Q0957+561: DETAILED MODELS OF THE GRAVITATIONAL LENS EFFECT

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ABSTRACT

Further observations and theoretical work on Q0957+561 are presented. The galaxy responsible for causing multiple gravitational lens images of Q0957+561 has been reobserved spectroscopically, and we give a more accurate redshift $z_d = 0.36$. In 1980 January, component B had brightened by 0.3 mag and was equal to A. We have further analyzed our CCD data on the surrounding cluster of galaxies, and we give a list of 180 objects (146 galaxies, 32 stars, and 2 QSO images) to a limiting magnitude $r = 24.5$. The data indicate that the center of luminosity of the cluster is about 23" west of the QSO images.

Detailed models of the gravitational lens effect have been computed. These models can explain all optical and radio data for the system. They predict time delays of up to 5 years between variations in the A and B images (with A preceding B). We believe the third image is rather faint and located close to the center of the lens galaxy. It is possible that the third image contributes to the observed galaxy core.

We discuss the possibility that this system may be used to measure H_0 . Given two time delays among the three images, such measurements may be successful if no further complications in the imaging arise.

Subject headings: cosmology — galaxies: redshifts — galaxies: clusters of — gravitation — quasars — relativity

I. INTRODUCTION

The pair of QSOs Q0957+561 A, B are now recognized to be gravitational images of a single object (Young *et al.* 1980, hereafter Paper I) following the discovery of the object in the Jodrell bank radio survey by Walsh and the subsequent optical identification of Q0957+561 by Walsh, Carswell, and Weymann (1979). In Paper I it was argued that the QSO at a redshift $z_s = 1.41$ is multiply imaged by a bright cluster galaxy ($z_d = 0.39 \pm 0.02$) aided by the cluster itself. Some simple models were able to explain the observed arrangement of the galaxy and images.

In this paper we present a more detailed analysis of this system. A study of the distribution of the galaxies to a limiting magnitude $r = 24.5$ on the CCD picture of Q0957+561 enables us to compute more accurate models of the gravitational lens effect. We have also investigated the possibility of using this system to measure the Hubble constant H_0 . Given observations of all three images (brightness, position) and time delays for brightness variations, there is a real possibility that H_0 can be determined.

II. THE GALAXY CLUSTER

a) The Object List

We obtained 6600 seconds of CCD data in the r passband as described in Paper I. This, with seeing of 1".2 FWHM, reaches magnitude $r = 25.5$ and shows ~ 400 objects in 12.3 square arcminutes. Many of these are evidently background objects; the faintest objects do not show as much of a tendency to be concentrated in the cluster core as the bright objects. A subset of 1000 s of the data with the best seeing (0".98) was taken, this having a limiting magnitude $r = 24.5$. A total of 180 objects were visible in this frame, which showed a marked tendency to be concentrated towards the cluster core.

The objects were analyzed with the procedures described in Kristian, Young, and Westphal (1980). Table 1 contains the analyzed object list; we found 146 galaxies, 32 stars, and the two QSO images.

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TABLE 1
Q0957+561 FIELD OBJECT LIST

No	R.A. Arcsec	Dec. Arcsec	Mag r	Class	Re	Sigma	Comments
1	-97.243	-48.747	22.456	G	0.387	0.972	
2	-96.068	29.044	21.376	G	0.544	0.871	
3	-95.626	-54.584	23.190	G	2.650	0.209	
4	-95.160	114.075	23.673	G	0.000	0.993	
5	-94.346	34.152	22.966	G	4.832	1.040	
6	-90.562	36.247	23.496	S			
7	-90.333	-25.239	19.265	G	1.890=Rc	3.260=Rt	0.85=x 4902=Co G3 *
8	-89.739	78.496	23.215	S			
9	-89.200	26.389	21.492	G	0.812	0.815	
10	-87.937	92.990	24.645	S			
11	-87.587	-38.265	23.410	G	0.032	1.672	
12	-86.752	97.499	23.771	S			
13	-86.695	-87.267	21.353	G	0.265	0.473	
14	-85.874	75.104	23.440	S			
15	-84.746	-81.403	22.477	G	1.682	0.565	
16	-84.281	-11.623	23.351	G	1.135	0.577	
17	-80.089	119.990	23.741	G	0.283	0.990	
18	-79.622	-65.255	23.472	G	0.361	1.657	
19	-79.207	44.832	19.975	G	2.604	0.856	G5
20	-78.706	71.164	20.730	G	1.335	0.118	
21	-78.385	78.447	20.302	S			
22	-78.299	97.107	22.945	G	0.082	0.904	
23	-75.879	13.434	19.381	S			
24	-75.332	89.489	23.652	G	0.000	1.344	
25	-74.901	-42.967	22.992	G	1.182	0.740	
26	-74.118	42.957	21.735	G	1.720	1.323	
27	-74.077	48.911	22.650	G	2.032	0.851	
28	-74.062	34.032	22.580	G	0.794	0.953	
29	-71.210	-83.418	24.039	G	0.180	0.561	
30	-70.221	-75.828	23.161	G	0.631	1.332	
31	-67.293	102.212	23.423	G	1.546	0.427	
32	-65.731	-8.243	21.890	G	0.190	2.336	
33	-64.760	-71.907	24.507	G	0.123	1.034	
34	-63.958	95.702	21.512	G	1.001	0.257	
35	-62.679	33.848	20.474	S			
36	-62.584	4.852	20.901	G	3.087	1.629	
37	-62.314	101.720	23.953	G	0.138	0.789	
38	-61.817	-79.233	23.972	G	0.040	0.602	
39	-61.350	117.445	22.338	G	0.018	1.627	
40	-61.016	20.447	21.620	G	0.662	1.297	
41	-60.101	64.190	23.638	G	0.095	1.216	
42	-55.096	-31.047	21.384	G	1.738	0.482	
43	-55.030	5.776	23.524	G	1.233	0.051	
44	-52.984	58.129	22.962	G	0.001	1.085	
45	-52.897	27.819	21.986	G	1.968	0.761	
46	-52.018	-61.438	22.313	G	0.625	0.885	
47	-50.839	95.812	22.882	G	4.093	0.049	
48	-49.061	62.455	22.651	G	0.764	0.829	
49	-49.028	45.449	22.437	G	0.196	1.012	
50	-48.299	-60.188	21.705	G	0.640	0.037	
51	-48.209	81.535	24.028	G	0.000	1.275	
52	-47.898	54.941	22.892	G	0.220	0.379	
53	-44.204	-8.059	21.798	G	1.042	0.510	
54	-44.117	36.164	19.883	G	2.530	0.107	G4
55	-42.010	-5.238	18.908	G	1.700=Rc	7.600=Rt	0.58=x 2004=Co G2 *
56	-41.548	65.372	23.229	G	0.033	1.433	
57	-40.663	17.134	21.219	G	0.795	0.050	
58	-36.844	70.071	24.364	S			
59	-36.392	-57.699	22.235	G	0.700	0.690	
60	-34.286	77.148	23.650	G	0.165	1.087	
61	-33.924	-48.532	21.345	G	1.683	0.943	
62	-33.453	54.336	23.211	G	0.356	0.917	
63	-32.533	-26.260	24.324	G	0.003	1.320	
64	-31.039	21.818	22.911	G	0.000	0.698	
65	-30.849	14.176	22.153	G	1.804	0.190	
66	-28.351	-47.019	22.751	G	0.537	1.348	
67	-27.584	9.852	18.798	S			
68	-24.877	108.177	23.713	G	0.444	1.404	
69	-22.515	-6.898	23.867	S			
70	-21.337	12.240	23.304	S			
71	-18.742	41.704	23.253	G	1.123	1.751	

