Frequency selectivity in laterally coupled semiconductor laser arrays

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A longitudinal-mode analysis of a system of laterally coupled waveguided resonators is presented in the coupled-mode approximation. It is shown that variations in the mirror reflectivity of the individual channels result in coupling between the supermodes of the structure. This may lead to mode suppression by modulation of the threshold gain of different Fabry–Perot modes.

Phase-locked semiconductor laser arrays have been the subject of widespread interest during the past few years.[1–6] The theoretical studies of laterally coupled semiconductor lasers included their electrical properties,[7] the effect of the gain distribution in the coupling mechanism,[8] and the description of the optical field in terms of the multichannel waveguide modes (supermodes).[5] In most of these studies, the laser modes were implicitly assumed to be identical to the propagating waveguide modes.[5,6] In a system of coupled waveguides with a *uniform reflectivity*, the proper oscillating modes are, indeed, the longitudinal (Fabry–Perot) resonances of the propagating supermodes of the structure. However, with the development of etching techniques that result in high-quality laser mirrors,[9] the reflectivity of the mirror at every channel can be varied in a controlled way by standard photolithography masking and selective coating. Consequently, an interesting question and one with possible practical ramifications is: "What are the resonator modes in the case of nonuniform facet reflectivity?" i.e., when each channel may have an arbitrary reflectivity. In such a case, it is clear that (1) the supermodes are not the oscillating modes and (2) strong selectivity among the oscillating modes is expected. To demonstrate these points in the simplest possible case, consider the two-channel resonator of Fig. 1, with propagation constants $\beta_1 = \beta_2$ and with reflectivities $r_1 = 0$, $r_2 = 1$. Imagine a fundamental (even) supermode incident from the left upon the right facet. Since no reflection takes place in channel 1, the reflected mode consists of an equal superposition of the even and the odd supermodes. It follows that the nonuniform reflectivity of the right facet results in coupling of the supermodes. To illustrate the second point, assume also that the difference in propagation delay is $(\sigma_1 - \sigma_2)L = \pi$ ($\sigma_{1,2}$ are the propagation constants of the even and odd supermodes, respectively). In this case an equal superposition of the even and odd modes can propagate such that at the left facet all the field is in the upper channel while at the right facet it is all in the lower channel. Such a superposition is self-reproducing after each round trip and is then a proper mode of the oscillator. What is more, it completely avoids the lossy ($r_1 = 0$) mirror, thus minimizing reflection losses. It will thus possess a higher Q than other possible modes, which are less successful in avoiding the lossy mirror.

In what follows, we will present an analysis of the general case of a two-channel resonator with arbitrary channel reflectivities in one facet. The generalization to the N-channel cases ($N > 2$) and arbitrary channel reflectivities on both sides is straightforward, but the basic properties of such systems can be visualized by studying the filter properties of the two-channel case.

The geometry of the laterally coupled two-channel laser is depicted schematically in Fig. 1. Each channel is assumed to support a single lateral mode, and the coupling coefficients k_{12} and k_{21} are assumed to be equal ($k_{12} = k_{21} = k$). The total electric field in the combined waveguide can be described approximately either in terms of the individual channel modes $E_i(x)$:

$$E(x, z) = a_1(z)\mathbf{E}_1(x) + a_2(z)\mathbf{E}_2(x), \tag{1}$$

where $a_i(z)$ ($i = 1, 2$) are the mode amplitudes, or in terms of the supermodes $w_i(x)$:

$$E(x, z) = b_1(z)w_1(x) + b_2(z)w_2(x). \tag{2}$$

The supermode propagation constants σ_i can be written (in the coupled-mode approximation) in terms of β_i as[10]

$$\sigma_{1,2} = \overline{\beta} \mp s, \tag{3}$$

where

$$\overline{\beta} = \frac{\beta_1 + \beta_2}{2},$$

$$s = (\Delta^2 + k^2)^{1/2},$$

$$\Delta = \frac{\beta_2 - \beta_1}{2}.$$

The descriptions of the field given by Eqs. (1) and (2) are equivalent. The field $E(x, z)$ can thus be described by either of the two vectors

