Regular Papers

Semiclassical Theory of Noise in Semiconductor Lasers—Part I

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Abstract—**A Van der Pol analysis of laser noise which includes the field intensity dependence of the refractive index is presented. The consequent amplitude phase coupling affects all laser spectra except the power fluctuations spectrum. An analytic expression for the linewidth broadening enhancement due to index variation is given.**

I. Introduction

RECENTLY, the subject of semiconductor laser noise has received considerable attention. The deviation of semiconductor laser noise characteristics from well accepted norms was demonstrated by Fleming and Mooradian, who made the first careful measurements of the field spectrum of a GaAlAs injection laser [1]. They found the spectrum to be Lorentzian and the linewidth to vary inversely with output power, as predicted by the modified Schawlow-Townes formula [2] for linewidth of a fundamentally broadened laser with partial inversion. The coefficient of the power dependence, however, was significantly larger than predicted by the modified Schawlow-Townes result. Henry explained this discrepancy by noting that the situation in a semiconductor laser is that of a detuned oscillator [3], and therefore, as shown by Lax [4], there is a field spectrum linewidth enhancement due to coupling between amplitude and phase fluctuations. Rather than adapting Lax's result to semiconductor lasers, however, Henry provided an elegant less formal model which showed the expected broadening enhancement is a factor $1 + \alpha^2$ larger than the modified Schawlow-Townes linewidth where α is defined as the ratio of real refractive index fluctuation to imaginary refractive index fluctuation in the active region. The key point in Henry's argument is that phase fluctuations can result from index variations during relaxation oscillations after a spontaneous event, as well as being caused directly by spontaneous emission. Early work by Haug and Haken [5] also noted a $1 + \alpha^2$ broadening enhancement, but it was incorrectly assumed by them that this term is negligible in semiconductor lasers.

Theoretical treatment of laser noise can be broadly grouped into those works which eliminate population dependences through an adiabatic approximation and those works which do not. Theories which employ an adiabatic approximation (e.g., Van der Pol treatment) have the advantage of simplicity, but their results are only valid for frequencies near the lasing frequency. A large body of work, including the recent work by Henry, is contained in this catagory. Of the more general theoretical works (i.e., nonadiabatic), those by Lax are most complete, considering the problem both classically and quantum mechanically [4], [6], [7]. An early work by McCumber on intensity fluctuations in lasers, however, makes clearer the distinction between adiabatic theories and nonadiabatic theories [8]. As noted by McCumber, prediction of relaxation resonance phenomena in the intensity spectrum is contingent upon inclusion of population dynamics in the theoretical model. This prediction is the main difference between the two catagories of laser noise theory.

In the more general nonadiabatic catagory, there is, to our knowledge, no treatment which considers the consequences of strong amplitude phase coupling. This now appears to be an important mechanism in determining semiconductor laser noise spectra. For this reason, we present in the companion piece to this paper, "Semiclassical Theory of Noise in Semiconductor Lasers—Part II" [9], a nonadiabatic analysis which includes amplitude phase coupling, i.e., one which includes carrier dynamics as well as the dependence of the refractive index on carrier density. To illustrate the connection between this dynamical analysis and the simplified analysis which results upon adiabatic elimination of the population variable, we first carry out (in Part I) a conventional Van der Pol analysis which includes the effects of amplitude phase coupling. Yariv and Caton have also made such an analysis, but have neglected this effect [10].

The key feature of any Van der Pol analysis is the form used for the active medium polarization. It expresses the polarization as a nonlinear function of the lasing field; the nonlinearity

Manuscript received June 21, 1982; revised January 21, 1983. This work was supported by the Office of Naval Research, the National Science Foundation, and Rockwell International.

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0018-9197/83/0600-1096$01.00

arises from saturation terms in the complex susceptibility. Such a relation is actually an equation of state which describes circumstances when equilibrium between the lasing mode intensity and the population inversion is achieved. Consequently, as noted earlier, the proceeding analysis applies only where such an equilibrium or quasi-equilibrium exists, namely, for that portion of the noise spectrum lying inside the frequency range $|\omega - \omega_m| < 1/\tau_R$ (where τ_R is the relaxation oscillation damping time and ω_m is the lasing frequency). The inclusion of fluctuations in this model is accomplished by adding a Langevin force term to the field equation. A quantum mechanically rigorous formulation of this problem would involve coupling the lasing mode and the lasing transition to the free radiation mode reservoir and the conductance and valence band reservoirs, and performing appropriate reservoir averages to develop mean operator equations of motion. As shown by Lax [11], the second moments of Langevin fluctuation operators can then be calculated using these mean equations of motion. Such a formulation of this problem has been given by Haug and Haken [5]. We take a less formal approach to calculate the Langevin force second moments which closely follows the treatment found in [8] and [10]. Of course, the ultimate justification of this procedure is agreement between the results of more rigorous treatments and those we obtain.