TABLE 1—Continued

No	R.A. Arcsec	Dec. Arcsec	Mag r	Class	Re	Sigma	Comments
72	-17.342	-84.742	24.713	S			
73	-15.332	-66.596	23.590	G	0.810	0.416	
74	-15.313	95.558	21.329	G	1.835	1.121	
75	-13.201	89.904	21.811	G	2.221	1.210	
76	-11.581	-79.277	23.439	G	0.316	0.963	
77	-10.700	-64.082	22.533	G	0.862	1.070	
78	-8.965	42.610	23.122	G	2.693	0.117	
79	-8.144	116.588	23.593	G	0.819	1.644	
80	-8.067	-86.101	21.491	G	0.912	0.077	
81	-7.984	-5.995	21.911	G	0.406	0.243	
82	-7.570	-34.328	24.076	G	0.718	0.521	
83	-1.227	6.051	17.930	Q			Q0957+561A
84	-0.929	40.012	23.577	G	0.000	0.913	
85	-0.345	-24.729	18.490	S			
86	-0.181	-47.200	22.930	G	0.243	1.290	
87	0.000	0.000	17.591	Q			Q0957+561B
88	0.141	0.994	18.491	G			G1 (No profile fit)
89	0.667	47.608	22.696	G	3.655	0.082	
90	1.317	26.620	20.322	S			
91	1.410	15.695	20.904	S			
92	1.597	-77.139	21.822	G	0.788	0.797	
93	3.306	14.292	22.640	G	0.795	0.041	
94	4.158	-45.442	20.604	G	1.429	0.995	
95	4.965	114.427	23.899	G	0.869	0.561	
96	5.542	-48.315	21.565	G	0.176	0.875	
97	7.510	3.127	20.969	G	0.821	0.423	
98	7.907	106.091	23.062	G	0.957	0.440	
99	11.052	-0.480	23.457	G	0.046	1.052	
100	11.056	-2.557	21.389	G	0.138	1.075	
101	11.446	5.075	21.554	G	0.609	1.032	
102	11.839	42.066	22.349	G	1.887	0.087	
103	12.989	-70.735	23.988	G	0.081	1.296	
104	16.125	-44.536	23.380	S			
105	17.264	98.399	23.047	G	0.693	2.005	
106	18.527	59.442	22.132	G	3.083	0.539	
107	18.573	6.180	22.438	G	2.966	0.744	
108	19.400	80.616	22.365	G	0.010	1.653	
109	19.461	2.702	21.827	G	2.768	0.233	
110	20.595	25.282	21.924	G	1.140	0.947	
111	20.803	100.374	22.046	G	0.425	1.898	
112	21.854	115.338	23.322	G	0.270	1.372	
113	21.923	-33.137	22.786	G	0.769	1.979	
114	22.004	-11.726	22.517	G	1.757	0.078	
115	22.486	-6.143	21.915	G	0.484	0.582	
116	22.713	-8.122	21.263	G	0.645	0.534	
117	22.781	-45.425	23.532	G	0.455	1.039	
118	25.338	14.508	21.191	G	0.707	0.525	
119	25.719	103.546	21.027	G	2.771	1.250	
120	26.512	67.532	20.988	S			
121	26.684	61.987	23.979	G	0.312	0.028	
122	29.228	17.774	20.516	G	1.429	0.327	
123	29.514	-48.384	24.394	G	0.000	0.652	
124	30.036	45.927	21.873	G	0.769	1.058	
125	30.369	-42.924	23.845	G	0.082	0.947	
126	30.911	38.456	21.744	G	1.060	1.493	
127	34.129	114.060	23.143	G	0.647	1.247	
128	34.163	16.848	22.881	G	1.352	0.885	
129	34.579	24.488	22.871	G	1.509	1.033	
130	37.105	30.305	23.295	G	1.318	0.046	
131	41.766	76.845	23.372	S			
132	43.503	-39.131	24.437	G	0.248	0.354	
133	44.705	-0.950	23.549	S			
134	45.482	-72.823	20.782	G	1.429	0.034	
135	45.655	44.083	20.909	G	0.794	1.123	
136	46.010	-16.740	23.408	G	0.931	0.449	
137	46.445	-11.142	23.466	G	0.193	0.646	
138	46.554	62.171	23.925	G	0.447	0.759	
139	46.874	-30.621	24.447	S			
140	47.646	-43.402	24.342	S			
141	48.089	49.288	21.597	G	0.985	0.757	
142	48.150	-39.432	23.775	G	0.028	0.766	
143	51.163	18.456	23.110	G	0.452	0.578	
144	51.387	93.863	20.942	G	0.662	1.046	
145	52.422	-45.720	21.563	S			

TABLE 1—Continued

No	R.A. Arcsec	Dec. Arcsec	Mag r	Class	Re	Sigma	Comments
146	53.275	108.578	21.849	G	3.027	0.302	
147	53.284	18.895	23.794	S			
148	54.454	-66.408	23.566	S			
149	54.526	42.201	22.585	G	0.078	0.802	
150	54.842	-37.391	23.821	G	0.007	1.374	
151	54.852	-41.328	24.949	S			
152	55.567	50.219	22.895	G	0.215	0.529	
153	58.247	-20.970	14.496	S			
154	61.475	65.719	22.191	G	2.806	0.364	
155	65.456	20.617	24.188	G	0.000	0.570	
156	66.729	-6.084	23.238	G	4.185	0.029	
157	68.039	-10.270	21.227	G	0.788	1.647	
158	68.375	-83.427	15.353	S			
159	68.455	-71.664	23.538	G	0.057	0.942	
160	71.097	-16.826	22.955	S			
161	76.120	-3.241	23.861	S			
162	77.862	105.465	21.880	G	1.745	1.026	
163	78.202	13.005	22.836	G	1.210	0.063	
164	79.004	-24.678	23.051	G	0.898	0.708	
165	80.155	-30.343	21.861	G	4.068	0.244	
166	82.883	54.463	24.010	G	0.683	1.107	
167	82.811	-12.190	21.095	G	0.756	0.060	
168	83.760	97.526	23.175	G	0.813	1.261	
169	83.947	-66.105	21.823	G	2.265	0.927	
170	84.072	56.519	23.159	G	0.165	1.335	
171	93.933	24.051	21.917	G	2.205	0.075	
172	96.008	104.461	21.550	G	1.067	0.997	
173	99.808	-70.355	24.420	S			
174	100.116	-84.753	20.901	G	0.737	1.176	
175	100.880	-42.929	19.838	S			
176	101.527	78.851	23.169	G	3.656	0.159	
177	101.982	80.291	23.690	G	2.121	0.024	
178	102.744	-59.767	23.854	G	0.000	1.187	
179	103.623	-65.723	23.342	G	0.258	0.680	
180	105.696	24.953	22.340	G	0.020	1.242	

*King model galaxy fit. See § II(a) for parameter description. All other galaxy fits are modified de Vaucouleurs models. Errors at magnitude 24 are about 0".2 in the (x, y) positions and 0.1 mag in the photometry (standard deviations). Errors for objects at other magnitudes scale with the reciprocal of the flux since the sky background contributes most of the noise for faint objects. The photometry may be afflicted with a zero point error of up to 0.03 mag, and the coordinate scale may be afflicted with a systematic error of up to 0.2%.

In Table 1 we give:

- The object positions in an arcsecond coordinate scale whose axes are oriented north (y) and east (x) and centered on Q0957+561 B.
- The object r -magnitudes. We have obtained $r=17.33$ for Q0957+561 A as in Paper I; all magnitudes, though differentially accurate, may be affected by a zero point error of up to 0.03 mag.
- The object classification (star or galaxy) using the Sebk (1979) algorithm.
- For the galaxies, profile information as follows. The faint galaxy profiles were fitted with a modified de Vaucouleurs model with a luminosity profile $L(r)$, where

$$L(r) = \int_0^\infty (s ds / \sigma_G^2) I_0(rs / \sigma_G^2) \exp[-(r^2 + s^2) / 2\sigma_G^2] \tilde{L}(s), \quad (1a)$$

$$\tilde{L}(s) = (L_0 / r_e^2) (b^8 / \pi 8!) \exp[-b(s/r_e)^{1/4}], \quad (1b)$$

$$b = 7.66924944 \dots \quad (I_0 \text{ is the modified Bessel function}).$$

There are three parameters in this model: the total luminosity L_0 , the effective radius r_e , and the "galaxy sigma" σ_G . The third parameter, σ_G , is not related to the seeing profile despite the appearance in equations (1) of a de Vaucouleurs model convolved with a Gaussian. It is simply a parameter which allowed us to obtain satisfactory model fits to faint galaxy profiles.

Equation (1) was convolved with a seeing profile before being fitted to the azimuthally averaged data. The seeing profile was fitted out to 3" radius (much larger than most of the objects) with the sum of two Gaussians of dispersions $\sigma_1 = 0".40$ and $\sigma_2 = 0".89$.

Galaxies G2 and G3 were not satisfactorily fitted by equations (1) but were better fitted with

$$L(r) = C_0 \exp(-r/r_T) / (1 + r^2/r_c^2)^x, \quad (2)$$

which is an analytic expression for a King model. Here we have four free parameters, viz., C_0 , r_T , r_c , x . Again, equation (2) was convolved with a seeing profile before fitting to the data. Ellipticities of the objects are ignored since radial luminosity profiles were used for the fits.

Table 1 also contains the parameters for the galaxy profile fits. These fits were used to obtain the galaxy magnitudes; they give a high weight to the significant part of the luminosity profile (which was usually fitted out to $4''$ – $5''$).