$$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \qquad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \tag{4}$$

the former being the channel-mode representation, and the latter, the supermode representation. These are related through the transformation

$$A = VB, \tag{5}$$

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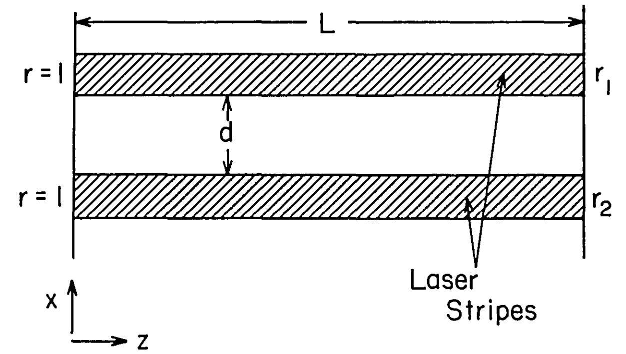

Fig. 1. Schematic description of the laterally coupled waveguided lasers.

where

$$V = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and $\theta = \tan^{-1}[(s - \Delta/s + \Delta)^{1/2}]$. We note that any operator matrix can can be easily transformed from one representation to the other [e.g., if matrix M operates on vector A in the channel-mode representation, the corresponding matrix in the supermode representation is $(V^{-1}MV)$, operating on B]. We find it easier to express the propagation of the field in the supermode representation since the propagation matrix is diagonal:

$$P(z) = \begin{bmatrix} e^{-i\sigma_1 z} & 0 \\ 0 & e^{-i\sigma_2 z} \end{bmatrix} . \tag{6}$$

The mirror reflectivity operator, on the other hand, is diagonal in the channel-mode representation:

$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} , \tag{7}$$

with r_1 and r_2 real numbers.[11] The diagonal form of the matrix R (in the channel representation) is due to the fact that there is no interchannel scattering at the mirror plane. (This assumption as well as the assumption that k_{12}, k_{21}, and r_i are real numbers is not essential to the derivation, and it can be relaxed, at the cost of complicating the algebra somewhat.) The oscillating mode of the resonator is that field configuration B (or A) that reproduces itself after one round trip, thus satisfying

$$\mathcal{L} \times B = B, \tag{8}$$

where $\mathcal{L} = P(L)V^{-1}RVP(L)$ is the round-trip matrix in the supermode representation. Equation (8) has nontrivial solutions for B if and only if

$$\det|\mathcal{L} - I| = 0. \tag{9}$$

Equation (9) is the secular equation of the resonant cavity. It can be regarded as two nonlinear equations involving three unknowns, g_1, g_2, and k_0, where $g_i = \mathrm{Im}(\beta_i)$ is the gain in channel i and $k_0 = 2\pi/\lambda$ is the wave number of the resonant field. In the case of equal reflectivity ($r_1 = r_2 = r$), it is easily verified that $V^{-1}RV$ is also diagonal, and, by using Eqs. (5)–(7) in Eq. (8), we get

$$[r\exp(-i2\sigma_1 L) - 1][r\exp(-i2\sigma_2 L) - 1] = 0. \tag{10}$$

Setting each bracket to zero in Eq. (10) determines the well-known lasing condition for each supermode:

$$\mathrm{Re}(\bar{\beta} \mp s)L = 2N\pi, \tag{11a}$$

$$r\exp[\mathrm{Im}(\beta \pm s)L] = 1. \tag{11b}$$

This shows that, in the case of uniform reflectivity, there is no built-in frequency selectivity in the resonant system, apart from the regular Fabry–Perot condition for the two uncoupled supermodes.