Noise fluctuations in the field will be separated into perturbations in field amplitude and field phase. Such a form for the field facilitates identification of three types of laser spectra, as discussed by Yariv and Caton [10]. These spectra are illustrated in Fig. 1. In Section II, the equations governing noise fluctuations are derived and solved. The Langevin force second moment will be found in Section III, and then used in Section IV to express the three laser spectra in terms of known quantities. In Section V, α as defined in this paper will be shown to be equivalent to the α defined by Henry, and an analytic expression will be given for α.

II. Noise Equations

The starting point of this analysis is Maxwell's equations:

$$\nabla \times \vec{E}(\vec{r}, t) = -\kappa \partial_t \vec{H}(\vec{r}, t) \tag{1}$$

$$\nabla \times \vec{H}(\vec{r}, t) = (\sigma + \epsilon \partial_t) \vec{E}(\vec{r}, t) + \partial_t [\vec{P}(\vec{r}, t) + \vec{p}(\vec{r}, t)]. \tag{2}$$

κ is the magnetic permeability, σ is the medium conductivity (later to become a generalized loss term), ϵ is the nonresonant dielectric constant, $\vec{P}(\vec{r}, t)$ is the component of polarization causing stimulated transitions, and $\vec{p}(\vec{r}, t)$ is a fluctuation term which will be treated as a random component of polarization causing spontaneous transitions. Solving for $\vec{E}(\vec{r}, t)$ yields

$$[\nabla^2 - \kappa\sigma\partial_t - \kappa\epsilon\partial_t^2]\, \vec{E}(\vec{r}, t) = \kappa \partial_t^2 [\vec{P}(\vec{r}, t) + \vec{p}(\vec{r}, t)] \tag{3}$$

where $\nabla(\nabla \cdot \vec{E}(\vec{r}, t)) \approx 0$ has been assumed. It is now assumed that a complete set of orthonormal spatial modes exist. These modes are solutions of the homogeneous wave equation without loss and are used to expand the electric field and the components of polarization as follows:

$$\vec{E}(\vec{r}, t) = \sum_n E_n(t)\, \vec{e}_n(\vec{r}) \tag{4}$$

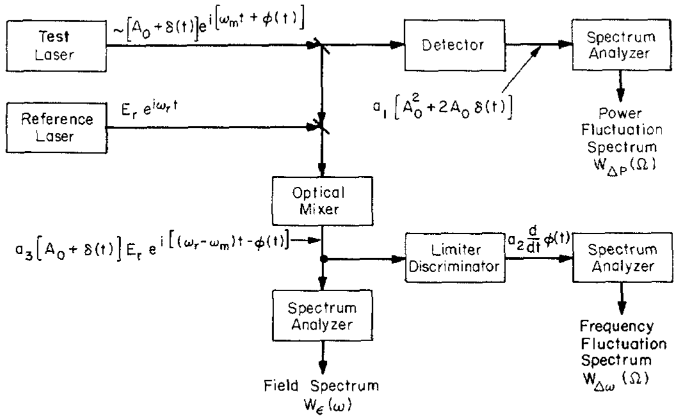

Fig. 1. Typical arrangement for measuring the various noise spectra of a laser oscillator.

