exact measure of the highest negative peak in the history of the input signal.)

If it is necessary to return the circuit to zero volts periodically, a simple transistor switch may be inserted across the storage capacitor. Fortunately, the drop in the switch is unimportant because the output is taken from the emitter follower.

Although the circuit shown in Fig. 2 has a gain of -1 (that is, the output is equal, but opposite, to the peak negative value of the input signal), it is not essential to the operation of the peak holding circuit that the gain be -1. In fact, considerable gain may be achieved at the expense of a corresponding degradation of the output signal.

The authors have built peak holding circuits employing operational amplifiers in the laboratory and with them have easily obtained the results mentioned above.

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A Note on the Ricatti Differential Equation*

The general Ricatti differential equation

$$\frac{dy}{dx} + A(x)y^2 + B(x)y = C(x) \tag{1}$$

is important in many problems of physical and technical research. It cannot be solved by exact integration, therefore an appropriate representation of the solution by a convergent sum is desirable.

Eq. (1) can be transformed by

$$y = \frac{a_1(x)}{a_2(x)} \tag{2}$$

into a system of two homogeneous differential equations of first order

$$\frac{d\mathbf{a}}{dx} = \mathbf{K}(x)\mathbf{a} \tag{3}$$

with

$$\mathbf{a}(x) = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \qquad \mathbf{K}(x) = \begin{bmatrix} 0 & C \\ A & B \end{bmatrix}.$$

The normalized solution of (3) is the matrizant[1] ($\mathbf{I}$ = unit matrix)

$$\mathbf{\Omega}_{x_0}^{x} = \begin{bmatrix} \omega_1 & \omega_2 \\ \omega_3 & \omega_4 \end{bmatrix} = \mathbf{I} + \int_{x_0}^{x} \mathbf{K}(\xi_1)d\xi_1 + \int_{x_0}^{x}\int_{x_0}^{\xi_1} \mathbf{K}(\xi_1)\mathbf{K}(\xi_2)d\xi_1 d\xi_2 + \cdots. \tag{4}$$

The solution of (1) is therefore

$$y(x) = \frac{y(x_0)\omega_1 + \omega_2}{y(x_0)\omega_3 + \omega_4}. \tag{5}$$

If A and B are constants then ω_i can be written in the form

$$\omega_1 = 1 + \sum_{n=1}^{\infty} A^n \int_{x_0}^{x}\int_{x_0}^{\xi_1} \cdots \cdot \int_{x_0}^{\xi_{2n-1}} C(\xi_1)C(\xi_3)\cdots C(\xi_{2n-1}) \cdot \exp\left[B\sum_{m=2}^{2n}(-1)^m \Delta\xi_m\right] d\xi_1 \cdots d\xi_{2n}$$

$$\omega_2 = \int_{x_0}^{x} C(\xi)e^{B\Delta\xi}d\xi + \sum_{n=1}^{\infty} A^n \int_{x_0}^{x}\int_{x_0}^{\xi_1} \cdots \cdot \int_{x_0}^{\xi_{2n}} C(\xi_1)C(\xi_3)\cdots C(\xi_{2n+1}) \cdot \exp\left[B\sum_{m=2}^{2n}(-1)^m \Delta\xi_m\right] d\xi_1 \cdots d\xi_{2n+1} \tag{6}$$

$$\omega_3 = \frac{1}{C(x)}\frac{d\omega_1}{dx}; \qquad \omega_4 = \frac{1}{C(x)}\frac{d\omega_2}{dx};$$

$$\Delta\xi_m = \xi_m - x_0.$$

The nth terms of the sums in (6) have the upper limits

$$|(\omega_1)_n| \leqq \left| e^{B\Delta x} A^n C_{\max}^n \frac{\Delta x^{2n}}{2n!} \right|;$$

$$|(\omega_2)_n| \leqq \left| e^{B\Delta x} A^n C_{\max}^{n+1} \frac{\Delta x^{2n+1}}{(2n+1)!} \right|. \tag{7}$$

$C_{\max}$ is the maximal value of C in the interval $\Delta x = x - x_0$.

Eq. (7) shows that it is always possible to get a quick convergence of the sums in (6) by choice of a sufficiently small Δx. Moreover the product rule of the matrizant[1] and the form of (5), respectively, allow the composition of the general solution of (1) by any number of interval steps Δx.

