LONGITUDINAL WAVE-PROPAGATION MODES ALONG A POSITIVE COLUMN

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Measurements have been made of the propagation of waves along the positive column of mercury discharge with characteristic velocities of the order of the speed of sound for the electron gas, $(3KT_e/m_e)^{1/2}$. These waves have been excited by appropriate couplers, in both dipole and rotationally symmetric modes.

The dipole propagation is most closely related to the Tonks-Dattner (T-D) resonances¹ which are, in fact, the cutoff conditions of these modes (wavelength measured along the column equals infinity). The most complete analysis of the T-D resonances has been made by Parker, Nickel, and Gould.² This macroscopic analysis, which utilizes a quasistatic approximation, $E = -\text{grad}\phi$, includes a pressure term of the form $\text{grad}[(3KT/m)n_e]$ in the force equation. The calculations, for which a previously calculated electron density profile³ is assumed, were carried out on a computer, and results show a standing wave distribution for the perturbed electron density. The wavelength involved corresponds roughly to that calculated from homogeneous plasma theory for longitudinal waves, namely $k^2 = (2\pi/\lambda)^2 = (\omega^2 - \omega_p^2)(3KT/m_e)^{-1}$.

One then can think of the T-D resonances as a longitudinal wave, traveling in a radial direction, which is reflected by the plasma-glass boundary and, if the plasma is dense enough, by the over-dense region in the center of the discharge tube. The boundary conditions at the surface of the discharge tube are such that the necessary external field is produced. It is only a short step to consider now the case where these longitudinal waves do not travel exclusively in the radial direction but have a component of propagation in the direction of the axis of the discharge. This is a qualitative description of the phenomena responsible for the measured propagation in the direction of the discharge-tube axis. Since the propagation velocities are comparable with the drift velocity of electrons from cathode to anode, it is

necessary to consider the effect of the drift. There exist, of course, bands of propagation in which there are many variations (number of nodes) in the radial direction. This is analogous to higher modes in wave-guide propagation. A similar picture is applicable to the rotationally symmetric mode.

The first experiments designed to excite propagation were performed using the fringing field of coaxial re-entrant cavities such as those often used with klystrons. These cavities have rotational symmetry and therefore do not excite the dipole mode. One such cavity was connected to a generator of approximately 0.1 W power and the other to a receiver. Both cavities fitted closely on (outside) the glass discharge tube and could be moved axially. For reason of technical convenience the early experiments were conducted leaving the measuring frequency constant. The current of the hot-cathode mercury-vapor discharge was varied, thus changing the electron density (over a wide range electron density is closely proportional to anode current). The amplitude of the received signal showed many distinct transmission bands as shown in Fig. 1(a).

Investigation of the transmission bands with a phase-sensitive receiver disclosed a large variation in the wavelength λ_z (measured in axial direction) within each transmission band. At that end of the band corresponding to the smaller value of $\omega^2/\bar{\omega}_p^2$ the wavelength became quite large, generally the order of the value in vacuum, whereas at the other end of the band, the wavelengths were smallest, about $\frac{1}{2}$ cm. The termination of the band at one end seems quite clearly to be due to a cutoff condition and, at the other end, probably due to Landau damping. The very dominating transmission of Fig. 1(a) which exists at values of $\omega^2/\bar{\omega}_p^2$ smaller than 0.2 is that which one also predicts from the analysis of a cold bounded plasma.

FIG. 1. Measuring frequency 1050 Mc/sec. (a) and (b) Logarithm of the received voltage obtained by transmitting between two couplers, as a function of $\bar{\omega}_p^2$. The scales for these curves are approximately the same. The transmission in (b) at $\omega^2/\bar{\omega}_p^2$ smaller than 0.15 is thought to be due to the rotationally symmetric mode which is coupled to because of influence of the earth's magnetic field. (c) Transmitted power using a wave guide oriented so that the E field in the guide is not parallel to the earth's magnetic field.