$$\vec{P}(\vec{r}, t) = \sum_n P_n(t)\, \vec{e}_n(\vec{r}) \tag{5}$$

$$\vec{p}(\vec{r}, t) = \sum_n p_n(t)\, \vec{e}_n(\vec{r}). \tag{6}$$

These expansions are substituted into (3) and then the inner product of (3) and $\vec{e}_n(\vec{r})$ is taken (combined vector and integral inner product)

$$\ddot{E}_n + \frac{1}{\tau_p}\dot{E}_n + \omega_n^2 E_n = -\frac{1}{\epsilon_o \mu^2}(\ddot{P}_n + \ddot{p}_n) \tag{7}$$

where $\tau_p \equiv \epsilon/\sigma$ is the photon lifetime, μ is the nonresonant index, ω_n is the resonant frequency of the nth mode. In general, the projection $P_n(t)$ of the polarization onto the nth mode is a superposition of the field projections $E_n(t)$. Under the assumption of single mode oscillation, however, $P_n(t)$ can be expressed as [12]

$$P_n = \epsilon_o(\chi^{(1)} + \chi^{(3)}|E_n|^2) E_n. \tag{8}$$

The imaginary part of $\chi^{(3)} (\chi_i^{(3)})$ represents gain saturation and the real part $(\chi_r^{(3)})$ leads to an intensity dependent index of refraction. Expressing P_n in this manner neglects nonsynchronous terms. The field is now expressed in terms of slowly varying amplitude and phase perturbations.

$$E_n = [A_o + \delta(t)]\, e^{i[\omega_m t + \varphi(t)]} \tag{9}$$

where ω_m is the lasing frequency. (8) and (9) are substituted into (7). Terms in $\ddot{\delta}(t)$ and $\ddot{\varphi}(t)$ are neglected since $\delta(t)$ and $\varphi(t)$ vary slowly in comparison with the lasing frequency; products of small quantities are also neglected. Simplification of (7) yields

$$2i\omega_m(\dot{\delta} + iA_o\dot{\varphi}) + \frac{2A_o^2\chi^{(3)}}{\mu^2}(2i\omega_m\dot{\delta} - \omega_m^2\delta) + \left[\omega_n^2 - \omega_m^2 + i\frac{\omega_m}{\tau_p} - (\chi^{(1)} + A_o^2\chi^{(3)})\frac{\omega_m^2}{\mu^2}\right] A_o = \Delta e^{-i\varphi} \tag{10}$$

$$\Delta e^{i\omega_m t} \equiv - \frac{1}{\epsilon_o \mu^2} \ddot{p}_n \tag{11}$$

where, for convenience, Δ has been defined as the slowly varying complex amplitude of the second derivative in time of the fluctuation $p_n(t)$. Δ will be modeled as a Langevin noise source. This choice is motivated by the classical result that total power from a radiating dipole is proportional to $\ddot{p}_n^2$ [13] and by the spontaneous emission spectral density being essentially "white" in comparison to the lasing linewidth. Langevin forces are delta correlated in time, and therefore, time dependent phase factors have no effect on their second moments [4]. Since the spectra calculated in Section IV will contain only second moments of Δ, we are justified in neglecting the phase factor $e^{-i\varphi}$ in (10) in what follows.

The amplitude and phase fluctuations as well as the Langevin noise term are assumed to have zero mean values. Therefore, a time average of (10) reduces it to equations which establish the operating point power and frequency

$$A_o^2 = \frac{\chi_i^{(1)} - \dfrac{\mu^2}{\omega_m \tau_p}}{-\chi_i^{(3)}} \tag{12}$$

$$\omega_m^2 = \frac{\omega_n^2}{1 + \dfrac{1}{\mu^2}(\chi_r^{(1)} + A_o^2 \chi_r^{(3)})}. \tag{13}$$

The equations governing δ and φ are contained in the time varying part of (10). Separating this part into its real and imaginary components yields

$$\left(1 + \frac{2A_o^2 \chi_r^{(3)}}{\mu^2}\right)\dot{\delta} - \frac{A_o^2 \omega_m \chi_i^{(3)}}{\mu^2}\delta = \frac{\Delta_i}{2\omega_m} \tag{14}$$

$$A_o \dot{\varphi} + \frac{2\chi_i^{(3)} A_o^2}{\mu^2}\dot{\delta} + \frac{\omega_m \chi_r^{(3)} A_o^2}{\mu^2}\delta = -\frac{\Delta_r}{2\omega_m} \tag{15}$$

where

$$\Delta \equiv \Delta_r + \iota \Delta_i. \tag{16}$$

The coefficient of $\dot{\delta}$ in (14) will be approximated as unity throughout the remainder of this paper. The neglected term is approximately equal to the unsaturated resonant index, and as such is typically several orders smaller than unity. These equations relate the Langevin noise force to the field amplitude and phase fluctuations. Equation (14) has the same form as the equation describing the velocity of a particle undergoing Brownian motion. In the case of Brownian motion, the coefficient of δ arises from the viscous force which acts on the particle. In a laser, this viscous force takes the form of gain saturation and restrains amplitude fluctuations. There is no such equivalent viscous force for phase fluctuations [see (15)] because the saturation process is dependent upon field intensity, not field amplitude [see (8)]. The third term in (15) represents amplitude fluctuations (δ) driving a term related to the refractive index of the active medium. This causes phase fluctuations in addition to those arising directly from spontaneous emission which involves Δ_r. In fact, it is this added phase fluctuation which causes broadening of the field spectrum linewidth beyond that predicted by the modified Schawlow-Townes expression.