Some of the classifications were checked by examining the entire 6600 s of CCD data. The classifications made from the 1000 s data set were mostly the same as classifications made from the full data set (which has higher signal-to-noise but lower resolution).

b) The Lens Galaxy

Our data do not give a good position for the brightest cluster galaxy (G1) responsible for the multiple gravitational lens images, since its core is blended with Q0957+561 B. Stockton (1980) has obtained data in much better seeing which shows it to be $0''.99$ north and $0''.14$ east of Q0957+561 B. We have used Stockton's position in Table 1. It is possible, however, that the position of the galaxy core is perturbed by the presence of a third QSO image. We shall discuss this in § IV.

On 1980 January 25.33 UT, new multichannel spectrophotometer observations of the QSO images and G1 were obtained. The resolution was $160 \text{ \AA}$ in the red and $80 \text{ \AA}$ in the blue. An aperture of diameter $7''$ was centered on Q0957+561 B, and a $5''$ aperture on Q0957+561 A. A total integration time of 1800 s was given to each object. The seeing was better than $1''$.

These data are plotted in Figure 1; Q0957+561 A has the same energy distribution as in Paper I, but B has brightened by 0.29 mag at $\lambda 4000$ so that it now is only just fainter than A (by 0.02 mag). B is now brighter than A in the red because of the contribution of G1. The break in the energy distribution of B is visible at $\lambda 5380$, somewhat

FIG. 1.—Multichannel spectroscopy of Q0957+561. Each image was observed for 1800 s. Emission lines in both images at $z_s = 1.41$ are clearly visible. The spectrum of image B has a break at $\lambda 5380$ due to the lens galaxy G1 with a redshift $z_d = 0.36$. This figure may be compared with Fig. 5 in Paper I showing that image B has brightened by 0.3 magnitudes whereas image A is unchanged. A flux of 0.1 mJy corresponds to $AB_s = 18.91$.

FIG. 2.—The energy distribution of the lens galaxy G1. To obtain this we subtracted 0.98 times Q0957+561 A from B to give the displayed data with their 2σ error bars. The break in the energy distribution at $\log_{10} \nu = 14.75$ is clearly visible. On fitting to a standard galaxy energy distribution (solid line) we find a redshift $z_d = 0.36 \pm 0.01$.

bluer than the value derived from the noisier data in Paper I. In order to obtain the spectrum of galaxy G1 alone, we have multiplied the data for A by 0.98 and subtracted them from the data for B. The result is shown in Figure 2, where the break at $\log_{10} \nu = 14.75$ is now quite obvious. The fit shown, of a standard brightest cluster galaxy, gives a redshift $z_d = 0.36 \pm 0.01$. This to be compared with $z_d = 0.39 \pm 0.02$ which we obtained in Paper I.

Infrared observations (Soifer *et al.* 1980) show the galaxy to be highly luminous and are in agreement with G1 being a brightest cluster galaxy at $z_d \sim 0.4$.

In Table 2 we give the equivalent widths and fluxes in the emission lines in Q0957+561 A, B as observed in 1979 May (Paper I) and 1980 January. It might seem reasonable to suppose that with time delays of ~ 5 years between the A and B components (a value which we shall calculate in § IV) that the continua may vary on this time scale but that the emission line strengths will not vary. Consequently we may expect the absolute fluxes in the emission lines to be the same in both sets of observations. This is the case for C IV $\lambda 1549$, but there are marginally significant changes in the absolute flux in the C III] $\lambda 1909$ and Mg II $\lambda 2799$ lines. We feel these differences could well be the compounding of statistical noise and some systematic errors in the spectrophotometry. The brightness ratio of the A and B components is best measured from the ratios of the emission line fluxes, from which we find a brightness ratio of 0.77 ± 0.03 if the absolute fluxes in the emission lines do not vary on a time scale of ~ 5 years.

TABLE 2
EMISSION LINES IN Q0957+561 A, B

LINE	1979 MAY		1980 JANUARY	
	A	B	A	B
Si IV $\lambda 1397$	...	...	15 ± 5	13 ± 5
			1.0 ± 0.3	0.9 ± 0.3
C IV $\lambda 1549$	77 ± 5	74 ± 5	77 ± 5	61 ± 5
	4.6 ± 0.3	3.5 ± 0.2	4.6 ± 0.3	3.6 ± 0.3
C III] $\lambda 1909$	62 ± 4	58 ± 6	51 ± 4	32 ± 4
	2.9 ± 0.2	2.1 ± 0.2	2.3 ± 0.2	1.5 ± 0.2
Mg II $\lambda 2799$...	58 ± 6	45 ± 6	39 ± 7	31 ± 6
	1.8 ± 0.2	1.5 ± 0.2	1.2 ± 0.2	1.1 ± 0.2

NOTE.—For each entry in this table the upper number gives the equivalent width of the line in angstroms and the lower number gives the flux in 10^{-14} ergs s^{-1} cm^{-2} .

On 1980 March 15.33 UT the double QSO was observed with the PFUEI prime focus CCD camera/spectrograph. A slit of width 2" was aligned with the two QSOs at p.a. 349°, and an exposure of 3600 s was made at a dispersion of 440 Å mm⁻¹ and a resolution of 35 Å (FWHM) in the wavelength range 4000–7000 Å. The seeing profile had an FWHM of 0".75.

In order to extract a relatively uncontaminated and noise-free spectrum of G1, we took the region of spectrum from 0".75 north of Q0957+561 B to 3" north of Q0957+561 B. This is shown in Figure 3. The data are collected into 19 Å bins and have not been smoothed otherwise (thus the points are statistically independent). Although the resolution is too low to resolve the Ca II H and K lines, the characteristic break is very pronounced at λ 5400 and confirms the new redshift $z_d = 0.36$ beyond any doubt.

c) The Cluster Parameters

From our deep survey studies (beginning in Kristian *et al.* 1980) we may compare the statistics of the number of objects in the Q0957+561 field with fields containing no clusters. We expect 30 stars and 60 galaxies in the field (on a 1000 s exposure), compared with 32 and 146 actually observed. Most of the noncluster galaxies are expected to be faint because the background number density–magnitude relation [$\log_{10} A(r) = 0.35r + C$] is steeper than for the cluster [$\log_{10} A(r) = 0.1r + C$]. However, several of the 10 brightest galaxies could be field galaxies projected on the cluster, and an improved study of the cluster must include redshifts of at least the brightest 10 galaxies (preferably the brightest 20) in Table 1. The presence of one or two bright field galaxies can seriously affect the determination of the cluster parameters, as we shall now see.

The surface density of galaxies in the cluster may be described by the probability of finding a galaxy at position (x, y) :

$$p(x, y) = p_0 \left\{ 1 + D / \left[1 + A(x - x_0)^2 + 2B(x - x_0)(y - y_0) + C(y - y_0)^2 \right]^{1/2} \right\}, \quad (3)$$

where p_0 is such that $\int p \, dx \, dy = 1$ (the integral being over the CCD frame). This represents the surface density $\sigma \propto r^{-1}$ (space density $\rho \propto r^{-2}$) allowing for a constant background and an elliptical shape.

FIG. 3.—Spectrum of G1 obtained with the prime focus CCD spectrograph on the 200 inch telescope. Each data bin is 19 Å and the resolution is 35 Å (FWHM). The characteristic break due to the Ca II H and K lines gives a redshift $z_d = 0.36$. Some spillover from Q0957+561 B produces the emission line Mg II λ 2799 at $z_s = 1.41$.

If there are N galaxies at positions (x_i, y_i) with luminosity L_i , then we determine the parameters x_0, y_0, A, B, C, D by maximizing the likelihood function

$$\ln W = \sum_{i=1}^N \ln p(x_i, y_i) \quad (4a)$$

or a luminosity weighted equivalent,

$$\ln W^* = \sum_{i=1}^N L_i \ln p(x_i, y_i), \quad (4b)$$

as appropriate.

If we assume the mass of a galaxy to be perfectly correlated with its observed r magnitude, we may obtain the center of mass of the cluster by calculating the center of luminosity. We have done this in two ways:

i) We measured the background galaxy density near the N, E, and S borders of the CCD frame (the cluster overlaps the W border). We then computed

$$x_0 = \frac{\sum_{i=1}^N L_i x_i - L_B \int_{\text{Frame}} x d^2 x}{\sum_{i=1}^N L_i - L_B}, \quad (5)$$

where L_B is the luminosity per unit area of the background galaxies. Since L_B is small compared with luminosity density in the cluster core, it does not matter that the measured L_B value is slightly larger than its true value (the CCD frame is not large enough to extend into a true "field galaxy" region).

ii) We obtained a value of $x_0 = (x_0, y_0)$ by maximizing $\ln W^*$ in (4b) with fixed, roughly correct, values for A, B, C, D , and iterating. The value of x_0 is not very sensitive to the precise values of the other parameters. Much of the cluster light is contributed by the brightest two galaxies (G1: $r=18.5$; G2: $r=18.9$), with the next eight brightest also being fairly important.