A second series of measurements has been conducted to investigate the dipole mode of propagation. "Dipole" refers here, of course, to one azimuthal variation. Using appropriate

couplers, also external to the plasma, similar experiments were carried out. The transmission results [(Fig. 1(b))] were, in general, similar to those for the rotationally symmetric mode with the exception of that band occurring at the lowest $\omega^2/\bar{\omega}_p^2$ values. The lowest band is the transmission mode one obtains by solution of the equation $\text{div grad}\{\varphi\epsilon_0[1-(\omega_p^2/\omega^2)]\}=0$ when electron temperature effects are neglected.⁵ This equation possesses a solution for a bounded plasma and also shows the presence of a backward wave. The inclusion of a finite electron temperature modifies this solution only slightly in the region of large wavelengths. The resonance predicted by Tonks¹ is, of course, that which one obtains here for $1/\lambda_z=0$. The higher order bands are not included in the analysis of a "cold" plasma.

The first two bands of the dipole mode were investigated with a phase-sensitive receiver. The measuring frequency here was varied between 150 and 400 Mc/sec, whereas the discharge current was held constant at 14.5 mA which corresponds to a value of $\bar{\omega}_p^2$ equal to 4.5×10^{18} . $\bar{\omega}_p^2$ is the average, over the cross section of the plasma column, of the square of the plasma frequency, and was measured by the microwave-cavity resonant-frequency-shift method.⁶ The results are shown in Fig. 2; as ordinate the wave number $\beta=2\pi/\lambda_z$ is normalized to the radius of the plasma column $r_w=0.8$ cm. Correction for the drift velocity of the electrons (from cathode to anode) which was determined by comparing $\omega-\beta$ curves measured by propagating in anode-cathode and cathode-anode directions has already been included in the ex-

FIG. 2. Theoretical and experimental $\omega-\beta$ curves for the first and second bands of dipole propagation. Correction for the drift velocity of the electrons has been made in the experimental curve. B stands for observed backward wave propagation.

perimental curve of Fig. 2.

The theoretical dispersion curve shown in Fig. 2 was calculated through modification of the computer program originally used for the cutoff ($\beta=0$) case.² The dispersion curve must be calculated for each zero-order radial electron-density profile of interest. The electron-density profiles³ can be characterized by one parameter, $r_w^2/\bar{\lambda}_D^2$, where $\bar{\lambda}_D^2$ is the average, over plasma cross section, of the square of the Debye length. The presence of a region of backward wave propagation for small βr_w is purely a boundary-condition problem. The numerical calculations show a region of backward propagation for the two lowest bands but none for the higher bands. The phase and group velocities in the forward wave-propagation region for large βr_w tend toward the speed of sound in the electron gas, $(3KT/m)^{1/2}$. In particular, $\Delta\omega^2/\Delta\beta^2$ approaches $3KT/m$ in this limit. It has not been possible to extend the measurements to such short wavelengths that $\omega^2 \sim \beta^2 \times 3KT/m$ because of difficulties in coupling to and out of the wave and because the Landau damping would then be extremely large.

A very interesting effect is shown in Fig. 1(c). The ordinate shows the energy transmitted through a wave guide which is mounted in the usual manner for T-D measurements, namely the discharge tube is inserted through two holes in the small sides of the guide. The axis of the discharge tube is then perpendicular to both the Poynting vector and the electrical field, TE_{10} mode assumed. Superimposed on the usual absorption pattern, the sharp minima, there is a second series of minima A' , B' , and C' . If the wave guide and discharge tube are oriented so that the electrical field in the wave guide is parallel to the earth's magnetic field, this second series disappears, while orientation of E perpendicular to B_{earth} yields maximum absorption. Further transmission tests using the various couplers have shown this secondary spectrum to be caused by energy absorbed by the rotationally symmetric mode; see also the cor-

relation to the modes A , B , and C . The magnetic field of the earth also produces strong coupling to the rotational mode when dipole exciters are used for transmission along the tube axis. This effect is most astounding when one considers the value of this field, about 0.5 G corresponding to an electron gyration frequency of 5 Mc/sec, and the values of measuring frequencies used, up to 4000 Mc/sec. The absorption of D' is thought to be caused by the first quadrupole mode.

A further interesting experiment influenced by these rotationally symmetric and dipole modes is that which uses high-frequency probes. Rectified current maxima were found at these particular electron densities, as well as strong transmission in these modes using a probe as a transmitter coupler. This is reasonable as these modes represent some of the possible oscillation patterns for a plasma in this simple configuration. This result is not in conflict with present thinking on this subject but rather emphasizes that the entire plasma should be considered, and not just that which surrounds the probe, in evaluating such experiments.

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