To solve (14) and (15), δ, φ, Δ_r, and Δ_i are taken to be sample functions, and therefore to be deterministic. This allows standard solution techniques to be applied to this system of differential equations. After the δ and the φ sample functions have been determined in terms of Δ_i and Δ_r, autocorrelation functions can be formed by ensemble averaging the product of two sample functions shifted in time. This procedure is first applied to (14) to determine the autocorrelation of $\delta(t)$. Solving by Laplace transform yields

$$\delta(t) = \frac{1}{2\omega_m}\int_0^t \Delta_i(\tau) e^{-\omega_1(t-\tau)}\, d\tau \tag{17}$$

$$\omega_1 \equiv -\frac{\chi_i^{(3)} A_o^2}{\mu^2}\omega_m \tag{18}$$

where $\delta(0) = 0$ has been assumed. The autocorrelation of $\delta(t)$ is now formed using (17):

$$\langle \delta(t+\tau)\,\delta(t)\rangle = \frac{1}{4\omega_m^2}\int_0^{t+\tau} d\lambda_1 \int_0^t d\lambda_2 \cdot \langle \Delta_i(\lambda_1)\,\Delta_i(\lambda_2)\rangle\, e^{-\omega_1(t+\tau-\lambda_1)}\, e^{-\omega_1(t-\lambda_2)}. \tag{19}$$

$\langle\ \rangle$ denotes ensemble averaging. Correlations of the real and imaginary parts of the Langevin force are given by

$$\langle \Delta_i(\lambda_1)\,\Delta_i(\lambda_2)\rangle = \langle \Delta_r(\lambda_1)\,\Delta_r(\lambda_2)\rangle = WD(\lambda_1 - \lambda_2) \tag{20}$$

$$\langle \Delta_i(\lambda_1)\,\Delta_r(\lambda_2)\rangle = 0 \tag{21}$$

where $D(x)$ is the delta function. Equation (21) results from spontaneous emission having no preferred phase (i.e., its phase is described by a uniform distribution function). Substitution of (20) into (19) and subsequent simplification yields

$$\langle \delta(t+\tau)\,\delta(t)\rangle = \frac{W}{8\omega_m^2 \omega_1} e^{-\omega_1|\tau|}(1 - e^{-2\omega_1 t}). \tag{22}$$

As t becomes large, the history of the system is forgotten (i.e., the initial conditions become unimportant) and the autocorrelation function achieves its stationary form

$$\langle \delta(t+\tau)\,\delta(t)\rangle = \frac{W}{8\omega_m^2 \omega_1} e^{-\omega_1|\tau|}. \tag{23}$$

The phase fluctuation autocorrelation is found by substituting the δ sample function solution into (15) and repeating the procedure described above.

$$\langle \varphi(t_1)\varphi(t_2)\rangle = \frac{W}{4\omega_m^2 A_o^2}(1+\alpha^2)\begin{cases} t_1 & t_1 < t_2 \\ t_2 & t_2 < t_1 \end{cases} \tag{24}$$

$$\alpha \equiv \frac{\chi_r^{(3)}}{\chi_i^{(3)}}. \tag{25}$$

In (24), exponentially decaying terms have been neglected in comparison to the term linear in time. The instantaneous frequency fluctuation is defined as $\Delta\omega \equiv \dot{\varphi}$. Its autocorrelation can be found using (24)

$$\langle \Delta\omega(t_1)\, \Delta\omega(t_2)\rangle = \frac{\partial^2}{\partial t_1 \partial t_2} \langle \varphi(t_1)\, \varphi(t_2)\rangle = \frac{W}{4\omega_m^2 A_o^2} (1 + \alpha^2)\, D(t_2 - t_1). \quad (26)$$

Since the amplitude fluctuation and frequency fluctuation have stationary autocorrelations, corresponding spectral density functions can be found using the Wiener-Khintchine relation. The results are

$$W_\delta(\Omega) = \frac{W}{4\omega_m^2(\Omega^2 + \omega_1^2)} \quad (27)$$

$$W_{\Delta\omega}(\Omega) = \frac{W}{4\omega_m^2 A_o^2} (1 + \alpha^2) \quad (28)$$

where $W_{\Delta\omega}(\Omega)$ is a "white" noise spectrum due to our treatment of the spontaneous emission spectral density as "white" in comparison to the lasing linewidth.