The two methods give the same result:

$$x_0 = (-36.3, 1.9) \quad (6a)$$

in the coordinate system of Table 1 (centered on Q0957+561 B). This uses all the galaxies in Table 1. We have begun to obtain redshifts for the brightest galaxies which we shall report on when this program is completed. We mention here that G2 is definitely in the cluster ($z=0.36$) and the peripheral galaxy G5 is not ($z=0.54$). G3 has a peculiar spectrum (it is a blue continuum), and we have not yet managed to obtain its redshift. It may not be a cluster member because of its large distance from the cluster center and its peculiar profile and color. Excluding G5 yields

$$x_0 = (-29.4, 0.1); \quad (6b)$$

and excluding G3 and G5 yields

$$x_0 = (-21.9, 3.4). \quad (6c)$$

Values from (6b) to (6c) are acceptable pending a set of redshifts for the other galaxies. Values nearer (6c) are preferable for the lens models (see § IV), which run into mild difficulties with (6b). The errors in these positions are not given since they are dominated by systematic effects (which galaxies are in the cluster?) rather than statistical effects. These values supersede the values given in Paper I which were based on histogram plots of galaxy number density in the x and y coordinates. Our new values differ from those in Paper I because we are now luminosity weighting the galaxies.

If the cluster has not had time to thermalize (and mass segregate) its core, we should measure the "core radius" of the cluster by number counts of galaxies (ignoring their luminosities). Actually some degree of relaxation in the core is likely (which may explain why G1 and G2 are so large) in which case the core radius is proportional to $(\text{mass})^{-1/2}$. We omit this complication and we will maximize (4a) to determine the size and ellipticity of the core. This does not much depend on the value of x_0 since errors in mean position add only quadratically to the measured core radii.

We find

$$A^{-1/2} = 50'', B \approx 0, C^{-1/2} = 27'', \quad (7)$$

when we consider galaxies with $r < 22.8$ only. Slightly smaller core radii result if we take $r < 22$ only, but the axis ratio of 0.54 is preserved. The cluster, thus, is oriented east-west and is appreciably flattened. The inclusion or exclusion of G3 has little effect on the core radii since we did not weight the galaxies by luminosity.

The mean luminosity density in the cluster core was 10 times that in the "background" at the edges of the frame when all observed galaxies were included. This translates to $D = 12$; most of the enhancement is due to G1 and G2 however. Since the true background is probably smaller than that used, the true density enhancement will be somewhat higher. The absolute value of the central luminosity density of the cluster is $r = 26.5 \text{ arcsec}^{-2}$, including all galaxies to $r = 24.5$.

For $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0$ we find $1 \text{ Mpc} = 179''.7$ at $z_d = 0.36$. Thus the core radii are 278 kpc (east-west) and 150 kpc (north-south), giving a geometric mean of the core radii of 204 kpc. This is in accord with values for nearby clusters (Bahcall 1975), but is half the size found by Dressler (1976). A more sophisticated statistical analysis by Sarazin (1980) finds core radii in agreement with Bahcall's values.

d) Absolute Coordinate Scale

The positions given in Table 1 are in coordinates centered on Q0957+561 B. We measured a glass copy of the O-plate from the Palomar Observatory Sky Survey using 21 position standards from the SAO. Measurements were made with the plate in position angles 0° and 180° , and the two sets of results were averaged.

Because the two QSO images were touching on the POSS plate, it was felt that the individual position measurements are unreliable. The images were of almost equal brightness (A was slightly brighter), and so we took the mean of the two positions as a reliable quantity:

$$\left. \begin{aligned} \alpha_{1950} &= 09^{\text{h}}57^{\text{m}}57^{\text{s}}.437 \pm 0.070, \\ \delta_{1950} &= +56^\circ08'19''.59 \pm 0''.50 \end{aligned} \right\} \begin{array}{l} \text{Mean of} \\ \text{Q0957+561 A, B.} \end{array}$$

A total of six objects in the CCD picture were visible on the POSS plate. These were all measured to verify the image scale of $0''.423$ per pixel on the CCD and to find our CCD coordinates (those in Table 1) are aligned north-south to an accuracy of 0.1° . The positions of the QSO images are now derived by using the observed separation of $6''.174 \pm 0''.01$ in the CCD data and position angle $348^\circ54' \pm 0.05^\circ$ (which agree with that derived from VLBI by Porcas *et al.* 1979).

We find

$$\begin{aligned} \text{Q0957+561 A: } \alpha_{1950} &= 09^{\text{h}}57^{\text{m}}57^{\text{s}}.364, \\ \delta_{1950} &= +56^\circ08'22''.62; \\ \text{Q0957+561 B: } \alpha_{1950} &= 09^{\text{h}}57^{\text{m}}57^{\text{s}}.510, \\ \delta_{1950} &= +56^\circ08'16''.56. \end{aligned}$$

The errors in these positions depend only on the error in the mean position given above. Comparing with the VLA map of Greenfield, Roberts, and Burke (1980), we find good agreement in declination and a small discrepancy in right ascension (they find A at $57^{\text{s}}29$).

III. FORMULATION OF GRAVITATIONAL LENS MODELS

a) Components in the Lens Models

We must consider three mass components in the lens models, viz.,

i) *The brightest cluster galaxy G1.* We shall treat this as a King model mass distribution with cutoff energy $\varepsilon_T = 12$. This was done in Paper I except now we consider a mass distribution with elliptical, rather than circular, isophotes. These isophotes are presumed, however, to have the same ellipticity at all radii.

ii) *The other cluster galaxies.* These are treated as point masses since they are mostly far removed from the light beam. If the cluster has perfect elliptical symmetry, then only the projected mass internal to the isophote which touches the observed image contributes to the lens effect (Bourassa and Kantowski 1975). Since the symmetry is not

perfect, due to the discreteness of the galaxies, there is a weak effect due to objects external to the pertinent isophote. We have, thus, included galaxies out to 1.5 core radii and down to $r=24$ in this component of the lens. Since the QSO images are ~ 0.6 core radii from the cluster center, this should be perfectly adequate to account for the weak effect of galaxies from 0.6 to 1.5 core radii from the cluster center. The condition $r < 24$ removes the faint galaxies, which are mostly interlopers having no effect, in the mean, on the lens angles.

We assume mass and luminosity to be perfectly correlated. The free parameter here is then the r magnitude of $1 M_{\odot}$ in the cluster.

iii) *Dark material in the cluster.* There are strong indications that most of the mass in great clusters of galaxies is invisible (it is not "missing" since its gravitational effects are clearly visible). Dynamical friction studies (White 1977) show that the mass is distributed smoothly rather than being lumped in with the galaxies. We allow for the presence of such dark matter by an elliptically symmetric, smooth mass distribution centered on the center of luminosity of the cluster and with the same core radii. The free parameter is the "velocity dispersion" of the dark matter which scales its mass density. Since the dark matter probably contains much more mass than the galaxies (by a factor of ~ 10), this velocity dispersion is actually that of the cluster.

b) The Gravitational Lens Calculations

We use the notation and formulae given by Bourassa and Kantowski (1975) in computing the multicomponent lens models. The results of those authors are easily extensible to the sum of any finite number of elliptically symmetric lens components, since the effects add linearly and all mass distributions may be projected onto a plane.

If the relativistic distances D_d , D_{ds} , and D_s are as in Bourassa and Kantowski (1975) and in Paper I, and if θ_Q is the true QSO position on the sky (in angular coordinates) and θ an image position, then

$$f(\theta) = -\theta + \theta_Q + (D_{ds}/D_s) \sum_i \alpha_i(\theta) = 0, \quad (8)$$

where α_i is the deflection angle caused by the i th component of the lens mass distribution. Given the functions α_i , we must solve (8) for image positions θ . This is best solved by Newton-Raphson iteration for which a "correction matrix"

$$\partial f / \partial \theta = -I + (D_{ds}/D_s) \sum_i \partial \alpha_i / \partial \theta \quad (9)$$

must be calculated (I is the identity matrix). The image amplification is given by the Jacobian of the transformation from θ_Q to θ in (8):

$$\mathcal{Q} = \|\partial f / \partial \theta\|^{-1}. \quad (10)$$

Bourassa and Kantowski (1975) give the formalism for elliptically symmetric mass distributions which we shall cast into a suitable form for our calculations as follows: For the G1 and dark mass components we use a surface mass distribution:

$$\sigma(b)/\rho_0 a = \sigma_*(b_*), \quad (11)$$

where ρ_0 = central mass density and a = structural length of major axis of lens [= (core radius)/3]. Here the impact parameter is

$$b^2 = (x - x_0)^2 + (y - y_0)^2 / \cos^2 \beta, \quad (12)$$

where (x_0, y_0) is the lens center and $\cos \beta$ its apparent axis ratio. We approximate $\sigma_*(b_*)$ (starred quantities are dimensionless) by

$$\sigma_*(b_*) = 10.249 \{1 + (0.385 b_*)^2\}^{-1/2} - 4.250 \{1 + (0.193 b_*)^2\}^{-1/2}. \quad (13)$$

This is a good approximation to a King model with $\varepsilon_T = 12$ (Paper I). It gives 1% accuracy for $b_* < 30$ and may be used to $b_* \sim 100$.