Applications of this method for the solution of the ionospheric recombination equation and the calculation of ionospheric reflection coefficients may be found in Volland.[2,3]

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Metal Contact Double Injection in GaAs*

Recent studies on the GaAs injection laser[1,2] have indicated certain problems associated with the localization of the region of population inversion to the immediate vicinity of the p-n junction. It has also been pointed out that very heavy doping is necessary for laser action.[3] However, in heavily doped material the absorption of light is large, raising the threshold current for laser action. These problems can be overcome simultaneously by using a metal-GaAs-metal structure operating in the double injection mode described by Lampert.[4] A metal of low work function is used to inject electrons at one surface and one of high work function to inject holes at the opposite surface. When sufficient double injection occurs to neutralize the bulk between the electrodes, the voltage required to sustain the current drops to a low value.

Experiments were conducted on semi-insulating GaAs [111] faces. The smooth etching side was prepared with a Bromine-methanol polish and coated with barium by vacuum evaporation. The opposite surface was mechanically polished, cleaned quickly in CP-4 etch and coated with platinum. Volt-ampere curves were taken at room temperature with a single sweep display.[5] A typical I-V trace is shown in Fig. 1. The negative resistance was not obtained with the reverse polarity.

It has been shown[6] that the barrier heights of contacts to GaAs made with metals of widely different work functions are very nearly the same. This effect has been attributed to surface states. It appears that in the present experiments the application of bias can supply on a dynamic basis the charge needed to shift the quasi-fermi levels to the band edges. In any case, the present author has been able to find no reasonable explanation for the observed negative resistance characteristics other than double injection.

In addition to avoiding highly doped material in or near the optically active region, double injection structures allow the injection mechanisms to be separated from the recombination and radiative mech-

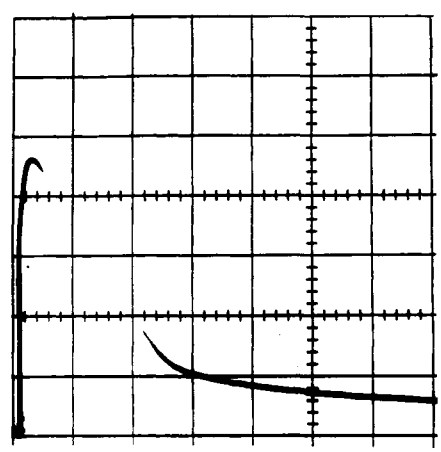

Fig. 1—I-V curve trace of Ba-GaAs-Pt double injection diode. Vertical scale 10 v/cm. Horizontal scale 20 ma/cm. Diode was 0.01-inch diameter and approximately 0.005-inch thick.

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anisms. In view of the present uncertainties in the details of the processes, this possibility should be most useful. It should be noted that a *p-i-n* structure[7] could also be used in the double injection mode. The concepts involved, of course, are not limited to GaAs.

The author wishes to express his appreciation to W. G. Spitzer for many discussions, H. M. Simpson for aid in the fabrication and R. Willardson and W. Allred for supplying the GaAs.

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Improvement of Energy Detection as a Result of Channel Subdivision*

It has been shown that an energy detector can be calibrated to read likelihood ratio; thus, for that class of signals identified by their energy content alone, the energy detector becomes an optimum receiver.[1] Suppose that x_i represents the ith sample of $x(t)=n(t)+s(t)$ where $n(t)$ is band limited to B cps white Gaussian noise and $s(t)$ is a signal characterized by its energy alone. If T_r equals the length of record, then M, the number of independent samples, is

$$M = 2T_rB. \tag{1}$$

Let y be the output of a linear integrator (adder) which follows a full-wave square law device,

$$y = \sum_{i=1}^{M} x_i^2. \tag{2}$$

The distribution of x_i^2 will depend upon whether or not a signal is present, *i.e.*, which of the possible states of nature, ω_0 or ω_1 is true. In either case, the mean and variance of the x_i^2 will be finite and are given by[2]

	ω_0	ω_1
$E[x_i^2]$	σ_n^2	$\frac{A^2}{2}+\sigma_n^2$
$\sigma^2[x_i^2]$	$2\sigma_n^4$	$2\left(\frac{A^4}{16}+A^2\sigma_n^2+\sigma_n^4\right)$,

where the signal is of the form

$$S(t) = \sqrt{2}\,A(\sin \omega t + \theta)$$

with

$$p_\omega(\omega) = \frac{1}{2B}, \qquad \text{for } \omega_A < \omega < \omega_A + B$$
$$= 0 \text{ elsewhere}$$

and

$$p_\theta(\theta) = \frac{1}{2\pi}, \qquad \text{for } 0 < \theta < 2\pi$$
$$= 0 \text{ elsewhere.}$$