III. The Langevin Force Autocorrelation Function

For the purposes of this calculation, we will distinguish between energy added to the lasing mode by spontaneous emission and by stimulated emission. The size of each component for steady-state operation is determined by the balance between dissipation and emission. If Ψ_s is the steady-state spontaneous energy in the mode, then Ψ_s/τ_p is the spontaneous dissipation rate. Balancing this with the spontaneous emission rate yields

$$\Psi_s = \hbar\omega_m E_{cv} \tau_p \quad (29)$$

where E_{cv} is the number of spontaneously emitted photons/s (spontaneous emission rate) into the mode.

For operating points well above threshold, the spontaneous emission rate is clamped, and therefore by (29), the spontaneous energy in the mode is clamped at a constant Ψ_s. In our model, this value is fixed for any operating point equal to or above threshold. Therefore, to relate Ψ_s to W (the second moment of the Langevin force), it is convenient to consider the threshold form of (7),

$$\ddot{E} + \frac{1}{\tau_p}\dot{E} + \omega_m^2 E = (\Delta_r + i\Delta_i)\, e^{i\omega_m t}. \quad (30)$$

This is (7) without the nonlinear saturation terms. These terms are negligible by assumption. $\omega_m \approx \omega_n$ has also been used in (30). Solving for $E(t)$ by Laplace transform yields

$$E(t) = \frac{1}{\omega_m} \int_0^t (\Delta_r(\tau) + i\Delta_i(\tau)) \cdot \exp\left[i\omega_m t - \frac{1}{2\tau_p}(t - \tau)\right] \sin\,[\omega_m(t - \tau)]\; d\tau \quad (31)$$

where $\omega_m^2 \gg 1/4\tau_p^2$ has been assumed. Using the procedure described in Section II, $\langle E(t)\, E^*(t)\rangle$ is formed using (31), then simplified using (20) and (21). Allowing $t \to \infty$, the steady-state spontaneous energy is given by

$$\Psi_s = \frac{\epsilon V}{2} \langle E(t)\, E^*(t)\rangle = \frac{W\epsilon V \tau_p}{4\omega_m^2} \quad (32)$$

$$V \equiv \int |\vec{e}_n(\vec{r})|^2\, d\vec{r} \quad (33)$$

where V is the mode volume. Equating expressions (29) and (32) for Ψ_s, then solving for W yields

$$W = \frac{4\hbar\omega_m^3 E_{cv}}{\epsilon V} \quad (34)$$

which, when used in (23)-(28), completely specifies them.

IV. Laser Spectra

A. Power Fluctuations Spectrum

This spectrum describes the output power fluctuations ($\Delta P = P - \bar{P}$). The spectral density of ΔP is given by

$$W_{\Delta P}(\Omega) = \int_{-\infty}^{+\infty} \langle \Delta P(t + \tau)\, \Delta P(t)\rangle\; e^{-i\Omega\tau}\, d\tau = A_o^2 \epsilon^2 V^2 \gamma^2 W_\delta(\Omega) = \frac{\hbar\omega_m \epsilon V A_o^2 \gamma^2 E_{cv}}{\Omega^2 + \omega_1^2} \quad (35)$$

where γ is the facet loss rate. This result is identical to that obtained by Yariv and Caton in an analysis which neglects amplitude phase coupling [10]. $W_{\Delta P}(\Omega)$ will be more closely scrutinized in Part II.

B. Frequency Fluctuations Spectrum

The spectral density of the instantaneous frequency fluctuation is given by (28). Using the Langevin normalization (34) in this expression yields

$$W_{\Delta\omega}(\Omega) = \frac{\hbar\omega_m E_{cv} v_g \ln(1/R)}{4P_o L} (1 + \alpha^2) \quad (36)$$

where P_o is the output power per facet, v_g is the group velocity, L is the cavity length, and R is the facet reflectivity. The quantity α^2 is a frequency fluctuation enhancement caused by the amplitude phase coupling.