Define a "velocity dispersion" with

$$\sigma_v^2 = 4\pi\tilde{G}\rho_0 a^2, \quad (14)$$

where $\tilde{G}$ is the constant of gravitation. In the spherically symmetric case σ_v is the actual central velocity dispersion of the object. In an elliptical case it may not be exactly equal to a measured velocity dispersion due to the complications of elliptical galaxy structures and the unknown true flattening and inclination of the galaxy.

The bend angle then becomes

$$\bar{\alpha} = \alpha_x - i\alpha_y = 2(\sigma_v^2/c^2) \cos \beta \int_0^{b_*} b'_* \sigma(b'_*) db'_* \{z_*^2 - b_*'^2 \sin^2 \beta\}^{-1/2}, \quad (15)$$

where $z_* = (x - x_0)/a + i(y - y_0)/a$ is the dimensionless position of the image in coordinates (x, y) aligned with the principal axes of the lens. It is trivial to apply a rotation operator for a lens at any orientation.

The partial derivative matrix is

$$(D_{ds}/D_s) \partial \alpha / \partial \theta = - \begin{bmatrix} G - F_r & F_i \\ F_i & G + F_r \end{bmatrix}, \quad (16)$$

where

$$G = -Q\sigma_*(b_*),$$

$$F = F_r + iF_i = \frac{(x - x_0) - i(y - y_0)/\cos^2 \beta}{(x - x_0) + i(y - y_0)/\cos^2 \beta} Q\sigma_*(b_*) \quad (17a)$$

$$-2Q \cos \beta \int_0^{b_*} b'_* \sigma_*(b'_*) db'_* z_* \{z_*^2 - b_*'^2 \sin^2 \beta\}^{-3/2}, \quad (17b)$$

$$Q = (\sigma_v^2/c^2)(D_{ds}D_d/D_s a). \quad (17c)$$

Again, rotation operators may be applied to the matrix (16) to align it in an arbitrarily rotated coordinate system.

Time delays for gravitational lenses have been considered by Cooke and Kantowski (1975). We shall use suitably modified versions of their equations.

The time delay due to geometrical path length increase is

$$c\Delta t_g = (1 + z_d)(D_{ds}D_d/D_s) \left| \sum_i \alpha_i(\theta) \right|^2 / 2; \quad (18)$$

and the time delay due to the light "falling" through the gravitational potential of our elliptical lens component is

$$c\Delta t_p = a(1 + z_d)(\sigma_v^2/c^2) 2 \cos \beta \operatorname{Re} \left\{ \int_0^{b_*} \tilde{m}_*(b'_*) (db'_*/b'_*) z_* \{z_*^2 - b_*'^2 \sin^2 \beta\}^{-1/2} \right\}, \quad (19a)$$

where

$$\tilde{m}_*(b_*) = \int_0^{b_*} b'_* db'_* \sigma_*(b'_*) \quad (19b)$$

and z_d is the redshift of the deflector. Note that whereas Δt_g is normalized to become zero for an undeflected ray, Δt_p

for the elliptical King model component becomes zero when passing through its center. Rays passing outside the center have a positive Δt_p , lagging behind the central ray.

A point mass M has simple formulae:

$$\bar{\alpha} = 4(\tilde{G}M/c^2)z^{-1}, \quad (20a)$$

$$z = (x - x_0) + i(y - y_0), \quad (20b)$$

$$G = 0, \quad (20c)$$

$$F = -4(\tilde{G}M/c^2)(D_{ds}D_d/D_s)z^{-2}, \quad (20d)$$

$$c\Delta t_p = (4\tilde{G}/c^2)(1 + z_d)M \ln(b/b_{\text{ref}}). \quad (20e)$$

Here Δt_p diverges as $b \rightarrow 0$, so we normalize $\Delta t_p = 0$ at some "reference" impact parameter b_{ref} . Since we are concerned only with differences in time delay between the various images, b_{ref} may be anything we like (such as 1).

In order to compute the image shapes we note, from the equations (8) and (9), that

$$\partial f / \partial \theta = -\partial \theta_Q / \partial \theta = -P. \quad (21a)$$

If we set

$$P = \begin{bmatrix} g - f_1 & f_2 \\ f_2 & g + f_1 \end{bmatrix} \quad (21b)$$

and

$$f = f_1 + if_2, \quad (21c)$$

then the image amplification is

$$\mathcal{Q} = \{g^2 - |f|^2\}^{-1}. \quad (21d)$$

If the source is a small circle, then the image is an ellipse with axis ratio

$$\cos \beta_I = |(g - |f|)/(g + |f|)|. \quad (21e)$$

The axes are at an angle ζ , where

$$e^{-2i\zeta} = f/|f|. \quad (21f)$$

If $\cos \beta_I < 1$, then ζ is the position angle of the major axis of the ellipse; and if $\cos \beta_I > 1$, then ζ is the position angle of the minor axis.

IV. Q0957+561 A, B

a) The Third Image

In Paper I we pointed out that, contrary to previous work based on point imaging masses, there cannot be just two images. For an isolated galaxy there will be either one or three images. (This conclusion was reached independently by Dyer and Roeder, 1980). For an arbitrary mass distribution $(2n+1)$ images must form, of which $(n+1)$ have positive parity and n have negative parity (are mirror-reversed from the original source). For Q0957+561 there are most likely to be three images (although in Paper I we showed a case with five images). Where, then, is the third image?

In Paper I we discussed two possibilities: (i) that Q0957+561 B is a close double with separation $0''.15$ and with an intensity ratio of $3/2$; (ii) that the third image is faint and superposed on the center of G1.

For an isolated galaxy acting as a lens, only case (i) is possible because of the near equality of the A and B images. Since the third image must exist, we shall consider the observational constraints on the ratio of its brightness, ρ , to the brighter B1 image and on its separation $\Delta\theta$ from the brighter B1 image.

i) *The VLA radio map at 6 cm* (Greenfield, Roberts, and Burke 1980)

In this map there is a spur from the B image extending north and ending on the galaxy center 1" north of Q0957+561 B. A later map (Burke 1980, private communication) shows this spur to be in reality a point source 1" north of Q0957+561 B and roughly centered on the galaxy.

Two constraints exist. If the third image is merged with B1, then it must be weak enough to leave B unresolved in the north-south direction. Now,

$$\sigma_B^2 = \sigma_A^2 + \Delta\theta^2 \rho (1 + \rho)^{-2} \leq 1.2 \sigma_A^2, \quad (22)$$

where σ_A is the north-south Gaussian dispersion of image A and $\sigma_A = 0''.26$.

The second constraint applies to large separations $\Delta\theta > 0''.5$. The surface brightness of the third image must not exceed that of the emission to the north of B. Thus we find $\rho \leq 0.08$. We suspect that some of this emission may be from the third image, and some of it from A's jet (p.a. 40°). We shall return to the question of A's jet after presenting some lens models (§ IVd).

ii) *The VLBI observations of Porcas et al. 1979*

Both A and B were reported unresolved at the 20 milli-arcsec level. The sources were weak, however, and the data had low signal-to-noise ratio. The most sensitive baseline had rms noise of 6 mJy in the visibility function, with a 30 mJy source. With a lobe spacing of 55 milli-arcsec, observations lasting 4 hours, and integrations of 20 minutes, the sensitivity to a double must be in the range $0''.025 < \Delta\theta < 0''.5$. The visibility function will oscillate between the sum and difference of the fluxes if B is a double source; this will be obvious only if the peak-to-peak amplitude is greater than $3\sigma = 18$ mJy, i.e., variations from 39 to 21 mJy. This corresponds to $\rho = 0.3$. The residual fringe rate spectrum, sensitive to a point source for $\Delta\theta > 0''.5$, gives a poorer constraint than the VLA map.

iii) *The optical photographs of Stockton (1980)*

The absence of image elongation in Q0957+561 B yields constraints similar to those of the VLA map. We notice that the surface brightness of the core of G1, being 1" north of image B, exceeds that of the pair of stars (objects 90 and 91 in Table 1) which have $r = 20.32$ and $r = 20.90$. In particular, the core of G1 seems rather hard and bright for a brightest cluster galaxy at an estimated $r = 18$ mag arcsec $^{-2}$, especially with surface brightness falling as $(1+z)^{-4}$ in Friedmann cosmologies, in addition to the K -correction. Either G1 has an unusually small, bright core for a brightest cluster galaxy or a significant contribution arises from the third image. The two effects are correlated; a small (high surface brightness) core gives a faint third image, and a large (faint) core gives a bright third image. The third image will merge with the galaxy core and displace it southward, since the tertiary image cannot be north of the galaxy core.