The probability density of y, given that ω_0 is true, $p[y/\omega_0]$ has a chi-squared distribution with M degrees of freedom; the probability density of y, given that ω_1 is true, $p[y/\omega_1]$, has not been determined. Application of the central limit theorem allows us to approximate both of these conditional probability functions by the normal probability density having mean and variance given by

	Mean	*Variance*
$p[y/\omega_0]$	$M\sigma_n^2$	$2M\sigma_n^4$
$p[y/\omega_1]$	$M\left[\frac{A^2}{2}+\sigma_n^2\right]$	$2M\left[\frac{A^4}{16}+A^2\sigma_n^2+\sigma_n^4\right]$.

If we make the substitutions,

$$h^2 = \left[\frac{A^4}{16} + A^2\sigma_n^2 + \sigma_n^4\right]$$
$$S = \frac{A^2}{2}$$
$$N = \sigma_n^2, \tag{4}$$

the likelihood ratio expressed in terms of y becomes

$$(y) = \frac{N}{h}\,\frac{\exp\left[-\frac{1}{2}\,\frac{[y-M(S+N)]^2}{2Mh^2}\right]}{\exp\left[-\frac{1}{2}\,\frac{[y-MN]^2}{2MN^2}\right]}. \tag{5}$$

For small signal-to-noise ratios, $h \cong N$, we may approximate $l(y)$ by

$$l(y) \cong \exp\left[-\frac{1}{2}\,\frac{-2MSy-M^2(S^2+2SN)}{2MN}\right]. \tag{6}$$

We wish to compare the likelihood ratio $l(y)$ to a threshold β. For the purposes of the present discussion we choose $\beta=1$. This corresponds to the maximum likelihood criteria, minimizes the total error probability and yields a particularly simple result. We say that a signal is present if

$$y \geq MS\left(1/2 + \frac{N}{S}\right). \tag{7}$$

The relation between $p(\bar{y}/\omega_0)$, $p(\bar{y}/\omega_1)$ and β for this choice or β is shown in Fig. 1.

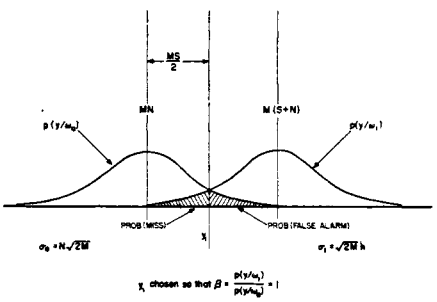

Fig. 1.

If the variable y is normalized with respect to the standard deviation, $\sqrt{2M'N}$, *i.e.*, $\hat{y}=y/\sqrt{2M'N}$ and $k\hat{y}_1$ set equal to $MS/2$, k provides an index of the reliability of detection where

$$k = \frac{\frac{MS}{2}}{\sqrt{2M'N}} = \frac{S}{N}\frac{\sqrt{M}}{2\sqrt{2}}. \tag{8}$$

Solving for T_r using (8) and (1),

$$T_r = \frac{4k^2}{B\left(\frac{S}{N}\right)^2} = \frac{4k^2B}{\left(\frac{S}{N_0}\right)^2}. \tag{9}$$

Suppose now that the original bandwidth B is divided into L equal-width subchannels making the individual channel bandwidth B/L cps and signal-to-noise ratio $L\ S/N$, (9) becomes

$$T_r = \frac{4k^2}{BL\left(\frac{S}{N}\right)^2}. \tag{10}$$

For a fixed value of T_r, the value of k for each of the L subchannels is

$$k_L = k_1\sqrt{L} \tag{11}$$

and

$$P_B \triangleq P\{\text{error in a bank of } L \text{ sub-channels}\}$$
$$= 1 - (1 - P_L)^L$$
$$\cong LP_L \cong \frac{\sqrt{L}}{\sqrt{2k_1}}\exp\left[-\frac{Lk_1^2}{2}\right]. \tag{12}$$

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* Received December 21, 1962.

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The Influence of Terrain Shielding on Radio Wave Propagation at 8000 Mc*

Terrain shielding measurements were made at the MIT Lincoln Laboratory, Millstone Hill, Mass. communication terminal during June 20–22, 1962. The transmitting terminal consisted of a 60-foot diameter X-band parabolic antenna mounted on an azimuth elevation mount. A crystal controlled transmitter was employed.

A truck was used to make measurements in the field. The receiving antenna con-

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