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EXPERIMENTAL DETERMINATION OF E-k RELATIONSHIP IN ELECTRON TUNNELING*

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We report here measurements of electron tunneling through thin AlN films in which the imaginary component of the propagation vector in the forbidden band has been determined as a function of energy from the dependence of the tunneling current upon insulator thickness. The relationship so derived agrees well with Franz's empirical relationship[1] for a material with the 4.2-eV forbidden-band energy of AlN. These results allow the prediction of voltage-current characteristics over the entire range of experimental variables with no arbitrary adjustable parameters, and also subject to several internal self-consistency checks. In each case complete consistency is observed. To the authors' knowledge, this represents the first unambiguous demonstration of such consistency in thin-film tunneling.

AlN films were made by treatment of freshly evaporated Al films in a N_2 glow discharge at a pressure of 200 μ for 3 min. Mg counter electrodes were subsequently evaporated through a mask. The entire procedure is similar to that used for Al_2O_3 samples.[2] Measurements made at room, liquid-nitrogen, and liquid-helium temperatures gave essentially identical results.

Previously, current-voltage data have been interpreted by fitting the observed characteristic of a structure with a particular insulator thickness to a theoretical model which assumed the dependence of k, the imaginary component of the propagation vector in the insulator forbidden band, on E, the energy below the conduction-band edge, to be given by

$$E=h^2k^2/2m_i^*. \qquad (1.1)$$

The entire behavior of electron tunneling is dominated by the exponential attenuation of the electron wave function in the forbidden gap of the insulator. Hence a detailed understanding of the process can only come when the energy-dependent attenuation coefficient k of the wave function is known accurately. By far the most direct and unambiguous method of obtaining the value of k is by the variation of the current density as a function of insulator thickness.

In the case where (1) the tunneling current I per unit area can be considered as proportional to the product of the tunneling probability and an effective number of electrons within one of the metals incident on the barrier presented by the insulator per unit area per second with energies near the metal Fermi level and with transverse momenta near zero, and (2) the barrier can be considered as trapezoidal, then, for low voltages, I is linear with applied voltage V and is given by[3]

$$I=A(V/x)\exp[-2\bar{k}(0)x], \qquad (1.2)$$

where $\bar{k}(0)$ is the average value of k encountered in the tunneling path corresponding to an incident electron with zero transverse momentum and energy equal to the metal Fermi energy under the condition of zero voltage applied to the structure. A is a slowly varying function of temperature defined by universal constants and the insulator E-k relationahip. The tunneling path length x is equal to the insulator thickness. When the logarithm of the product of zero-bias resistance and sample capacitance is plotted as a function of insulator thickness, the resulting slope is $2\bar{k}(0)$. A plot of this type for AlN films is shown in Fig. 1. The insulator thickness was found from the sample capacitance assuming a dielectric constant of 8.5.

For applied voltages higher than the average energy spread of tunneling electrons, the current can be written as[3]

$$I=(B/x^2)\exp[-2\bar{k}(qV)x], \tag{1.3}$$

where q is the electron charge, B is a slowly varying function of voltage and temperature involving universal constants and the insulator E-k relationship, and x can be taken as the insulator thickness for applied voltages less than the barrier energy of the positive electrode. (The energy-band representation of a thin-film structure with a trapezoidal barrier is shown in Fig. 2.)

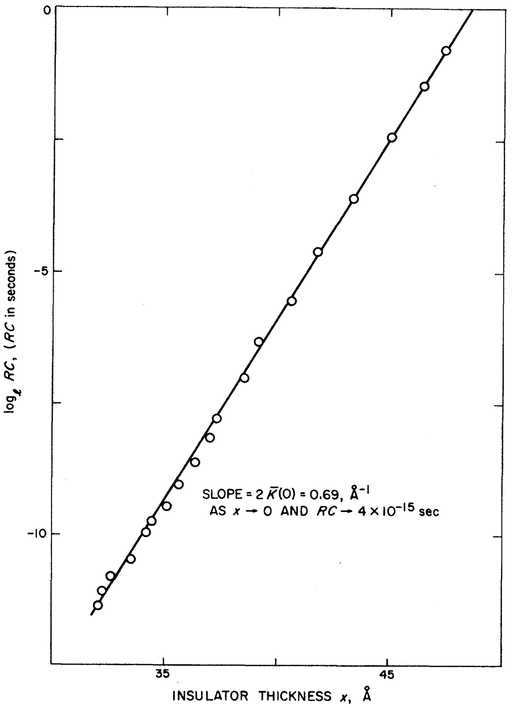

FIG. 1. $\mathrm{Log}_e RC$ versus insulator thickness.