In any case, for large $\Delta\theta$ we can only demand $\rho \leq 0.12$. The VLA data would allow up to half the observed surface brightness of the "galaxy core" to be third image contribution. Stockton (1980, private communication) has been kind enough to send us some B ($\lambda 4400$) passband pictures of Q0957+561 in which the galaxy is not visible. In this band the galaxy will be 2 mag fainter relative to the QSO images than in r ($\lambda 6500$). Although there is some uncertainty, inspecting the brightness of nearby stars to calibrate the depth of the exposure would demand $\rho \leq 0.07$, the same limit as the VLA map. Of course, since the third image is so close to the galaxy center, the ray of light may intercept a dust cloud which would affect the blue far more than the red.

b) *Free Parameters*

The major free parameters are (i) the velocity dispersion of G1: σ_V^G ; (ii) the velocity dispersion of the cluster: σ_V^C ; (iii) the core radius of G1: $r_c^G = 3a_G$. There is, in addition, some sensitivity of the imaging to (iv) the cluster center x_0 ; (v) the ellipticity of G1: e_G ; (vi) the fractions of mass in dark matter and galaxies in the cluster.

We must fit: (i) the brightness ratio A to B; (ii) the image separation of $6''.17$; (iii) the distance ratio from G1 of A and B; (iv) the skew in the A-G1-B line.

Also, we must not violate the constraints given above on the third image. We must also arrange for the extended radio lobes to be singly imaged.

c) *G1 Only Models*

We shall first attempt to explain the lens effect with galaxy G1 alone. This was done in Paper I, but we shall now use an elliptical rather than a spherical galaxy model. The observed ellipticity of the galaxy is $e_G \sim 0.15$ from Paper I. This parameter must be varied to fit the image skewness.

It is not clear yet just what the intensity ratio of the A and B images is (probably in the range 0.7–0.8) because of possible time delayed variations. The following model, however, simulates the observed situation as well as can be determined at present:

$$\sigma_V^G = 470 \text{ km s}^{-1}, \quad \epsilon_G = 0.60,$$

$$a_G = 1.60 \text{ kpc}, \quad \psi_G = 37^\circ,$$

where ψ_G is the angle of the major axis of G1 (in the x, y coordinate system where the x axis is 0° and the y axis is 90°).

In coordinates running east (x) and north (y) and *centered on G1* (the logical theoretical coordinates; the observational coordinates given in Table 1 are centered on Q0957+561 B since we are not sure where the center of G1 is)

$$\theta_Q = (-0.38, 2.375),$$

with the three images:

Image A:

$$\theta_A = (-1.41, 5.18), \quad \mathcal{Q}_A = 1.64, \quad \cos \beta_A = 0.50, \quad \zeta_A = 16^\circ 3;$$

Image B1:

$$\theta_{B1} = (-0.13, -1.07), \quad \mathcal{Q}_{B1} = -1.04, \quad \cos \beta_{B1} = 0.27, \quad \zeta_{B1} = 84^\circ 4;$$

Image B2:

$$\theta_{B2} = (-0.08, -0.71), \quad \mathcal{Q}_{B2} = 0.50, \quad \cos \beta_{B2} = 0.23, \quad \zeta_{B2} = 78^\circ 5;$$

Time delays:

$$\Delta t_A - \Delta t_{B1} = 2.56 \text{ yr},$$

$$\Delta t_{B1} - \Delta t_{B2} = 0.46 \text{ yr},$$

so that a brightness change is seen first in B2, then B1, and last in A.

The main defects of this fit are:

- i) The galaxy velocity dispersion is very high.
- ii) The galaxy ellipticity is much too high (although it is possible that the mass distribution in the galaxy has much higher ellipticities than the light).
- iii) Image B has $\Delta\theta = 0''.36$ and $\rho = 0.49$. The duplicity of B should be easily detected, but is not observed in the VLA or VLBI data.

We can adjust the brightness of the image B2 nearly independently of the other quantities (positions of A, B1 relative to G1, image skew, brightness of A, etc.) by adjusting the structural length a_G . As a_G is increased, B2 and B1 move together and brighten. As a_G is decreased, B1 fades to $\mathcal{Q}_{B1} = -0.5$ and B2 moves toward G1 and fades dramatically. We were unable to find a solution with $|\mathcal{Q}_{B1}/\mathcal{Q}_A| > 0.3$ (with B2 faint) and with the correct distance ratio of A and B from G1 (5:1). In summary, within the assumptions of our model, the observed imaging cannot be fit with an elliptical galaxy G1 alone.

d) G1 plus Cluster Models

We shall first compute the red magnitude of 1 $\mathcal{M}_\odot$ in the cluster that will yield a particular “velocity dispersion.” From (14), and using the relation

$$\Sigma_0 = 6\rho_0 a \tag{23a}$$

for a projected King model (Σ_0 is the central surface density), we find

$$\Sigma_0 = 3.69 \times 10^{10} (\sigma_V/1000 \text{ km s}^{-1})^2 (\mathcal{M}_\odot \text{ arcsec}^{-2}) \tag{23b}$$

for a major axis structural length $a_C = r_C/3 = 93 \text{ kpc}$, and $z_d = 0.36$, with $H_0 = 60$ and $q_0 = 0$.

Since the observed central luminosity density in the cluster is $r=26.5 \text{ arcsec}^{-2}$, the red magnitude of $1 \mathcal{M}_\odot$ is

$$r(\mathcal{M}_\odot) = 52.9 + 5 \log_{10}(\sigma_V / 1000 \text{ km s}^{-1}) \quad (24)$$

The mass of G1 (at $r=18.5$) would then be

$$\mathcal{M}(\text{G1}) = 5.8 \times 10^{13} (\sigma_V / 1000 \text{ km s}^{-1})^2, \quad (25)$$

which is typical of the large virial masses obtained for galaxies in large clusters.

Since only 5–10% of the mass of great clusters is likely to be in the galaxies, we set $1 \mathcal{M}_\odot$ to have $r(\mathcal{M}_\odot) = 50.0$.

In adjusting our lens models, we found it remarkably difficult to fit both the skew in the image line and the high brightness of B relative to A. Fitting either one is reasonably easy, but fitting both strained the capacity of the present models.

We settled upon the following representative case; it must not be looked upon as a unique or well-determined solution at this time, but merely an example:

Cluster center: $x_0 = (-23, 0)$

Structural length: $a_C = 90 \text{ kpc}$

Axis ratio: $\cos \beta_C = 0.55$

Position angle of major axis: $\psi_C = 25^\circ$

Velocity dispersion: $\sigma_V^C = 1100 \text{ km s}^{-1}$

Galaxy structural length: $a_G = 0.8 \text{ kpc}$

Axis ratio: $\cos \beta_G = 0.8$

Position angle of major axis: $\psi_G = 37^\circ$

Velocity dispersion: $\sigma_V^G = 260 \text{ km s}^{-1}$

(angles are measured in the x, y coordinate frame; they are not standard celestial position angles), where we have assumed $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0$. The velocity dispersion σ_V^C is due to both galaxy and dark mass; but since the structure is appreciably flattened, the observed velocity dispersion will be considerably less than σ_V^C . With this model we place an image at

$$\theta_Q = (-10.236, 3.432)$$

(in coordinates centered on G1 which we use for all theoretical imaging); we find the images:

Image A:

$$\theta_A = (-1.37, 5.08), \mathcal{A}_A = 5.82, \cos \beta_A = 0.64, \zeta_A = -65^\circ 0;$$

Image B1:

$$\theta_{B1} = (-0.14, -1.07), \mathcal{A}_{B1} = -4.10, \cos \beta_{B1} = 0.90, \zeta_{B1} = -11^\circ 2;$$

Image B2:

$$\theta_{B2} = (0.00, -0.29), \mathcal{A}_{B2} = 0.28, \cos \beta_{B2} = 0.62, \zeta_{B2} = 75^\circ 1;$$

Time delays:

$$\Delta t_A - \Delta t_{B1} = -5.88 \text{ yr } (-2.83 \text{ yr geometrical only}),$$

$$\Delta t_{B1} - \Delta t_{B2} = 0.91 \text{ yr } (+0.52 \text{ yr geometrical only}),$$

so that A varies first, then B2, and lastly B1.

