result when a large zinc source is used, or alternatively, when a small source and a long diffusion time is used.

No significant change was noticed in the case of cadmium diffusion in either InSb or InAs. The result given in condition 2 of the table is for a cadmium–indium alloy used as a source. When pure cadmium is used, care must be taken or the polished surface of the InSb will be damaged by alloying. However, no lattice change was found even when the cadmium source was large enough to cause some surface damage.