In the case of unequal reflectivities ($r_1 \neq r_2$), we proceed as before, and, with the aid of Eq. (3), we obtain

$$\left\{\frac{\exp(2i\sigma_1 L)}{r\left[1 - \delta\left(\frac{\Delta}{s}\right)\right]} - 1\right\}\left\{\frac{\exp(2i\sigma_2 L)}{r\left[1 + \delta\left(\frac{\Delta}{s}\right)\right]} - 1\right\} = K_s, \tag{12}$$

where

$$r = \frac{r_1 + r_2}{2}, \qquad \delta = \frac{r_1 - r_2}{2r}, \qquad K_s = \frac{\delta^2\left(\frac{k}{s}\right)^2}{1 - \delta^2\left(\frac{\Delta}{s}\right)^2} .$$

We note immediately that if the reflectivity in the channel with a higher propagation constant is higher, the even supermode has a higher effective round-trip reflectivity. In order to treat Eq. (12), we define

$$x_i = \frac{\exp[-\mathrm{Im}(\sigma_i)L]}{r\left[1 \mp \delta\left(\frac{\Delta}{s}\right)\right]}, \tag{13a}$$

$$\varphi_i = 2\pi N_i - 2 \times \mathrm{Re}\{\sigma_i\} \times L, \tag{13b}$$

where $i = 1, 2$ and the upper sign in Eq. (13a) corresponds to $i = 1$. These quantities represent the inverse modal gain and phase shift for the coupled-supermodes round trip. With these values, Eq. (12) can be written as

$$[x_1\exp(\varphi_1) - 1][x_2\exp(\varphi_2) - 1] = K_s. \tag{14}$$

The supermode coupling factor K_s is related to the coupling coefficient k and to the mismatch parameter Δ. In the weak-coupling range $|k/\Delta| \ll 1$, and assuming that $\delta < 1$, we get $K_s \ll 1$, which means that the supermodes are weakly coupled. This results in resonances close to those given by Eq. (11) but with the introduction of a small modulating term on the right-hand side of Eq. (11b) that depends on the dispersion curves in the coupled waveguides. In the range of nearly matched waveguides, $|\Delta/k| \ll 1$ and $K_s \sim \delta^2$. In this case the supermodes are strongly coupled, and frequency selectivity is obtained provided that $\mathrm{Re}\{2s/\bar{\beta}\}$ changes noticeably (by dispersion) between the different Fabry–Perot modes. As an example, Fig. 2 shows the

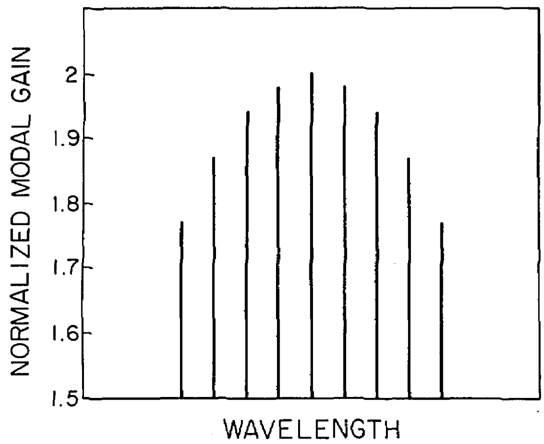

Fig. 2. Cavity resonances for the coupled supermodes with normalized modal gain $x_1 = x_2$, coupling factor $K_s = 1$, $L = 300\ \mu m$, $s = 25\ cm^{-1}$, $(\Delta s)_{FP} = 1\ cm^{-1}$, and $\overline{\beta} = 2.5 \times 10^5\ cm^{-1}$.

solution of Eq. (14) for the case of equal gain, $x_1 = x_2$, assuming that $K_s = 1$ and that the other parameters have typical values of GaAs/GaAlAs laser arrays [a change in s over different Fabry–Perot modes caused by changes in $\Delta\beta$ by dispersion, $(\Delta s)_{FP} \sim 1\ cm^{-1}$, was assumed]. The analysis presented here may suggest that the frequency selectivity and tunability reported in the past[12] are related to this effect. Finally, it is worthwhile to mention that a phase shift in one of the mirrors can also be implemented experimentally by making the two laser stripes of different lengths. This results in a complex value of K_s and δ, permitting an additional degree of freedom for device design. (A similar device was implemented before.[13] In this case a laser was laterally coupled to a passive waveguide in order to suppress unwanted longitudinal modes.)

In conclusion, we have presented a longitudinal-mode analysis of a system of two laterally coupled waveguided lasers. It is shown that when the mirror reflectivities in the two laser channels are different, the supermodes couple to each other, and frequency selectivity results.

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