For higher applied voltages, $\bar{k}(qV)$ can be found by taking the ratio of currents I_1 and I_2 from samples of different thickness x_1 and x_2:

$$2\bar{k}(qV)=\frac{1}{x_1-x_2}\ln\frac{I_2x_2^2}{I_1x_1^2}.$$

This average wave vector may then be related to energy E (referred to the insulator forbidden-band edge), through the two barrier energies φ_1 and φ_2. Since $\bar{k}$ is an integral of $k(E)$ over the energy range involved, $k(E)$ itself can be written as a voltage derivative of $\bar{k}$:

$$k(\varphi_2-qV)=\frac{1}{q}\frac{\partial}{\partial V}(\varphi_1-\varphi_2+qV)\bar{k}(qV).$$

Applying this expression to $\bar{k}(qV)$ for both polarities of applied bias allows one to determine $\varphi_1-\varphi_2$ and construct the $k(E)$-vs-E dependence.

The results of this procedure are shown in Fig. 2. The actual magnitudes of the barrier energies follow from the intercept of the curve

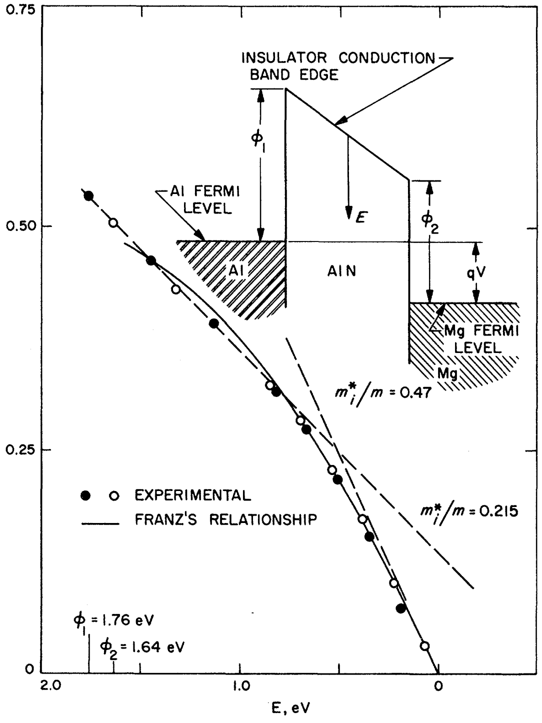

FIG. 2. E-k relationship for aluminum nitride. ○ and ● represent experimental relationships obtained with a positive voltage applied to the magnesium and aluminum electrodes, respectively. Solid line represents Franz's empirical relationship. E represents energy below insulator conduction-band edge. Included is an energy-band diagram of a structure with a positive voltage V applied to the Mg electrode.

on the E axis, defining the zero of E. Near $E=0$ the curve can be represented by the standard parabolic approximation with an effective mass $m^*=0.49$. For larger energies a marked deviation of the type expected for a finite forbidden-band energy is observed. Consistency is exhibited by the fact that the data for both polarities of applied bias yield the same $k(E)$-vs-E dependence. The actual dependence allows the computation of A in (1.2) and the expected RC intercept in Fig. 1 becomes 3×10^{-15} sec. The actual intercept is 4×10^{-15} sec. Theoretical volt-ampere characteristics constructed from the foregoing results agree with measured values within the uncertainties involved.

Using the optical value of the dielectric constant of AlN, the corrections to this procedure expected from image forces have been estimated, and found to be small. The E-k relationship obtained from various thicknesses was found to agree within the accuracy of the data. This would not be the case, nor would the calculated intercept in Fig. 1 be correct, if deviations from the trapezoidal shape were appreciable over the thickness range covered. Slight nonuniformities in the thickness can also affect the procedure, but result predominantly in an effective thickness different from the thickness measured by capacitance by a constant factor.[4] This produces only a change in the scale of x, and hence of k, but does not otherwise alter the procedure or the form of the results.

The barrier energies deduced from the foregoing procedure can be checked independently by (1) the occurrence of cusps in the temperature coefficient of the current,[5] and by (2) the photoemission method.[6] The results of these measurements are in satisfactory agreement and are shown in Table I. Samples with Al counter electrodes gave essentially an identical E-k relationship but exhibited a φ_2 approximately 0.5 eV higher than those with Mg counter electrodes indicating no appreciable number of surface states.

Table I. Barrier energies (in eV) as determined by photoemission, cusps in the temperature coefficient of the current, and the procedure described in the text (labeled E-k).

	E-k	Temp cusp	Photo
φ_1(Al)	1.75		1.6
φ_2(Mg)	1.6	1.6	

We therefore conclude that it is possible to obtain directly from thin-film tunneling data sufficient information to characterize the process in a straightforward manner with no arbitrary assumptions or adjustable parameters. Although obvious refinements can undoubtedly be achieved, we believe this represents a much more sound approach to the interpretation of experiments of this type.

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