In this solution we have allowed B2 to become as bright as 7% of B1, and we have let B1 be 0.70 of A. Image B2 would then contribute most of the radio emission just north of Q0957+561 B. Since image B2 would also be bright enough to pull the apparent center of G1 a little south, we have placed B1 1".07 from G1 rather than 0".99 (as measured by Stockton). In fact, a little experimentation showed that the structural length of the galaxy a_G may be varied, leaving the features of the imaging unchanged except to cause B2 to fade and move toward G1 as a_G is lowered. Thus we may adjust B2 to essentially any brightness we like, while preserving the aspect of A and B1. The galaxy core radius $3a_G = 2.4$ kpc for the given solution is typical of brightest cluster galaxies (Hoessel 1980).

We should remark that all distances and time delays were computed for $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0$.

If we adjust the galaxy core radius to $3a_G = 1.8$ kpc, then the A and B1 images are essentially unchanged but the B2 image solution becomes:

$$\theta_{B2} = (0.00, -0.18), \quad \varpi_{B2} = 0.10, \quad \cos \beta_{B2} = 0.72, \quad \zeta_{B2} = 69^\circ 6,$$

with $\Delta t_{B1} - \Delta t_{B2} = 1.04$ yr.

Stockton (1980) measured the core radius of the galaxy G1 to be $3a_G = 1.34 \text{ kpc} = 0".24$ under the assumption that image B2 did not contribute significantly to the luminosity in the galaxy core. This value must be considered to be the lowest possible value that the core radius of G1 is allowed to have. For this small a core radius the B2 image has $\varpi_{B2} = 0.03$ and is located at $(0, -0".11)$. This is the faintest allowed third image within the framework of the present models and makes B2 only 0.7% of the brightness of image B1. This yields an r magnitude of 22.9 and a radio flux of 0.21 mJy (if B2 is 30 mJy). Dust clouds can diminish the optical flux still further. The effects of the discrete stellar nature of the galaxy can have a significant effect (this will be discussed in a later paper in detail) and can perturb the flux of B2 by large amounts (factors of 10 or more) if the QSO is a sufficiently small source.

Let us examine these solutions in more detail. We have taken the observed configuration of radio lobes and jets observed by the VLA and projected them onto the source plane to find the true configuration. The source plane is shown in Figure 4a. We show, schematically, the quasar (Q), its jet running NE, and the three lobes C, D, and E. Also plotted is the "singular curve"; any object lying inside this curve is imaged three times and those lying outside are imaged only once. The lobe-QSO line in the source plane is very much more nearly a straight line than is observed. Notice that only the QSO lies inside the singular curve; all the lobes are imaged only once.

Now turn to the image plane in Figure 4b. We see the primary images of the QSO, lobes, and jet. We also see secondary and tertiary images of the QSO and a short piece of its jet. The image of the jet runs almost due north

FIG. 4a

FIG. 4b

FIG. 4.—(a) True configuration of Q0957+561 in the lens model given in the text. We show the QSO (Q), radio lobes (C, D, E) and a jet running NE from the QSO. Sources outside the closed curve are imaged once; sources inside this critical curve are imaged three times. The axes are in arcseconds with the origin centered on G1. The cluster center is west of G1 at $(-23, 0)$. (b) Observed configuration of Q0957+561. The circles in the source plane are imaged into ellipses in the image plane. Only the QSO and a short piece of its jet are imaged three times. Galaxy G1 is at the origin, and the small, closed curve around it is the image of the critical curve shown in part (a) of this figure. The asterisk denotes the location of the QSO as imaged by the cluster of galaxies alone.

TABLE 3
Q0957+561 IMAGE MODEL

Feature	Source Position	Image Position	Amplification A	Axis Ratio $\cos \beta_1$	Angle ξ
A.....	(-10.24, 3.43)	(-1.37, 5.08)	5.82	0.64	-65°0
B1.....	(-10.24, 3.43)	(-0.14, -1.07)	-4.10	0.90	-11°2
B2 ^a	(-10.24, 3.43)	(0.00, -0.29)	0.28	0.62	75°1
	(-10.24, 3.43)	(0.00, -0.18)	0.10	0.72	69°6
C.....	(-6.88, 5.06)	(4.34, 7.20)	5.03	0.40	-62°4
D.....	(-8.48, 4.50)	(1.66, 6.93)	5.43	0.45	-58°7
E.....	(-12.74, 2.42)	(-5.76, 1.91)	8.63	0.31	-81°5

^aTwo solutions for B2 are given. The first solution has a galaxy core radius $3a_G = 2.4$ kpc, the second has $3a_G = 1.8$ kpc.

from B1, joining the tertiary image at the singular curve in the image plane. The combination of B2 and these pieces of jet are proffered as an explanation for the emission north of Q0957+561 B seen in the VLA map. In fact, the contribution from the jet will not be very large since its surface brightness is rather low as judged by the surface brightness of the jet from Q0957+561 A. We would expect new VLA maps to show a point source near the center of G1 with a faint bridge joining it to Q0957+561 B.

It is still possible that the galaxy is also a radio source, that this forms the emission north of Q0957+561 B, and that the B2 image is rather faint. In the case of our second solution above, the B2 image would contribute only 30% of the flux in the “jet” (although this percentage may be adjusted to any value by changing a_G). The details of the imaging of the lobes remain unchanged in this case. Measurement of the brightness ratio of B2 to B1 (when B2 is found) will accurately measure the core radius of the galaxy G1.

Table 3 has details of the imaging of both the QSO and the various lobes. It may be seen that the magnification of the primary image is ~ 6 , with lobe E somewhat greater. Imaging by the cluster (whose center is to the west) would stretch the images north-south. Reimaging by the galaxy stretches them east-west again so that they are not very elongated. Lobe E is west rather than north of the galaxy G1, so both G1 and the cluster tend to stretch it north-south, which is why it is more elongated than the other lobes.

In summary, it is possible to construct a plausible model whereby Q0957+561 is a single object, multiply imaged by a gravitational lens made up of a brightest cluster galaxy, modified by the rest of the cluster. The data, however, do not yet provide sufficient constraints to accurately define a unique model.

V. DISCUSSION

a) Measurement of the Hubble Constant Using a Gravitational Lens

An important feature of a gravitational lens is that the time delays for intensity variations among the images are proportional to H_0^{-1} . (Refsdal 1964). This is obvious for the geometrical part of the time delay. For the time delay due to gravitational potentials it is also true since we deduce masses using line-of-sight velocities and angular scales on the sky. Measuring the delay times can yield a value of H_0 when an isolated galaxy is responsible for the gravitational lens effect. For this one needs only the two bright images, provided reasonable models for the mass distribution in the galaxy can be deduced from the image brightnesses and positions. With the galaxy cluster adding to the lens effect, we need at least three images (and two time delays) to have any hope of measuring H_0 . We shall discuss in a future paper various effects which can perturb the ideal situation now to be described.

The critical unknown parameters are the true position of the QSO on the sky (as it would be seen without the galaxy and cluster present) and, of course, H_0 . These three parameters need three time delays (from four images) for unambiguous determination. Two images (with one time delay) are probably insufficient; the imaging process must then be understood well enough to know the true position on the sky without using the time delay information.