C. Field Spectrum

Before the field spectrum can be calculated, the field autocorrelation function must be obtained. Amplitude fluctuations make a negligible contribution to the field autocorrelation function due to the damping effect of gain saturation. Therefore, considering only phase fluctuations, the field autocorrelation is given by

$$\langle E(t + \tau)\, E^*(t)\rangle = A_o^2 \langle e^{i[\varphi(t+\tau) - \varphi(t)]}\rangle = A_o^2 \int_{-\infty}^{+\infty} f(\Delta\varphi)\, e^{i\Delta\varphi}\, d\Delta\varphi \quad (37)$$

$$\Delta\varphi \equiv \varphi(t + \tau) - \varphi(t) \quad (38)$$

where $f(\Delta\varphi)$ is the phase difference distribution function. The last integral in (37) can be recognized as the characteristic function of $f(\Delta\varphi)$ with unity argument. Since $f(\Delta\varphi)$ is a Gaussian distribution function [4], the field autocorrelation function is given by

$$\langle E(t+\tau)E^*(t)\rangle = A_o^2 e^{-1/2\langle[\Delta\varphi]^2\rangle} \tag{39}$$

Substitution of (24) and (34) into (39) and subsequent application of the Wiener-Khintchine relation yields the field spectrum

$$W_\epsilon(\omega) = \frac{A_o^2\Delta\omega}{(\omega-\omega_m)^2 + \left(\frac{\Delta\omega}{2}\right)^2} \tag{40}$$

$$\Delta\omega \equiv \frac{\hbar\omega_m E_{cv} v_g \ln(1/R)}{4P_o L}(1+\alpha^2). \tag{41}$$

With $\alpha = 0$, (41) is the modified Schawlow-Townes linewidth formula.

V. Discussion

To demonstrate that the field spectrum linewidth broadening factor discussed by Henry is exactly the broadening factor $1+\alpha^2$ calculated above, α, as defined in this paper, will be shown to be equivalent to the α defined by Henry as

$$\alpha \equiv \frac{\Delta n'}{\Delta n''} \tag{42}$$

where $\Delta n'$ and $\Delta n''$ are carrier induced fluctuations of the real and imaginary parts of the refractive index.

Below threshold (13) reduces to the following:

$$\omega_m - \omega_n = -\frac{\omega_n \chi_r^{(1)}}{2\mu^2} \tag{43}$$

where $\chi_r^{(1)} << \mu^2$ has been used. Changes in $\omega_m - \omega_n$ below threshold result primarily from changes in the carrier density with pumping. Above threshold $\omega_m - \omega_n$ is clamped at $(\omega_m - \omega_n)_t$ where

$$(\omega_m - \omega_n)_t = -\frac{\omega_n(\chi_r^{(1)})_t}{2\mu^2} = -\frac{\omega_n}{2\mu^2}(\chi_r^{(1)} + A_o^2\chi_r^{(3)}). \tag{44}$$

Substituting for A_o^2 using (12) and rearranging terms yields

$$\alpha = \frac{(\chi_r^{(1)})_t - \chi_r^{(1)}}{(\chi_i^{(1)})_t - \chi_i^{(1)}} \tag{45}$$

$$(\chi_i^{(1)})_t \equiv \frac{\mu^2}{\omega_m \tau_p} \tag{46}$$

where the definition of $(\chi_i^{(1)})_t$ follows directly from (12). $\chi_r^{(1)}$ and $\chi_i^{(1)}$ can be viewed as functions of pumping or, alternately, as functions of the unsaturated carrier density. Assuming the latter, (45) can be simplified by Taylor expansion of $\chi_r^{(1)}$ and $\chi_i^{(1)}$ to first-order about threshold

$$\alpha \approx \frac{\frac{d\chi_r^{(1)}}{dn}\Delta n}{\frac{d\chi_i^{(1)}}{dn}\Delta n} = \frac{\Delta n'}{\Delta n''}. \tag{47}$$

This equation shows that Henry's broadening term α and the broadening term α defined in this paper are equivalent.