However, with all three images and two time delays there is sufficient information, in principle, to measure H_0 . Consider the three images and coordinate system shown in Figure 5. Let (ξ_Q, η_Q) be the true QSO position. First we must model the lens to reproduce the image locations and intensities. We then remove the fraction of the time delays due to the gravitational potentials. In practice this might be quite tricky since roughly half the effect is gravitational

and the effect of the galaxy cluster is substantial. The three bend angles are

$$\alpha_1^2 = \xi_Q^2 + \eta_Q^2, \quad (26a)$$

$$\alpha_2^2 = \xi_Q^2 + (\eta_Q + \theta_{21})^2, \quad (26b)$$

$$\alpha_3^2 = (\xi_Q - \theta_{31} \sin \phi)^2 + (\eta_Q - \theta_{31} \cos \phi)^2. \quad (26c)$$

If δ_{21} is the time delay between images 2 and 1 (in Fig. 5) and is positive for image 1 to vary first, then

$$\delta_{21} = (c\Delta/2H_0)(\alpha_2^2 - \alpha_1^2), \quad (27a)$$

where $c\Delta/H_0 = (1+z_d)D_d D_{ds}/D_s$. Similarly,

$$\delta_{31} = (c\Delta/2H_0)(\alpha_3^2 - \alpha_1^2). \quad (27b)$$

Thus,

$$\delta_{21} = (c\Delta/2H_0)(2\eta_Q\theta_{21} + \theta_{21}^2), \quad (28a)$$

$$\delta_{31} = (c\Delta/2H_0)(\theta_{31}^2 - 2\xi_Q\theta_{31}\sin\phi - 2\eta_Q\theta_{31}\cos\phi). \quad (28b)$$

Given the delays δ_{21} , δ_{31} , we may invert these formulae to solve for (ξ_Q, η_Q) :

$$\eta_Q = \frac{1}{\theta_{21}} \left(\frac{H_0 \delta_{21}}{c\Delta} \right) - \frac{\theta_{21}}{2}, \quad (29a)$$

$$\xi_Q = \frac{\theta_{31}}{2\sin\phi} + \frac{\theta_{21}}{2\tan\phi} - \left(\frac{H_0}{c\Delta} \right) \left(\frac{\delta_{31}}{\theta_{31}\sin\phi} + \frac{\delta_{21}}{\theta_{21}\tan\phi} \right). \quad (29b)$$

As H_0 varies, ξ_Q and η_Q move along a straight line on the sky. Measuring H_0 involves understanding the gravitational

FIG. 5.—Geometry used to determine H_0 . Three images (1,2,3) are arranged as shown with the true QSO position at (ξ_Q, η_Q) . The bend angles of the three images are $\alpha_1, \alpha_2, \alpha_3$. The true QSO position required to produce a given arrangement of images depends on the value of H_0 . The figure schematically shows the case $H_0 = 60$. For other values of H_0 , the true QSO position will lie somewhere along the line passing through (ξ_Q, η_Q) and labeled with values for H_0 of 0, 50, and 100. The end point of this line, $H_0 = 0$ is the intersection of the dotted lines, which are the perpendicular bisectors of the lines joining the images. For different values of H_0 , the bending angles and time delays will vary as discussed in the text. To measure H_0 one must understand the imaging well enough to independently locate (ξ_Q, η_Q) on this track.

lens well enough to constrain ξ_Q and η_Q on this line. There is a check in that the model must have the true QSO position fall on this line. It should also be clear in what circumstances H_0 cannot be measured. If

$$(\delta_{21}/\theta_{21})^2 + (\delta_{31}/\theta_{31} \sin \phi + \delta_{21}/\theta_{21} \tan \phi)^2 \quad (30)$$

is too small, then the point (ξ_Q, η_Q) moves only small distances for large changes in H_0 . Zero sensitivity results, of course, only when $\delta_{21} \rightarrow 0$ and $\delta_{31} \rightarrow 0$. Maximum sensitivity occurs when $(\delta_{31}/\sin \phi)$ or $(\delta_{21}/\tan \phi)$ become large (without canceling). Clearly a strong skew ϕ in the image line is not to be desired. An example of perfect sensitivity results if $\delta_{31} = \delta_{21} = \delta$, $\phi = 0$, and $\theta_{31} = \theta_{21} = \theta$. Then

$$\delta = \delta_{21} = (c\Delta/2H_0)(\alpha_2^2 - \alpha_1^2) = (c\Delta/2H_0)\theta^2. \quad (31)$$

In this case we need not model the cluster at all; H_0 is determined independently of (ξ_Q, η_Q) . The double quasar may have approximate parameters $\phi = 21^\circ$, $\theta_{31} = 5''$, $\theta_{21} = 1''$, and $\Delta = 0.19$. Then

$$\xi_Q'' = 8''.28 - 35''.56(H_0/60)[(\delta_{21}/1 \text{ yr}) + (\delta_{31}/4.66 \text{ yr})], \quad (32a)$$

$$\eta_Q'' = -0''.5 + 13''.67(H_0/60)(\delta_{21}/1 \text{ yr}). \quad (32b)$$

We shall be unable to determine H_0 if $(\delta_{21}/1 \text{ yr}) \sim -(\delta_{31}/4.66 \text{ yr})$ and $(\delta_{21}/1 \text{ yr}) \lesssim 0.5$. This all assumes no errors in δ_{31} and δ_{21} ; in most cases the errors are dominated by uncertainty in locating (ξ_Q, η_Q) . The schematic situation is shown in Figure 5. The H_0 track begins at the intersection of the perpendicular bisectors of the lines between images 1 and 2 and images 2 and 3. This corresponds to $H_0 = 0$; any nonzero set of time delays must then be produced in an infinitely large universe. The track then moves off to infinity along some direction defined by δ_{21} and δ_{31} with the markers, linearly spaced, giving H_0 . Notice that our model given in § IVd has extremely good sensitivity to H_0 since $(\delta_{31}/4.66 \text{ yr}) \approx (\delta_{21}/1 \text{ yr}) \sim -0.6$. We obtain a movement of $5''$ in (ξ_Q, η_Q) for a change in H_0 from 60 to 66.

b) Further Work Required

The crucial observations needed to improve the gravitational lens models are

- i) *The location and intensity of the third image.* This may already have been seen, as we point out in § IVa, but verification may require spectroscopy with the Space Telescope. The image brightness must be measured in the emission lines (relative to the emission line fluxes in the other images) for reasons to be discussed in the next paper.
- ii) *The two time delays among the three images.* Of these, our model gives the delay from A to B1 as 5 years, and from B1 to B2 only 1 year.
- iii) *The velocity dispersions of G1 and the cluster of galaxies.* The dispersion must be obtained to an accuracy of a few percent. One might be able to get by with a velocity dispersion for either G1 or the cluster since the image splitting from A to B1 would give the other dispersion.
- iv) *The center of mass of the cluster of galaxies.* This may be done by finding the center of luminosity of the galaxies (cluster membership of the brightest galaxies must be verified with redshifts). A better method would be to use X-ray observations, if the cluster turns out to be an X-ray source. The X-ray observations could also provide independent information on the cluster shape and core radius.

c) Summary

We have reobserved the galaxy G1 with Multichannel and CCD spectroscopy, and we present an improved redshift $z_d = 0.36$. The image Q0957+561 B has been observed to brighten by 0.3 mag at optical wavelengths since the observations of Paper I so that the two images were of equal brightness in 1980 January. The cluster of galaxies surrounding Q0957+561 has been measured, and we give a list of positions and red magnitudes for 146 galaxies in the CCD picture.

We have constructed new gravitational lens models with an elliptical galaxy for G1, point masses for the other galaxies in the cluster, and an elliptical mass distribution of dark matter in the cluster. Present observations probably rule out image models where Q0957+561 B is a close double with components in the brightness ratio 3:2. The third image is probably $\lesssim 7\%$ of the brightness of the main B image and lies close to the center of G1. It may be visible in the VLA map and in Stockton's high-resolution optical picture, or it may require Space Telescope observations to be found. The detailed model, although not unique, explains satisfactorily all presently available data on Q0957+561.

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REFERENCES

- Bahcall, N. A. 1975, *Ap. J.*, **198**, 249.
 Bourassa, R. R., and Kantowski, R. 1975, *Ap. J.*, **195**, 13.
 Cooke, J. H., and Kantowski, R. 1975, *Ap. J. (Letters)*, **195**, L11.
 Dressler, A. 1976, Ph.D. thesis, University of California, Santa Cruz.
 Dyer, C. C., and Roeder, R. C. 1980, *Ap. J. (Letters)*, **238**, L67.
 Greenfield, P. E., Roberts, D. H., and Burke, B. F. 1980, *Science*, **208**, 495.
 Hoessel, J. T. 1980, *Ap. J.*, in press.
 Kristian, J. A., Young, P. J., and Westphal, J. A. 1980, preprint.
 Porcas, R. W., Booth, R. S., Browne, I. W. A., Walsh, D. and Wilkinson, P. N. 1979, *Nature*, **282**, 385.
 Refsdal, S. 1964, *M.N.R.A.S.*, **128**, 307.
 Sarazin, C. L. 1980, *Ap. J.*, **236**, 75.
 Seabok, W. L. 1979, *A.J.*, **84**, 1526.
 Soifer, B. T., Neugebauer, G., Matthews, K., Becklin, E. E., Wynn-Williams, C. G., and Capps, R. 1980, *Nature*, **285**, 91.
 Stockton, A. 1980, *Ap. J. (Letters)*, in press.
 Walsh, D., Carswell, R. F., and Weymann, R. J. 1979, *Nature*, **279**, 381.
 White, S. D. M. 1977, *M.N.R.A.S.*, **179**, 33.
 Young, P. J., Gunn, J. E., Kristian, J., Oke, J. B., and Westphal, J. A. 1980, *Ap. J.*, **241**, 507 (Paper I).

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