We will now use (45) to find an analytic expression for α in terms of known quantities. The simple rate equation for photon density is given by

$$\frac{dP}{dt} = A(n - n_o)P - \frac{P}{\tau_p} \tag{48}$$

where P is the photon density, n is the carrier density, A is the gain coefficient, n_o is the carrier density necessary to achieve transparency, and τ_p is the photon lifetime. The steady-state form of (48) is a relation between photon lifetime and gain, which upon substitution into (46) yields

$$(\chi_i^{(1)})_t = \frac{A\mu^2}{\omega_m}(n_t - n_o) \tag{49}$$

where n_t is the threshold carrier concentration. This functional dependence between $\chi_i^{(1)}$ and n should also apply away from threshold. Therefore, (49) can be used to express the denominator in (47) in terms of known parameters. Equation (43) is used to calculate the numerator in (47).

$$\frac{d\chi_r^{(1)}}{dn} = -2\mu^2\frac{1}{\omega_n}\frac{d\omega_m}{dn} = 2\mu^2\frac{1}{\mu}\frac{d\mu}{dn}. \tag{50}$$

The second equality follows from considering frequency shifts due to index variation in a simple Fabry-Perot resonator. Substitution of (50) and the general form of (49) into (47) results in the following expression for α:

$$\alpha = \frac{2\omega_m}{A\mu}\frac{d\mu}{dn}. \tag{51}$$

For example, $d\mu/dn = 1.2 \times 10^{-20}$ cm^3 has been deduced from measurements of dynamic wavelength shift due to modulation in the InGaAsP system [14]. Using $\omega_m = 1.2 \times 10^{15}$ rad/s, $\mu = 3.5$, and $A = 10^{-6}$ cm$^3 \cdot$ s^{-1} [14] yields $\alpha = 8.2$. The error in this value is at least 50 percent due to the uncertainty in A.

VI. Conclusion

In this, the first of two papers on semiconductor laser noise, we have presented a Van der Pol analysis which includes amplitude phase coupling through an intensity dependent index of refraction. This coupling resulted in a field spectrum linewidth enhancement of $1+\alpha^2$ and an increase in the magnitude of the frequency fluctuation spectrum by the same factor. Amplitude fluctuations and the corresponding power fluctuations spectrum were, not surprisingly, unaffected by this coupling. The quantity α, which in the present analysis is the ratio of the real and imaginary parts of the third-order susceptibility, has been shown to be equivalent to Henry's broadening term α. An analytic expression for α in terms of easily measured quantities was also derived.

The Van der Pol approach to laser noise assumes gain and index to be instantaneous functions of intensity. This neglects time constants associated with the saturation process, and therefore confines the theoretical results to a relatively narrow frequency range around the lasing frequency. In Part II, a more general treatment will be presented which includes saturation time constants. It will introduce the carrier density as a dynamic variable in addition to the field amplitude and phase, and therefore allow effects due to driven and spontaneous carrier fluctuation and relaxation resonance to be included. The main effect of that analysis will be to predict a resonance in the power fluctuations spectrum and frequency fluctuations spectrum, and fine structure in the field spectrum.

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Kerry Vahala, photograph and biography not available at the time of publication.

Amnon Yariv (S'56–M'59–F'70) was born in Tel Aviv, Israel, on April 13, 1930. He received the B.S., M.S., and Ph.D. degrees, all in electrical engineering, from the University of California, Berkeley, in 1954, 1956, and 1958, respectively.

He was employed by Bell Laboratories, Murray Hill, NJ, in 1959, during the early stages of the laser effort. He then joined the staff of the California Institute of Technology, Pasadena, in 1964, as an Associate Professor of Electrical Engineering, and became a Professor in 1966. He also took part in the discovery of a number of early solid-state laser systems, the formulation of the theory of parametric quantum noise and the prediction of parametric fluorescence, the invention of the technique of mode-locked ultrashort-pulse lasers and FM lasers, the introduction of GaAs and CdTe as IR electrooptic and window materials, and the proposal and demonstration of semiconductor based integrated optics technology. His present research efforts are in the areas of nonlinear optics, IR electrooptical materials, recombination mechanisms in semiconductors, and thin-film optics. He authored or coauthored numerous papers in professional journals as well as two books: *Quantum Electronics* (New York: Wiley, 1967) and *Introduction to Optical Electronics* (New York: Holt. Rinehart and Winston, 1971). He is also an Associate Editor of the IEEE Journal of Quantum Electronics and *Optics Communications*.

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